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REGULAR ARTICLE

Analysis of Nano-Scale Effects on the Vibration of Functionally Graded Plates Using a Higher-Order Shear Deformation Theory

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This paper presents a comprehensive investigation into the free vibration behavior of functionally graded sandwich (FGSW) nanoplates, accounting for small-scale effects via Eringen's nonlocal elasticity theory. A novel inverse hyperbolic shear deformation theory (IHSDT) is developed to accurately model shear deformation and rotary inertia without requiring shear correction factors. The material properties of the plate's face sheets are graded through the thickness according to a power-law distribution, while the core can be homogeneous or graded, defined by various layering schemes. The governing equations are derived using Hamilton's principle and solved analytically for simply supported boundary conditions using the Navier solution technique. A detailed parametric study quantifies the influence of the power-law index (p), the nonlocal parameter (μ), and the material layering scheme on the non-dimensional fundamental frequencies. Results demonstrate that increasing the nonlocal parameter consistently reduces the fundamental frequency due to a stiffness-softening effect. Crucially, the optimal layering scheme for maximizing frequency depends on the scale; symmetric sandwich configurations outperform pure functionally graded material (FGM) plates at the nanoscale. This study provides critical insights and benchmark data for the design and optimization of FGSW structures in micro- and nano-electromechanical systems (MEMS/NEMS).

Keywords: Functionally graded materials (FGM), Nano-scale elasticity theory, Higher-order shear deformation theory (HSDT), Free vibration, Scale effect, Power-law index, Layering scheme.

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1. INTRODUCTION

The concept of functionally graded materials (FGMs) was first introduced and widely developed in Japan, where Koizumi [1] highlighted the pioneering activities and applications of these advanced composites. Since then, extensive research has been conducted to understand and model their behavior. Reddy [2] contributed one of the earliest analytical frameworks for analyzing FGM plates, while Thai and Kim [3] provided a comprehensive review of theories for the modeling and analysis of both plates and shells, emphasizing the evolution from classical to refined shear deformation models. In parallel, Mantari et al. [4] advanced higher-order shear deformation theories for laminated and sandwich plates, extending their applicability to FGMs. The introduction of nonlocal elasticity theory by Eringen [5] further enriched the field by enabling the study of size-dependent effects, which are particularly important for FGMs at the micro- and nanoscale. Building upon these foundations, recent works have focused on more specific applications and refined modeling techniques. Messas et al. [6] explored nanoscale effects in FGM structures through nonlocal theories, while Rebai et al. [7] investigated the influence of idealization models on the deflection of FGM plates. Extending this line of inquiry, Messas et al. [8] analyzed the vibration behavior of nano-FGM plates, with particular emphasis on the role of homogenization models and nanomaterial parameters. Most recently, Rebai and Tidjani [9] examined the buckling behavior of FGM plates, considering the combined effects of homogenization approaches, geometric parameters, and porosity, thereby offering a more holistic perspective on the stability of these structures.

This work aims to develop a robust nonlocal model for the free vibration analysis of FG sandwich nanoplates using a new Inverse Hyperbolic Shear Deformation Theory (IHSDT). The study systematically explores the interplay between material gradation (p), small-scale effects (μ), and architectural design (layering scheme) on the dynamic response. The findings are intended to serve as a benchmark and a guide for the design of next-generation micro- and nano-scale FGM devices.

2. MATHEMATICAL FORMULATION

2.1 Geometry and Material Properties

A rectangular functionally graded sandwich (FGSW) nanoplate of length a, width b, and thickness h is considered, with Cartesian coordinates (x, y, z) centered at the mid-plane ($z = 0$).

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The plate is composed of three layers: two graded face sheets and a core homogeneous. The effective property $P(z)$ (E, ρ) follows a power-law distribution; the material properties of the materials through the thickness of the plates are calculated by:

$$\begin{cases} P(z) = P_{\mathrm{m}} + (P_{\mathrm{c}} - P_{\mathrm{m}})\left(\frac{z-h_0}{h_1-h_0}\right)^p & h_0 \le z \le h_1 \\ P(z) = P_{\mathrm{c}} & h_1 < z < h_2.. \\ P(z) = P_{\mathrm{m}} + (P_{\mathrm{c}} - P_{\mathrm{m}})\left(\frac{z-h_3}{h_2-h_3}\right)^p & h_2 \le z \le h_3 \end{cases}$$

Where P_m and P_c are the metal and ceramic properties, and p is the power-law index. Poisson's ratio ν is assumed constant.

2.2 Displacement Field (IHSDT)

Using the Inverse Hyperbolic Shear Deformation Theory (IHSDT), displacements are expressed as:

$$u(x,y,z,t) = u0(x,y,t) - z\frac{\partial w(x,y,t)}{\partial x} + f(z)\varphi_x(x,y,t)$$
$$v(x,y,z,t) = v0(x,y,t) - z\frac{\partial w(x,y,t)}{\partial y} + f(z)\varphi_y(x,y,t)$$
$$w(x,y,z,t) = w0(x,y,t)$$

Where (u_0,v_0,w_0) are mid-plane displacements, φ_x, φ_y are shear rotations, and $f(z)$ is a shear shape function chosen to satisfy traction-free boundary conditions.

2.3 Strains and Constitutive Relations

Linear strain–displacement relations define bending, membrane, and shear strains. Nonlocal elasticity (Eringen) modifies Hooke's law:

$$(1 - \mu\nabla^2)\sigma_{ij} = s_{ij}$$

where σ_{ij}, s_{ij} are, respectively, the nonlocal and local stress tensors, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the second Laplace operator, $\mu = (e_0 a)^2$(nm^2) is the nonlocal parameter, in which e_0 is a material constant which is determined via experimental or atomistic dynamic for each material, and a is an internal characteristic length which relates to the distance of the molecules, lattice parameter, and granular size.

2.4 Governing Equations

From Hamilton's principle:

$$0 = \int_0^T (\delta\Pi - \delta T)\mathrm{d}t$$

where $\delta\Pi$ is the variation of the strain energy and δT is the variation of the kinematic energy of the plates.

For simply supported edges, displacement fields are expanded in Fourier series (Navier method), reducing the problem to the eigenvalue form:

$$([\mathrm{K}] - \omega^2[\mathrm{M}])\ \{\Delta\} = 0$$

where [K] and [M] are stiffness and mass matrices, and {Δ} is the unknown displacement vector.

The non-dimensional natural frequency is defined as:

$$\bar{\Omega} = \omega(a^2/h)\sqrt{\rho_0/E_0}$$

3. RESULTS AND DISCUSSION

The vibrational characteristics of functionally graded (FG) sandwich plates are investigated numerically based on the nonlocal elasticity theory. The following material properties and geometric parameters are employed for all analyses:

Matrix (Aluminum): E_m = 70 GPa, ρ_m = 2707 kg/m^3
Ceramic (Alumina): E_c = 380 GPa, ρ_c = 3800 kg/m^3
Poisson's ratio: ν = 0.3 (assumed constant)
Plate dimensions: $a = b = 10$, $h = 1$, resulting in a side-to-thickness ratio $a/h = 10$.

The study focuses on elucidating the effects of the material gradation index (p), the nonlocal parameter (ζ), and the core-to-face sheet layering sequence on the non-dimensional fundamental frequency (Ω).

3.1 Validation and Convergence

The accuracy and convergence of the present numerical model are first established by considering a benchmark case. For all six layering schemes investigated, setting the power-law index $p = 0$ and the nonlocal parameter $\zeta = 0$ consistently yields a fundamental frequency of Ω = 0.84785. This value corresponds exactly to the fundamental frequency of a homogeneous, isotropic square plate based on classical local elasticity theory, under simply supported boundary conditions. This agreement validates the present formulation and its numerical implementation.

3.2 Effect of Power-Law Index (p) and Layering Scheme

The influence of the material gradation index p on the fundamental frequency is paramount and is analyzed for fixed values of the nonlocal parameter. The data for four distinct values of ζ are presented in Figs. 2-5 and their corresponding Tables 2-5.

3.2.1. Case Study: $\zeta = 0.5$

Table 2 – Non-dimensional fundamental frequency (Ω) for various layering schemes vs. power-law index p ($\zeta = 0.5$)

p	[1,0,1]	[2,1,2]	[2,1,1]	[1,1,1]	[2,2,1]	[1,2,1]
0	0.84785	0.84785	0.84785	0.84785	0.84785	0.84785
0.5	1.46095	1.41447	1.37507	1.37278	1.32489	1.30458
1	1.61282	1.56762	1.52035	1.52237	1.46379	1.44412
5	1.74997	1.73551	1.68841	1.70543	1.64167	1.63167
10	1.74795	1.74984	1.70642	1.72761	1.66593	1.66023

For a moderate nonlocal effect ($\zeta = 0.5$), the frequency increases sharply from the homogeneous case ($p = 0$) to $p = 1$ for all schemes. The [1,0,1] scheme (pure FGM) shows the highest stiffness, while the [1,2,1] scheme (metal-rich core) shows the lowest.

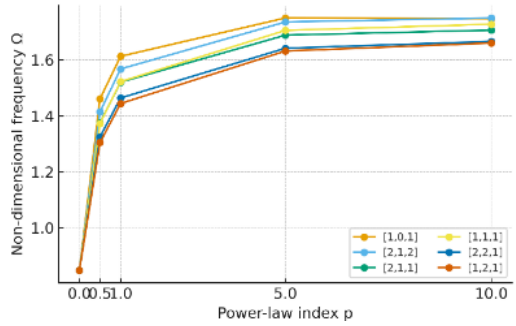


Fig. 2 – Variation of fundamental frequency with power-law index p for different layering schemes ($\zeta = 0.5$)

At higher p values ($p \geq 5$), the frequencies begin to converge but maintain a consistent hierarchy based on ceramic content. The [2,1,2] scheme, which has ceramic-rich face sheets, performs nearly as well as the pure FGM plate at $p = 10$.

3.2.2. Case Study: $\zeta = 1.0$

Table 3 – Non-dimensional fundamental frequency (Ω) for various layering schemes vs. power-law index p ($\zeta = 1.0$)

p	[1,0,1]	[2,1,2]	[2,1,1]	[1,1,1]	[2,2,1]	[1,2,1]
0	0.84785	0.84785	0.84785	0.84785	0.84785	0.84785
0.5	1.43935	1.39732	1.35937	1.35866	1.31256	1.29429
1	1.57710	1.53887	1.49402	1.49854	1.44295	1.42663
5	1.68337	1.68311	1.64004	1.66095	1.60249	1.59822
10	1.68023	1.69255	1.65336	1.67863	1.62264	1.62310

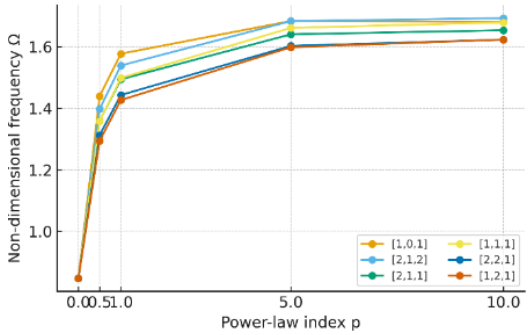


Fig. 3 – Variation of fundamental frequency with power-law index p for different layering schemes ($\zeta = 1.0$)

With the nonlocal parameter doubled to $\zeta = 1.0$, a general softening effect is observed, evidenced by lower frequency values across all p compared to Table 2. The same trends persist, but the performance gap between schemes slightly narrows. Notably, for the [2,1,2] scheme, the frequency at $p = 10$ (1.69255) now exceeds that of the [1,0,1] scheme (1.68023), suggesting that for stronger nonlocal effects, a symmetric sandwich configuration can outperform a pure FGM plate at very high ceramic contents.

The softening effect of the nonlocal parameter is further accentuated at $\zeta = 1.5$. The frequencies for all schemes are lower than their counterparts in Tables 2 and 3.

3.2.3. Case Study: $\zeta = 1.5$

The crossing trend observed in Fig. 3 becomes more pronounced. The symmetric schemes [2,1,2] and [1,1,1] now demonstrate a clear advantage over the [1,0,1] scheme for $p \geq 5$, indicating that at smaller scales (higher ζ), the distribution of material (layering scheme) becomes as critical as the total ceramic content.

Table 4 – Non-dimensional fundamental frequency (Ω) for various layering schemes vs. power-law index p ($\zeta = 1.5$)

p	[1,0,1]	[2,1,2]	[2,1,1]	[1,1,1]	[2,2,1]	[1,2,1]
0	0.84785	0.84785	0.84785	0.84785	0.84785	0.84785
0.5	1.41869	1.38077	1.34419	1.34497	1.30055	1.28424
1	1.54365	1.51164	1.46901	1.47579	1.42298	1.40976
5	1.62705	1.63518	1.59560	1.61978	1.56598	1.56674
10	1.61982	1.64055	1.60496	1.63360	1.58257	1.58835

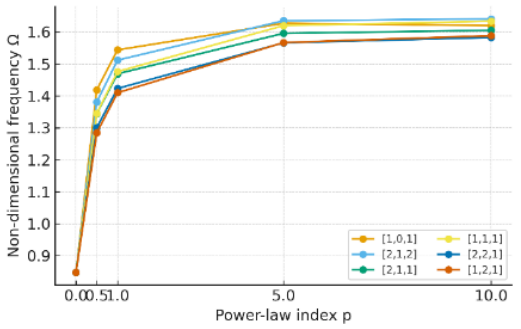


Fig. 4 – Variation of fundamental frequency with power-law index p for different layering schemes ($\zeta = 1.5$)

3.2.4. Case Study: $\zeta = 2.0$

Table 5 – Non-dimensional fundamental frequency (Ω) for various layering schemes vs. power-law index p ($\zeta = 2.0$)

p	[1,0,1]	[2,1,2]	[2,1,1]	[1,1,1]	[2,2,1]	[1,2,1]
0	0.84785	0.84785	0.84785	0.84785	0.84785	0.84785
0.5	1.39888	1.36480	1.32951	1.33169	1.28887	1.27442
1	1.51225	1.48580	1.44522	1.45404	1.40382	1.39347
5	1.57604	1.59113	1.55460	1.58152	1.53186	1.53705
10	1.56549	1.59306	1.56058	1.59200	1.54532	1.55575

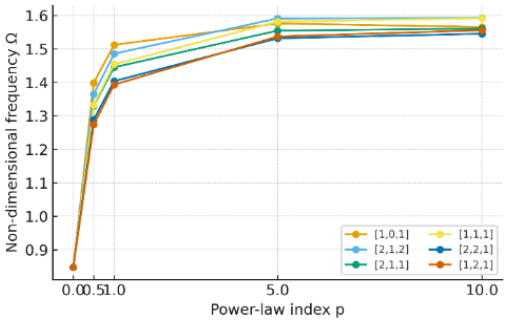


Fig. 5 – Variation of fundamental frequency with power-law index p for different layering schemes ($\zeta = 2.0$)

For a strong nonlocal effect ($\zeta = 2.0$), the superiority of symmetric sandwich configurations is unequivocal. The [2,1,2] scheme provides the highest frequency for $p \geq 1$, followed closely by [1,1,1]. The pure FGM plate ([1,0,1]) yields the lowest frequencies among all ceramic-rich configurations for $p > 1$. This demonstrates a significant shift in optimal design: while a pure FGM is best for local theory, a symmetric sandwich plate is preferable for nonlocal micro/nano-scale plates.

3.3 Combined Interaction of p and ζ

The complete interaction between the power-law index and the nonlocal parameter is best visualized in 3D space. Fig. 6 summarizes this combined effect for all six layering schemes.

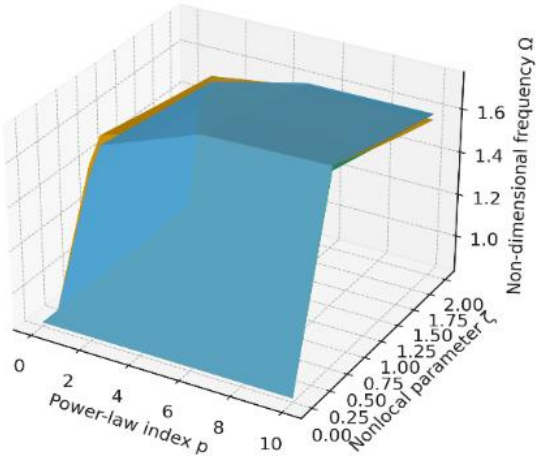


Fig. 6 – Three-dimensional surface plots of non-dimensional fundamental frequency with respect to the material index p and nonlocal parameter ζ, for different layering schemes

The 3D surfaces clearly illustrate the two competing phenomena:

Stiffening Effect: Along the p-axis, frequency increases as the structure becomes richer in stiffer ceramic.

Softening Effect: Along the ζ-axis, frequency decreases due to the nonlocal scale effect that reduces effective stiffness.

The curvature of the surfaces shows that the rate of frequency increase with p is drastically reduced as ζ grows. Furthermore, the crossing of the surfaces for different schemes visually confirms the findings from the 2D analyses: the optimal layering scheme for maximizing fundamental frequency is not constant but depends critically on the combination of p and ζ. For large ζ (nanoscale plates), a symmetric sandwich design ([2,1,2], [1,1,1]) is objectively superior to a pure FGM design ([1,0,1]). This analysis is crucial for the design and optimization of FG micro-/nano-plates, as it demonstrates that both material composition and structural configuration must be tailored to the specific scale of the application.

4. CONCLUSIONS

This study successfully developed a comprehensive mathematical model for analyzing the free vibration of nonlocal FGM plates using an HSDT. The proposed HSDT model, combined with Eringen's nonlocal theory, is robust and effective for capturing scale effects.

The power-law index p significantly influences the fundamental frequency, with an optimal value often existing for maximum frequency.

The material layering scheme is a powerful design parameter, with symmetric schemes generally providing higher stiffness and natural frequencies.

The nonlocal parameter ζ has a consistent stiffness-softening effect, leading to a substantial decrease in natural frequencies. This effect is paramount for the analysis of micro- and nano-scale devices.

The interaction between the gradation profile, layering, and scale effects is complex and non-negligible, emphasizing the need for the unified approach presented here.

The presented results serve as a valuable benchmark. The model can be extended to study buckling, bending, and forced vibration under various environmental conditions.

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Аналіз наномасштабних ефектів у вібраціях функціонально-градієнтованих пластин із застосуванням теорії зсувних деформацій вищого порядку

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У статті представлено всебічне дослідження вільних коливань нанопластин сендвіч-типу з функціонально-градієнтних матеріалів (ФГМ), з урахуванням масштабних ефектів на основі теорії нелокальної пружності Ерінгена. Розроблено нову теорію зсувних деформацій на основі обернених гіперболічних функцій (ОГФ), яка дозволяє точно моделювати зсувні деформації та інерцію обертання без необхідності застосування поправочних коефіцієнтів для зсуву. Властивості матеріалу зовнішніх шарів пластини змінюються за товщиною відповідно до степеневого закону, тоді як заповнювач може бути як однорідним, так і градієнтним, залежно від обраної схеми пошарової структури. Основні рівняння виведено з використанням принципу Гамільтона та розв'язано аналітично для випадку шарнірного закріплення країв за допомогою методу Нав'є. Детальний параметричний аналіз дозволив кількісно оцінити вплив показника степеневого закону (p), параметра нелокальності (μ) та схеми пошарової структури на безрозмірні основні частоти коливань. Результати показують, що збільшення параметра нелокальності призводить до систематичного зниження основної частоти внаслідок ефекту зменшення жорсткості. Важливо відзначити, що оптимальна схема пошарової структури для максимізації частоти залежить від масштабу: на нанорівні симетричні сендвіч-конфігурації демонструють кращі характеристики, ніж пластини, виготовлені виключно з функціонально-градієнтного матеріалу (FGM). Це дослідження надає важливі наукові дані та еталонні результати, необхідні для проєктування й оптимізації конструкцій типу FGSW у мікро- та наноелектромеханічних системах (MEMS/NEMS).

Ключові слова: Функціонально-градієнтні матеріали (ФГМ), Теорія пружності на нанорівні, Теорія зсувних деформацій вищого порядку (ТЗДВП), Вільні коливання, Масштабний ефект, Показник степеневого закону, Схема шарування.