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REGULAR ARTICLE

Enhancing Four-Point Probe Measurements for Heterogeneous Materials

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A van der Pauw-like nonlocal four-probe method is proposed and analyzed for studying the transverse electrical characteristics of non-uniform normal and superconducting materials. The conventional four-point probe design with four equally spaced, collinear electrodes, current applied through the outer pair, and voltage measured between the inner pair, assumes a uniform current distribution and isotropic conductivity. In contrast, the developed nonlocal electrical measurements take into account the heterogeneity through a generalized resistive network model with four contact pads: two current and voltage contacts on the top surface and two corresponding contacts on the bottom surface. The inclusion of scattering possibilities between all pairs of nodes, which in the classical domain can be replaced by corresponding resistances, leads to a realistic representation of the potential distribution and naturally explains the occurrence of both positive and negative nonlocal resistances observed experimentally. The temperature dependence of the nonlocal four-probe resistance is shown to be extremely sensitive to small differences in the superconducting parameters of layered structures. Distinctions in the critical temperature or transition width produce characteristic peak-dip features in the four-probe resistance, allowing direct identification of such inhomogeneities. The proposed nonlocal four-probe technique provides a simple yet powerful tool for resolving spatial variations in conductivity and superconducting transitions in thin films and multilayer heterostructures. Compatibility with existing measurement methods makes it a promising technique for both fundamental research and applied diagnostics in studies of the electronic properties of normal and superconducting materials.

Keywords: Four-probe method, Nonlocal approach, Inhomogeneous medium, Superconducting properties.

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1. INTRODUCTION

The four-point probe (4PP) technique, also known as four-terminal sensing, has a history spanning more than a century of development in electrical metrology. Its application to materials characterization began in the early twentieth century. In 1915, Wenner introduced the four-point configuration for measuring earth resistivity in geophysical investigations [1]. His arrangement of four equally spaced, collinear electrodes, with current applied through the outer pair and voltage measured between the inner pair, became the prototype for modern semiconductor characterization [2]. The transition of the four-point probe method from geophysics to semiconductor physics occurred in the 1950s, driven by the rapid growth of the transistor industry. A major advance was achieved in 1958 when van der Pauw formulated a general solution for the potential distribution in thin conductive layers of arbitrary shape, thereby eliminating the geometric constraints of earlier techniques [3, 4]. Later studies refined and extended the van der Pauw approach, clarifying its relation to the Montgomery method for rectangular samples through conformal mapping techniques [5]. Further developments introduced corrections for finite contact size, finite thickness, and material anisotropy [1, 2]. In recent decades, the method has been adapted for micro- and nanoscale and scanning-probe applications, enabling high-resolution resistivity mapping [6].

The 4PP technique offers decisive benefits that have established it as the preferred method for accurate electrical resistance measurements in scientific and industrial practice. Its principal advantage is the elimination of contact and lead resistances from the measured results. This constitutes a fundamental improvement over the two-point method, in which the additional resistances are inseparable from the sample resistance and can cause significant errors [6]. Additional advantages include the applicability of the 4PP method to a wide range of materials and resistance levels, from highly conductive metals to moderately resistive semiconductors. It is particularly effective for thin films and

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surface layers, where the two-dimensional sheet resistance is the principal parameter of interest [1, 2].

At the same time, both the linear array and van der Pauw configurations fundamentally assume that the sample is electrically homogeneous and isotropic. This assumption is embedded in the theoretical derivations relating the measured voltage-current ratio to the sample resistivity or sheet resistance. When the assumption is violated, interpretation of the results becomes considerably more complex, as the measured resistance represents a nontrivial weighted average over spatially varying conductivity. The weighting function depends on the local current density distribution, which itself is determined by the conductivity landscape in a self-consistent manner [1]. Consequently, meaningful extraction of local resistivity values from 4PP measurements requires additional modeling or complementary techniques. In inhomogeneous materials, the current preferentially flows through highly conductive regions, effectively shunting current away from more resistive domains. As a result, the measured resistance is dominated by the most conductive ways, obscuring contributions from less conductive regions [7-9]. When current is injected through a stack comprising layers of differing resistivity, its distribution depends on their relative conductances, and the measured resistance represents a superposition of the individual layer contributions. Accurate determination of layer-specific resistivities in such systems requires modeling of the current paths or complementary measurements with varying probe geometries or configurations.

In this paper, we propose and analyze a nonlocal four-probe method, similar to the van der Pauw approach [3, 4], for studying the transverse electrical characteristics of inhomogeneous normal and superconducting materials, primarily multilayer heterostructures. Unlike the traditional 4PP method, which assumes uniform current distribution and isotropic conductivity, the developed approach accounts for inhomogeneity using a generalized resistive network model.

2. REVISED MULTI-RESISTANCE MODEL

The proposed configuration of nonlocal electrical measurements involves four contact pads: two current and voltage contacts at the top (labeled 2 and 4) and at the bottom (1 and 3) surfaces, as shown in Fig. 1.

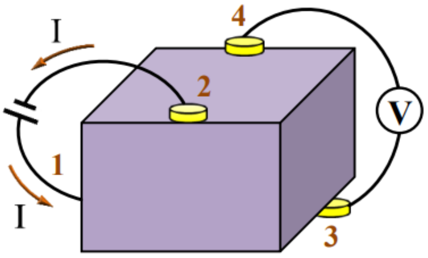
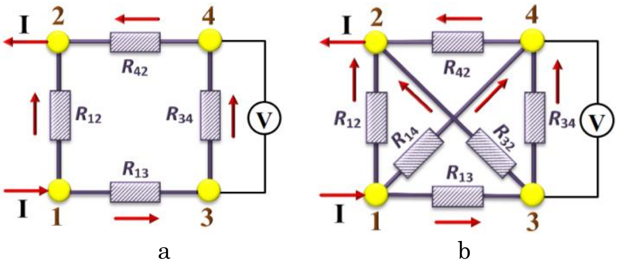


Fig. 1 – Schematics of a four-probe configuration corresponding to the nonlocal transverse electrical measurements with a current injected via the contacts 1 and 2 and the voltage drop measured between the contacts 3 and 4

To determine the currents within this measurement scheme, one should know the scattering probabilities between pairs of nodes, which in the classical domain can be replaced by the corresponding resistances R_{nm} (n, m = 1, 2, 3, 4, $n \neq m$). The current components I_{nm} are obtained using Kirchhoff's circuit laws, which state that the sum of currents entering each node must vanish and the directed sum of voltage drops around any closed loop is zero. The total current I is fixed by the external current source, while the measured voltage drop V_{34} equals the product of the current I_{34} flowing between nodes 3 and 4 and the resistance R_{34}. The measured nonlocal resistance is therefore given by

$$R_{12,34} = V_{34}/I \qquad (1)$$

Although the dependence of $R_{12,34}$ on the injected current I is often nontrivial to interpret, such *nonlocal* measurements can reveal subtle transport phenomena that may otherwise remain undetected [10, 11]. The main issue discussed below concerns the form of the equivalent circuit that describes the distribution of electrical potentials within a four-terminal structure. This circuit is usually represented as consisting of four resistors connecting adjacent terminals, as illustrated in Fig. 2a [12-14]. Under this assumption, the nonlocal resistance (1) can be expressed as $R_{12,34} = R_{12}R_{34}/(R_{12} + R_{13} + R_{34} + R_{42})$. In this case, $R_{12,34}$ is always positive, and the result simplifies to $R_{12,34} = R/4$ for identical resistances $R_{12} = R_{13} = R_{34} = R_{42} = R$. However, these conclusions contradict experimental observations, where negative $R_{12,34}$ values have been reported in nonlocal 4PP geometries [15-17]. The discrepancy arises from the oversimplified equivalent circuit in Fig. 2a, which neglects the diagonal interconnections between nodes 1–4 and 2–3. A more realistic model incorporating the missing links, and thus increasing the number of resistive elements from four to six, is shown in Fig. 2b.



a b

Fig. 2 – Circuit diagrams with four (a) and six (b) resistances for a comprehensive description of $R_{12,34}$ results obtained by non-local four-probe through-sample electrical measurements

In the general case, the resulting system of linear equations for the partial currents I_{nm} can be solved numerically. A simplified analytical expression relevant for practical applications can be obtained by assuming only two distinct resistance values: $R_{13} = R_{24} = R_s$, representing near-surface transport between contacts located on the same surface, and $R_{12} = R_{14} = R_{32} = R_{34} = R_b$, corresponding to bulk transport across the sample. Under these assumptions, the nonlocal resistance $R_{12,34}$ is given by

$$R_{12,34}(T) = \frac{1 - R_s(T)/R_b(T)}{4\left(1 + R_s(T)/R_b(T)\right)} R_b(T) \,. \qquad (2)$$

Here, the possible temperature dependence of the individual resistances $R_s(T)$ and $R_b(T)$ is explicitly included. Eq. (2) shows that $R_{12,34}$ becomes negative when $R_s(T) > R_b(T)$ and positive in the opposite case, independent of the specific physical realization of the two resistive channels. We refer to these as the *surface* R_s and *bulk* R_b resistances. Since this paper deals with the fundamental aspects of the proposed method, we will limit ourselves to this simplified approximation with two different parameters in the normal state. The relationship (2) explains the occurrence of negative four-probe resistance in systems as tunnel junctions composed of two metallic electrodes separated by an insulating barrier, where the electrode resistance exceeds that of the tunneling layer [15]. A similar effect may appear in cross-wire and other low-resistance devices [16], or at complex interfaces formed by high-T_c superconducting and ferromagnetic thin films [17].

It is important to stress the extreme sensitivity of the nonlocal resistance $R_{12,34}$ to the difference between R_s and R_b, analogous to the operation of a Wheatstone bridge. Even a small deviation between the surface and bulk resistances produces a finite $R_{12,34}$, whose sign directly reflects their ratio.

More striking manifestation occurs in the superconducting transition region where the temperature dependences $R_s(T)$ and $R_b(T)$ may exhibit slightly different critical parameters, which strongly influence the resulting $R_{12,34}(T)$ behavior. In the following analysis, both $R_s(T)$ and $R_b(T)$ are approximated by identical functional forms $R(T) = R^*\left(1+\tanh\left((T-T_c)/\delta T_c\right)\right)$ characterized by the midpoint T_c and the width δT_c of the transition, and the resistance R^* at $T = T_c$. When all normal-state and superconducting parameters coincide, the four-probe resistance tends to zero. A difference only in the normal-state resistances leads to a conventional $R(T)$ dependence across the normal-to-superconducting (N-S) transition, which can, under certain conditions, invert its sign, see below. Figs. 3-5 illustrate how small variations in superconducting parameters, particularly the transition widths, modify the expected nonlocal $R_{12,34}(T)$ curves.

3. SUPERCONDUCTING TRILAYERS

First, we assume that the three regions of the superconducting heterostructure have identical resistance in the normal state, and the difference arises solely from the superconducting transition temperatures. It follows from Eq. (2) that their shape of the measured nonlocal resistance is governed by the competition between the surface and bulk contributions, confirming the exceptional sensitivity of the proposed nonlocal method to subtle variations in transport properties. According to Eq. (2), the resistance $R_{12,34}(T)$ becomes positive within the transition region, when the surface layers exhibit higher critical temperatures T_c^s than the bulk T_c^b, and vanishes outside this temperature interval (see Fig. 3). Conversely, $R_{12,34}(T)$ becomes negative when $T_c^s < T_c^b$.

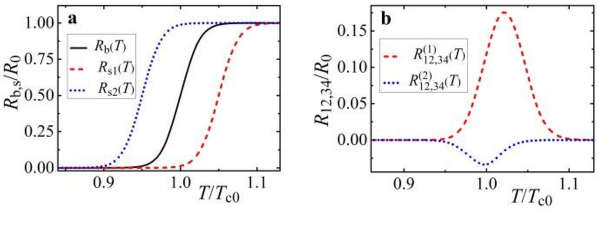


Fig. 3 – Effect of the difference in the bulk $T_c^b = T_{c0}$ and surface critical temperatures $T_c^{s1} = 1.05T_{c0}$, $T_c^{s2} = 0.95T_{c0}$, while the transition widths $\delta T_c^{s1} = \delta T_c^{s2} = \delta T_c^b = 0.02T_{c0}$: (a) temperature-dependent surface $R_s(T)$ and bulk $R_b(T)$ resistances, (b) expected $R_{12,34}(T)$ curves; the normal-state resistances $R_{s1}^{(N)} = R_{s2}^{(N)} = R_b^{(N)}$

A second scenario arises when the critical temperatures of the surface and bulk regions coincide, but the transition widths δT_c^s and δT_c^b differ. In this case, the inequality between $R_s(T)$ and $R_b(T)$ reverses near the midpoint of the N-S transition, producing a characteristic *peak–dip* structure in the $R_{12,34}(T)$ dependence (see Fig. 4).

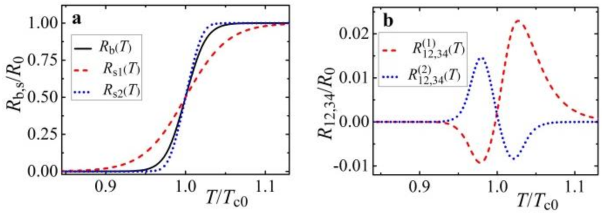


Fig. 4 – Effect of the difference in the widths of the N-to-S transition $\delta T_c^b = 0.02T_{c0}$, $\delta T_c^{s1} = 0.05T_{c0}$, $\delta T_c^{s2} = 0.01T_{c0}$, while $T_c^b = T_c^{s1} = T_c^{s2} = T_{c0}$: (a) temperature-dependent surface $R_s(T)$ and bulk $R_b(T)$ resistances, (b) expected $R_{12,34}(T)$ curves; the normal-state resistances $R_{s1}^{(N)} = R_{s2}^{(N)} = R_b^{(N)}$

Finally, if the surface $R_s^{(N)}$ and bulk $R_b^{(N)}$ resistances differ already in the normal state, the nonlocal resistance above the superconducting transition is positive when the bulk resistance exceeds that of the surface ($R_b^{(N)} > R_s^{(N)}$), and negative in the opposite case. This behavior is consistent with the well-known experimental results for three-layer tunnel structures [15] and other systems with layered conductivity [16, 17].

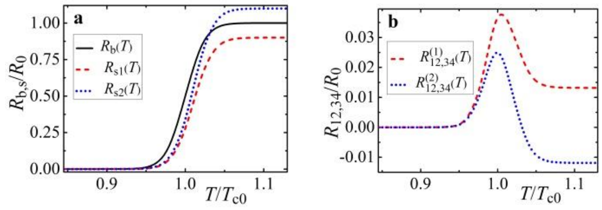


Fig. 5 – Effect of the difference in the normal-state resistances $R_b^{(N)} = R_0$, $R_{s1}^{(N)} = 0.9R_0$, $R_{s2}^{(N)} = 1.1R_0$; $T_c^b = T_{c0}$, $T_c^{s1} = T_c^{s2} = 1.01T_{c0}$, $\delta T_c^{s1} = \delta T_c^{s2} = \delta T_c^b = 0.02T_{c0}$: (a) temperature-dependent surface $R_s(T)$ and bulk $R_b(T)$ resistances, (b) expected $R_{12,34}(T)$ curves

4. CONCLUSIONS

In this work, we have developed and analyzed a van der Pauw-like nonlocal four-probe method for studying transverse electrical characteristics in normal and superconducting heterostructures. The proposed approach generalizes the conventional 4PP technique by incorporating both surface and bulk conduction paths into an equivalent six-resistance network that naturally explains the occurrence of positive and negative nonlocal resistances. The introduction of diagonal coupling resistances between the contact nodes allows a more realistic description of current flow and potential distribution in three-dimensional samples.

Analytical consideration and numerical modeling show that the sign and magnitude of the measured nonlocal resistance $R_{12,34}(T)$ are governed by the relative values of the surface and bulk resistances at the temperature T. The proposed nonlocal method thus represents a sensitive and versatile diagnostic tool for probing spatial variations in electrical transport, interfacial properties, and superconducting transitions in complex structures. Its conceptual simplicity and compatibility with existing 4PP measurement setups make it attractive for both fundamental studies and technological applications, including thin-film characterization, multilayer device optimization, and contact resistance analysis.

It should also be noted that the proposed four-point geometry is not limited to resistance measurements and can be extended to studies of electrical noise in heterostructures, in particular $1/f$ noise [18].

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Удосконалення чотирьохзондових вимірювань неоднорідних матеріалів

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Запропоновано та проаналізовано нелокальний чотирьохзондовий метод дослідження поперечних електричних характеристик неоднорідних нормальних та надпровідних матеріалів, подібний до відомого методу Ван дер Пау. Традиційна схема чотирьохточкової методики з чотирма рівновіддаленими колінеарними електродами і струмом, що подається через зовнішню пару, та напругою, що вимірюється між внутрішньою парою, передбачає однорідний розподіл струму та ізотропну провідність. На противагу цьому, розроблений нелокальний підхід до електричних вимірювань враховує неоднорідність через узагальнену модель резистивної мережі з чотирма контактними площадками: два контакти струму та напруги на верхній поверхні та два відповідні контакти на нижній поверхні. На відміну від традиційної чотирьохточкової методики зондування, яка припускає однорідний розподіл струму та ізотропну провідність, розроблений підхід враховує неоднорідність за допомогою узагальненої моделі резистивної мережі. Врахування можливих розсіювань між усіма парами вузлів, які в класичному підходіможна замінити відповідними опорами, призводить до реалістичного представлення розподілу потенціалу та природно пояснює виникнення як позитивних, так і негативних нелокальних опорів, що спостерігаються експериментально. Показано, що температурна залежність нелокального чотирьохзондового опору надзвичайно чутлива до невеликих розбіжностей у надпровідних параметрах шаруватих структур. Відмінності в критичній температурі або ширині переходу створюють характерні пік-провал структури у характеристиках чотирьохзондового опору, що дозволяє безпосередньо ідентифікувати такі неоднорідності. Запропонований нелокальний чотирьохзондовий метод є простим, але потужним інструментом для виявлення просторових варіацій провідності та надпровідних переходів у тонких плівках та багатошарових гетероструктурах. Сумісність з існуючими вимірювальними методами робить його перспективною методикою як для фундаментальних досліджень, так і для прикладної діагностики в дослідженнях електронних властивостей нормальних та надпровідних матеріалів.

Ключові слова: Чотиризондовий метод, Нелокальний підхід, Неоднорідне середовище, Надпровідні властивості.