JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. **18** No 2, 02034(6pp) (2026)

ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том **18** № 2, 02034(6cc) (2026)

REGULAR ARTICLE

Magnetic Resistance in Magnetically Ordered "Symmetrical" and "Asymmetrical" Sandwiches with Ultra-Thin Layers

Yu.M. Shabelnyk[1,*], Yu.O. Shkurdoda[1], , L.V. Dekhtyaruk[2], A.M. Chornous[1], I.I. Slyusarenko[2],
Yu.A. Kolesnichenko[3], D.I. Saltykov[4], V.Ye. Yanhol[1], T.A. Nalivayko[5]

[1] *Sumy State University, 40007 Sumy, Ukraine*
[2] *Taras Shevchenko National University of Kyiv, 01601 Kyiv, Ukraine*
[3] *B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine, 61103 Kharkiv, Ukraine*
[4] *Sumy State Pedagogical University named after A.S. Makarenko, 40002 Sumy, Ukraine*
[5] *Kharkiv National Automobile and Highway University, 61002 Kharkiv, Ukraine*

(Received 10 February 2026; revised manuscript received 19 April 2026; published online 29 April 2026)

Theoretically, using two-current [1] and resistive models [2, 3], the dimensional effects (dependences of the magnetoresistive ratio on the thickness d_{m2} of the top magnetic layer) of giant magnetoresistance in "symmetric" $F_{\alpha_m>1}/\mathrm{S}/F_{\alpha_m>1}$ and "asymmetric" $F_{\alpha_{m1}>1}/\mathrm{S}/F_{\alpha_{m2}>1}$ sandwiches have been investigated. It is shown that in regions of small (large) values of the thickness d_{m2} of the top magnetic layer relative to the thickness d_{m1} of the bottom magnetic layer, the giant magnetoresistance effect is negligible. This is due to the fact that when the inequalities $d_{m2} \ll d_{m1}$ $(d_{m2} \gg d_{m1})$ hold, the specified effect is small due to the shunting of the resistance of the top layer (the resistance of the base layer) by the resistances of the base magnetic layer and the non-magnetic interlayer (the resistances of the top layer and the spacer). In the absence of the shunting effect, i.e., when the thickness of the top magnetic layer coincides with the total thickness of the spacer and the base magnetic layer, the magnetoresistive ratio reaches its maximum value.

Analytical calculations of the parameters of electron spin-polarized transport were performed for both "symmetric" $F_{\alpha_m>1}/\mathrm{S}/F_{\alpha_m>1}$ and "asymmetric" $F_{\alpha_{m1}>1}/\mathrm{S}/F_{\alpha_{m2}>1}$ magnetically ordered sandwiches with ultrathin layers.

Keywords: Magnetically ordered systems, Sandwich, Base layer, Shunting effect.

DOI: 10.21272/jnep.18(2).02034

PACS numbers: 75.47.De, 75.70.Cn, 81.40.Rs, 85.70.Kh

1. INTRODUCTION

Multilayer nanoscale structures currently belong to a new class of spintronic materials due to the unique combination of their magnetic and electrical properties. The most well-known effect observed in metallic multilayer structures is the giant magnetoresistance (GMR) effect, which is caused by the spin-dependent scattering of charge carriers within the metal layers and at their interfaces, depending on the type of magnetic ordering in the three-layer conductor. Research into the causes of this phenomenon has led to the development of novel materials with broad potential for practical applications: spin valves, structures with magnetic tunnel junctions, spin nanotransistors, and others. In recent years, such materials have found increasingly widespread application in computing, electronics, and the automotive industry, where they are used as hard disk read heads, magnetic field sensors, MRAM memory modules and logic gates, ABS sensors, and more.

Research on nanoheterostructures is actively underway both in our country and abroad and is comprehensive in nature. From a practical standpoint, researchers' efforts are focused on developing new systems with specified technical characteristics, ensuring the stability of these properties, and improving the technologies used to produce them. Resolving these issues is complicated by the presence of many fundamental aspects related to the peculiarities of indirect exchange interactions, the specifics of spin-dependent electron scattering, as well as size effects. Despite a significant number of experimental and theoretical studies of thin-film materials with spin-dependent electron scattering, a number of questions remain unresolved. Thus, there is a need to develop and validate simple theoretical models of size effects in the magnetoresistive properties of magnetically ordered structures. The development of theoretical models [4, 5] can

---

[*] y.shabelnyk@aph.sumdu.edu.ua

Cite this article as: Yu.M. Shabelnyk, et al., *J. Nano- Electron. Phys.* 18 No 2, 02034 (2026) https://doi.org/10.21272/jnep.18(2).02034

address the challenge of predicting the behavior of the magnetoresistance in three-layer magnetically ordered systems with varying, in particular, the thickness of the magnetic metal layers.

In view of the above, the aim of this work is a theoretical study of the dependence of the magnetoresistive ratio on the thickness of the top magnetic layer of a three-layer film within the framework of the two-current and resistive models, and the calculation of the parameters of electron spin-polarized transport in a sandwich structure under the assumption that the interlayer is ultrathin.

2. STATEMENT OF THE PROBLEM. GENERAL EXPRESSION FOR THE MAGNETORESISTIVE RATIO AND ITS ANALYSIS

The giant magnetoresistance effect [6, 7] in a magnetically ordered three-layer film is usually quantified by the magnetoresistive ratio δ, which is determined by the change in the specific resistance $\left(\overline{\rho_{\text{ap}}}(0)-\overline{\rho_{\text{p}}}(H)\right)$ of the sandwich as a result of its remagnetization by an external magnetic field of intensity H, normalized to the resistance $\overline{\rho_{\text{p}}}(H)$:

$$\delta=\frac{\overline{\rho_{\text{ap}}}(0)-\overline{\rho_{\text{p}}}(H)}{\overline{\rho_{\text{p}}}(H)}=\frac{\overline{\rho_{\text{ap}}}(0)}{\overline{\rho_{\text{p}}}(H)}-1 \qquad (1)$$

Here, $\overline{\rho_{\text{ap}}}(0)$ is the specific resistance of the conductor averaged over the thickness of the sandwich in the absence of an external magnetic field, i.e., when the sandwich exhibits an antiferromagnetic configuration (the directions of the spontaneous magnetization vectors **M** in the magnetic layers of the metal are antiparallel), $\overline{\rho_{\text{p}}}(H)$ – the sample's specific resistance averaged over the thickness of the magnetically ordered three-layer film in the presence of an external magnetic field, i.e., when the sandwich exhibits a ferromagnetic configuration (the directions of the spontaneous magnetization vectors **M** in the metal's magnetic layers coincide).

Consider a three-layer magnetically ordered film consisting of magnetic metal layers with thicknesses d_{mj} $(j=1,2)$ separated by a non-magnetic spacer with a thickness d_n. Let us assume that for the mean free path lengths $l^s_{\text{int}\,j}$ of electrons in the transition regions between the non-magnetic spacer and the magnetic metal layers (interfaces), the following inequalities hold: $l^s_{\text{int}\,j}<<l^s_{mj},l_n$ (l^s_{mj},l_n – mean free path lengths of spin-polarized electrons in j-magnetic layer and in the non-magnetic interlayer, respectively), $s=\pm(\uparrow\downarrow)$ – spin indices that determine the sign of the projection of the electron spin onto the direction of the spontaneous magnetization vector **M** in the magnetic layers of the sandwich and $l^s_{\text{int}}<<\sqrt{Dt_D}$ (D – mutual diffusion coefficient, t_D – diffusion time) [8, 9]. In this case, the transition regions of the metal can be modeled as geometric planes, such that the thickness of the sandwich will be equal to $d=d_{m1}+d_n+d_{m2}$.

It is well known that the GMR effect is observed when interactions occur between the magnetic layers of the metal via spin-polarized charge carriers. However, this is possible only when the interlayer in the sandwich is coarse-grained, i.e., when the following inequality holds: $\beta_n<<d_n/l_n$ [10], where $\beta_n=\frac{l_n}{L_n}\frac{R_n}{1-R_n}$ is the grain boundary parameter, l_n is the mean free path of charge carriers in the non-magnetic layer, L_n is the average width of the crystallites in the spacer plane, and R_n is the probability of diffuse scattering of charge carriers at grain boundaries [11].

Assuming that the dominant mechanism of the giant magnetoresistance effect is asymmetric spin-dependent scattering of charge carriers in the volume of the magnetic layers, the magnetoresistive ratio (MRR) (1) can be written in the form [12]:

$$\delta=\frac{(\alpha_{m1}-1)(\alpha_{m2}-1)\,\rho^{+}_{m2,m1}\,d_{m2,m1}}{\alpha_{m1}\left(1+\rho^{+}_{m2,m1}\,d_{m2,m1}+\rho^{+}_{n,m1}\,d_{n,m1}\right)\left(1+\rho^{-}_{m2,m1}\,d_{m2,m1}+\rho^{-}_{n,m1}\,d_{n,m1}\right)}\cong$$

$$\cong\begin{cases}\dfrac{(\alpha_{m1}-1)(\alpha_{m2}-1)}{\alpha_{m1}\left(1+\rho^{+}_{n,m1}d_{n,m1}\right)\left(1+\rho^{-}_{n,m1}d_{n,m1}\right)}d_{m2,m1}, & d_{m2,m1}<<\sqrt{\prod\limits_{s=\pm}\left(1+\rho^{s}_{n,m1}d_{n,m1}\right)}, \qquad (2,\text{a})\\[2ex] \dfrac{(\alpha_{m1}-1)(\alpha_{m2}-1)}{\alpha_{m2}d_{m2,m1}\sum\limits_{s=\pm}\left(1+\rho^{s}_{n,m1}d_{n,m1}\right)}\left\{1-\dfrac{\sum\limits_{s=\pm}\left(1+\rho^{s}_{n,m1}d_{n,m1}\right)}{d_{m2,m1}}\right\}, & d_{m2,m1}<<\sqrt{\prod\limits_{s=\pm}\left(1+\rho^{s}_{n,m1}d_{n,m1}\right)}, \qquad (2,\text{б})\end{cases}$$

In equations (2a) and (2b): $\alpha_{mj}=\frac{\rho^{-}_{mj}}{\rho^{+}_{mj}},(j=1,2)$ – asymmetry parameters describing the asymmetry in the scattering of spin-polarized electrons within the volume of the metal's magnetic layers; $\rho^{s}_{m2,m1}\,d_{m2,m1}\equiv\frac{\rho^{s}_{m2}}{\rho^{s}_{m1}}\frac{d_{m2}}{d_{m1}}$, ρ^{s}_{mj} – the resistivity s-spin channel in a magnetic layer of the metal with a thickness of d_{mj}; $\rho^{s}_{n,m1}\,d_{n,m1}\equiv\frac{\rho^{s}_{n}}{\rho^{s}_{m1}}\frac{d_n}{d_{m1}}$ $\rho^{+}_{n}\equiv\rho^{-}_{n}$, – the resistivity of the non-magnetic interlayer (spacer) with a thickness of d_n.

It follows from equations (2a) and (2b) that, in the region of small values of the thickness of the top magnetic layer, the resistances of the base magnetic layer and the non-magnetic interlayer act as shunt resistances for the resistance of the top magnetic layer, and the magnitude of the GMR effect is negligible (see formula (2a)). This effect is also negligible in the region of large values of the magnetic overcoat layer thickness, since in this case the resistances of the magnetic overcoat layer and the spacer act as shunt resistances for the resistance of the base magnetic layer (see equation (2b)).

If the following equality holds:

$$d_{m2,m1} = \sqrt{\prod_{s=\pm}\left(1+\rho^{s}_{n,m1}d_{n,m1}\right)}\,, \qquad (3)$$

i.e., when the thicknesses of the top and base magnetic layers are comparable in order of magnitude $\left(d_{m2} \sim d_{m1}\right)$ the shunting effect will be absent and the MRV δ (1) reaches its maximum value, which is equal to:

$$\delta_{\max} = \frac{\left(\alpha_{m1}-1\right)\left(\alpha_{m2}-1\right)}{\left(\sqrt{\alpha_{m1}\left(1+\rho^{-}_{n,m1}d_{n,m1}\right)}+\sqrt{\alpha_{m2}\left(1+\rho^{+}_{n,m1}d_{n,m1}\right)}\right)^{2}}\,. \qquad (4)$$

The resulting formula is significantly simplified when the non-magnetic layer is ultrathin, i.e., the inequalitie $d_n << d_{mj}$ hold (in the limiting case, we can assume that $d_n = 0$). In this case, formula (4) takes the form:

$$\delta_{\max} = \frac{\left(\alpha_{m1}-1\right)\left(\alpha_{m2}-1\right)}{\left(\sqrt{\alpha_{m1}}+\sqrt{\alpha_{m2}}\right)^{2}}\,. \qquad (5)$$

3. CALCULATION OF ELECTRON TRANSPORT PARAMETERS IN “SYMMETRICAL” SENDVICHES $F_{\alpha_m>1}$ / S/$F_{\alpha_m>1}$ WITH AN ULTRA-THIN LAYER

In the case of a "symmetric" sandwich of the type $F_{\alpha_m>1}$ / S/$F_{\alpha_m>1}$, the magnetic layers of the sample are of the same type $\left(\alpha_{m1}=\alpha_{m2}=\alpha_m\right)$, and equation (5) reduces to the well-known equation:

$$\delta_{\max} = \frac{\left(\alpha_m-1\right)^2}{4\alpha_m}\,. \qquad (6)$$

A similar result is obtained in the case where the interaction of spin-polarized charge carriers with the external boundaries (ferromagnet–medium, ferromagnet–substrate) of the sandwich and with the boundaries separating the metal layers does not lead to the scattering of spin-polarized electrons. In this case, taking into account the spin polarization of the charge carriers, the following generalized equalities hold [5, 13]: $q^s_j = 1$, $P^s_{ji}+Q^{-s}_{ij} = 1$, $\left(j \neq i = 1,2\right)$ (for a sandwich with the *ap* configuration), $q^s_j = 1$, $P^s_{ji}+Q^s_{ij} = 1$ (for a conductor with the *p* configuration). Here, q^s_j is the probability of specular reflection of a spin-polarized electron *j* – by the outer boundary of the sample (Fuchs specularity parameter [14]), P^s_{ji} and Q^s_{ij} – the probability of reflection of spin-polarized electrons by the sample interface and their passage into the adjacent conductor layer without scattering, respectively.

To calculate the parameter α_m, which determines the asymmetry in the scattering of spin-polarized electrons within the magnetic layers, we rewrite formula (6) as the equation:

$$\alpha_m^2 - 2\left(1+2\delta_{\max}\right)\alpha_m + 1 = 0\,, \qquad (7)$$

Next, using the example of a "symmetric" sandwich structure Co/Cu/Co, we will demonstrate that in this structure, the majority (effective) charge carriers are those whose spin direction coincides with the direction of the magnetization vector. It is well known that the presence of interfaces between magnetic and non-magnetic metals, which have different band structures, leads to the formation of a potential jump at the sandwich interfaces and, as a result, to a decrease in the probability of electrons passing through the interfaces. Since the band structure of a ferromagnetic metal is spin-dependent, the probability of a charge carrier passing through the interface depends on the direction of its spin relative to the direction of the local magnetization vector. A comparison of the band structure Cu with the minority subband Co indicates a significant mismatch between them, as a result of which minority charge carriers will, with a high probability, be scattered at the sandwich interfaces [13, 14-16]. At the same time, the band structure of Cu almost coincides with that of the majority subband of Co, and majority electrons pass through the interfaces of the three-layer film practically unimpeded [13, 14-16] and are effective carriers responsible for the effect (the concept of “inefficiency” [17]).

Solving equation (7) with respect to α_m, taking into account the above, namely that the inequality $\alpha_m > 1$ holds, we obtain the following formula for calculating the volume asymmetry parameter:

$$\alpha_m = 1 + 2\left(\delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max}+1\right)}\right), \qquad (8)$$

and, accordingly, using the experimentally measured value $\delta_{\max}$ when studying the dependence of the MRR on the thickness of the covering ferromagnetic metal layer, the volume asymmetry parameter can be calculated.

To determine the specific resistance of a “symmetric” magnetically ordered sandwich and the specific resistances of the two spin conduction channels in the magnetic metal layers, we will use the “β-model”, according to which the specific resistance of the two spin channels is equal to [15]:

$$\rho_m^{\pm} = \frac{2\rho_m}{1 \pm \beta_m}\,, \tag{9}$$

where $\rho_m = \dfrac{\rho_m^+ \rho_m^-}{\rho_m^+ + \rho_m^-}$ and ρ_m^s are the total specific resistance of the sample and the specific resistances *s*-spin channel in the ferromagnetic layer, respectively. The phenomenological parameter β_m can clearly be referred to as the channel asymmetry parameter, i.e., the parameter that determines the difference in the specific resistances of the two spin channels in the ferromagnet.

By expanding formula (9), we find the relationship between the parameters β_m and α_m:

$$\beta_m = \frac{\alpha_m - 1}{\alpha_m + 1}\,, \tag{10}$$

or between the parameters α_m and β_m:

$$\alpha_m = \frac{1 + \beta_m}{1 - \beta_m}\,. \tag{11}$$

Taking into account formula (8), the parameter β_m takes the form:

$$\begin{gathered}\beta_m = \frac{\delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}}{1 + \delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}} \equiv \\ \equiv 1 - \frac{1}{1 + \delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}}\end{gathered}\,. \tag{12}$$

Knowing the channel difference parameter of the specific resistances β_m and the total resistance of the magnetic metal ρ_m, we can calculate the specific resistances of the two spin channels. To do this, we substitute expression (12) into formula (9), and for the specific resistances of each spin channel, we obtain the following formulas:

$$\frac{\rho_m^+}{\rho_m} = 2\frac{1 + \delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}}{1 + 2\delta_{\max} + 2\sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}} \equiv 1 + \frac{1}{\alpha_m}\,, \tag{13}$$

$$\frac{\rho_m^-}{\rho_m} = 2\left(1 + \delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}\right) \equiv 1 + \alpha_m\,, \tag{14}$$

from which it follows that the difference between the two spin channels will be equal to:

$$\begin{aligned}&\frac{\Delta\rho_m}{\rho_m} = \frac{\rho_m^- - \rho_m^+}{\rho_m} = \\ &= 4\frac{\left(1 + \delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}\right)\left(\delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}\right)}{1 + 2\delta_{\max} + 2\sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}} \equiv \\ &\equiv 2\frac{\rho_m^+}{\rho_m}\left(\delta_{\max} + \sqrt{\delta_{\max}\left(\delta_{\max} + 1\right)}\right) = \frac{\alpha_m^2 - 1}{\alpha_m}\end{aligned} \tag{15}$$

4. CALCULATION OF ELECTRON TRANSPORT PARAMETERS IN “ASYMMETRICAL” SANDWICHES $F_{\alpha_{m1}>1}/\mathrm{S}/F_{\alpha_{m2}>1}$ WITH AN ULTRATHIN LAYER

If electrons are the major charge carriers in the metal’s magnetic layers, with their spin directions coinciding with the direction of the magnetization vectors, but the magnetic layers are made of different ferromagnetic materials, then the amplitude value of the MRR will be determined by formula (5). In this case, the number of problem parameters must be reduced. To do this, in formula (6), we must substitute $\alpha_m \to \alpha_{\text{eff}}$, and equate the result to formula (6), yielding:

$$\frac{\left(\alpha_{\text{eff}} - 1\right)^2}{4\alpha_{\text{eff}}} = \frac{\left(\alpha_{m1} - 1\right)\left(\alpha_{m2} - 1\right)}{\left(\sqrt{\alpha_{m1}} + \sqrt{\alpha_{m2}}\right)^2}\,. \tag{16}$$

Solving equation (16) with respect to α_{eff}, taking into account that the volume asymmetry parameter is greater than one, the effective parameter α_{eff}, which describes the asymmetry in the scattering of charge carriers within the volume of the metal’s magnetic layers, can be written as:

$$\begin{gathered}\alpha_{\text{eff}} = 1 + \frac{2\sqrt{\left(\alpha_{m1} - 1\right)\left(\alpha_{m2} - 1\right)}}{\left(\sqrt{\alpha_{m1}} + \sqrt{\alpha_{m2}}\right)^2} \times \\ \times \left\{ 1 + \sqrt{\alpha_{m1}\alpha_{m2}} + \sqrt{\left(\alpha_{m1} - 1\right)\left(\alpha_{m2} - 1\right)} \right\}\end{gathered}\,. \tag{17}$$

In this case, the amplitude numerical value of the MRR for “asymmetric” sandwiches will be determined by the following formula:

$$\delta_{\max}^{\text{eff}} = \frac{\left(\alpha_{\text{eff}} - 1\right)^2}{4\alpha_{\text{eff}}}\,, \tag{18}$$

from which the quantity α_{eff} can be expressed in terms of the experimentally measured quantity $\delta_{\max}^{\text{eff}}$:

$$\alpha_{\text{eff}} = 1 + 2\left(\delta_{\max}^{\text{eff}} + \sqrt{\delta_{\max}^{\text{eff}}\left(\delta_{\max}^{\text{eff}} + 1\right)}\right), \tag{19}$$

Comparing formulas (17) and (19), we obtain an equation for the parameters α_{m1} and α_{m2} of the volumetric asymmetry in the sandwich:

$$\begin{gathered}\delta_{\max}^{\text{eff}} + \sqrt{\delta_{\max}^{\text{eff}}\left(\delta_{\max}^{\text{eff}} + 1\right)} - \frac{\sqrt{\left(\alpha_{m1} - 1\right)\left(\alpha_{m2} - 1\right)}}{\left(\sqrt{\alpha_{m1}} + \sqrt{\alpha_{m2}}\right)^2} \times \\ \times \left\{ 1 + \sqrt{\alpha_{m1}\alpha_{m2}} + \sqrt{\left(\alpha_{m1} - 1\right)\left(\alpha_{m2} - 1\right)} \right\} = 0\end{gathered}\,. \tag{20}$$

To determine the parameters α_{m1} and α_{m2}, two experiments must be conducted sequentially to establish the dependence of the MRR on the thickness d_{m2} in “symmetric” and “asymmetric” sandwiches. First, in the “symmetric” sandwich $F_{\alpha_m>1}/\mathrm{S}/F_{\alpha_m>1}$, measure the dependence of the MRR on the thickness d_{m2}, determine $\delta_{\max}$, and calculate the parameter α_{m1} using formula (8). Next, while retaining the same base ferromagnetic layer, deposit a non-magnetic interlayer and a top magnetic layer made of a different ferromagnetic material. Having obtained an “asymmetric” sandwich, investigate the dependence of MRR on thickness d_{m2} and determine $\delta_{\max}^{\mathrm{eff}}$. Substitute the found values α_{m1} and $\delta_{\max}^{\mathrm{eff}}$ into the equation with respect to α_{m2}, and from the resulting equation, the parameter α_{m2} can be determined.

To determine the specific resistances of the two spin channels and the difference between them in the “asymmetric” sandwich, we will again use the “β-model.” Since the sandwich is “asymmetric,” we must first find the effective parameter β_{eff}. To do this, equation (16), using formula (11), must be expressed in terms of the parameters β_{eff}, β_{m1}, and β_{m2} and written as a quadratic equation in β_{eff}, whose solution can be given as:

$$\beta_{\mathrm{eff}}=\frac{1}{2}B\left\{1+\sqrt{1-\frac{4}{B^2}}\right\},\qquad B=2+\frac{1-\beta_{m1}\beta_{m2}+\sqrt{\left(1-\beta_{m1}^2\right)\left(1-\beta_{m2}^2\right)}}{2\beta_{m1}\beta_{m2}}. \qquad (21)$$

Next, we need to express the right-hand side of equation (18) in terms of β_{eff}, solve the resulting equation for β_{eff}, and set the result equal to equation (21), yielding:

$$1+\sqrt{1+8\delta_{\max}^{\mathrm{eff}}}+2\delta_{\max}^{\mathrm{eff}}\left\{2-B1+\sqrt{1-\frac{4}{B^2}}\right\}=0. \qquad (22)$$

To find the parameters of channel asymmetry and the specific resistances of the spin channels, we must first find $\delta_{\max}$ for the “symmetric” sandwich and calculate $\beta_m\equiv\beta_{m1}$ using formula (12), then find $\delta_{\max}^{\mathrm{eff}}$ for the “asymmetric” one. Substituting the found values β_{m1} and $\delta_{\max}^{\mathrm{eff}}$ into formula (22), we obtain an equation from which we find the parameter β_{m2}. Knowing the parameters β_{m1} and β_{m2} from formula (9), we can find the specific resistances of the spin channels and the difference (resistance difference) between them.

CONCLUSIONS

Thus, the dependence of the magnetoresistive ratio on the thickness of the top magnetic layer is non-monotonic. In the regions of small and large numerical values of the top magnetic layer, the giant magnetoresistance effect is negligible due to the presence of a shunting effect. In the absence of the shunting effect (the thickness of the top magnetic layer coincides with the thickness of the non-magnetic interlayer and the base magnetic layer, provided that the interface thicknesses are negligible compared to the thicknesses of the magnetic and non-magnetic metal layers and can be neglected), the magnetoresistive effect reaches its maximum value. The experimentally determined maximum (amplitude) numerical value of the magnetoresistive ratio as a function of the MRR δ on the thickness d_{m2} of the top magnetic layer of the sandwich allows for the theoretical calculation of the parameters of electron spin-polarized transport in “symmetric” $F_{\alpha_m>1}/\mathrm{S}/F_{\alpha_m>1}$ and “asymmetric” $F_{\alpha_{m1}>1}/\mathrm{S}/F_{\alpha_{m2}>1}$ sandwiches.

ACKNOWLEDGMENTS

The authors acknowledge the support of the Erasmus+ project (2025-1-SK01-KA171-HED-000344037) and the WPI Fellowship Program for Ukrainian Researchers, and its director, Prof. Norbert Mauser.

FUNDING

This work was funded by the NATO Program "Science for Peace and Security" (project number G6131) and the State Programs of the Ministry of Education and Science of Ukraine (project number 0224U033036).

REFERENCES

1. N.F. Mott, *Proc. Roy. Soc.* **A153**, 699 (1936) https://doi.org/10.1098/rspa.1936.0031.
2. A. Fert, I.A. Campbell, *Phys. Rev. Lett.* **21**, 1190 (1968) https://doi.org/10.1103/PhysRevLett.21.1190.
3. A. Fert, I.A. Campbell, *J. Phys. F.: Metal. Phys.* **6**, 849 (1976) https://doi.org/10.1088/0305-4608/6/5/025.
4. O.I. Mokhonko, Yu.M. Shabelnyk, Yu.O. Kolesnichenko, D.I. Saltykov, I.I. Slyusarenko, S.V. Hakhovych, Yu.O. Shkurdoda, L.V. Dekhtyaruk, A.M. Chornous, *Nanosystems, Nanomaterials, Nanotechnologies* **23**, 619 (2025) https://doi.org/10.15407/nnn.23.03.0619.
5. L.V. Dekhtyaruk, *Metallofiz. Noveishie Tekhnol.* **30**, 219 (2008).
6. G. Binasch, P. Grunberg, F. Saurenbach et al., *Phys. Rev.* **39**, 4828 (1989) https://doi.org/10.1103/PhysRevB.39.4828.
7. M.N. Baibich, J.M. Broto, A. Fert et al., *Phys. Rev. Lett.* **61**, 2472 (1988) https://doi.org/10.1103/PhysRevLett.61.2472.
8. L.V. Dekhtyaruk, Yu.A. Kolesnichenko, *Fizika Metallov i Metallovedenie* **75** No 5, 21 (1993).
9. L.V. Dekhtyaruk, I.M. Pazukha, S.I. Protsenko, I.V. Cheshko, *Phys. Solid State* **48**, 1831 (2006) https://doi.org/10.1134/S1063783406100015.
10. O.A. Bilous, L.V. Dekhtyaruk, A.M. Chornous, *MNT* **23**, 43 (2001).
11. A.F. Mayadas, M. Shatzkes, *Phys. Rev. B* **1**, 1382 (1970) https://doi.org/10.1103/PhysRevB.1.1382.
12. Yu.O. Shkurdoda, L.V. Dekhtyaruk, A.M. Chornous, A.G. Basov, A.P. Kharchenko, *Proceedings of the 6th International Conference Nanomaterials: Applications and Prop-*

erties, NAP 2016, 02MFPM05 (2016) https://doi.org/10.1109/NAP.2016.7757284.

13. O.I. Tovstolytkin, M.O. Borovii, V.V. Kurylyuk, Yu.A. Kunytskii, *Physical Foundations of Spintronics* (Vinnytsia: Nilan – LTD: 2014) [in Ukrainian].
14. J. Mathon, *Phenomenological Theory of Giant Magnetoresistance. Magnetic Thin Films and Industrial Applications* Vol. 71 (Springer-Verlag: 2001).
15. O.V. Tretyak, V.A. Lviv, O.V. Barabanov, *Physical Fundamentals of Spintronics* (Kyiv: VPC “Kyiv University”: 2002).
16. E.Y. Tsymbal, D.G. Pettifor, In *Solid State Physics* (Ed. by H. Ehrenreich, F. Spaepen) (San Diego CA: Academic, 56, 113: 2001).
17. A.B. Pippard, *Proc. Roy. Soc.* **224**, 273 (1954) https://doi.org/10.1098/rspa.1954.0157.

Магнітоопір в магнітовпордякованих «симетричних» та «несиметричних» сендвічах з ультратонкими прошарками

Ю.М. Шабельник[1], Ю.О. Шкурдода[1], Л.В. Дехтярук[2], А.М. Чорноус[1], І.І. Слюсаренко[2],
Ю.О. Колесніченко[3], Д.І. Салтиков[4], В.Є. Янголь[1], Т.А. Наливайко[5]

[1] *Сумський державний університет, 40007 Суми, Україна*
[2] *Київський національний університет імені Тараса Шевченка, 01601 Київ, Україна*
[3] *Фізико-технічний інститут низьких температур імені Б. І. Веркіна НАН України, 61103 Харків, Україна*
[4] *Сумський державний педагогічний університет імені А.С. Макаренка, 40002 Суми, Україна*
[5] *Харківський національний автомобільно-дорожній університет, 61002 Харків, Україна*

Теоретично, з використанням двострумової [1] і резисторної моделей [2, 3] досліджені розмірні (залежності магніторезистивного відношення від товщини d_{m2} накривного магнітного шару) ефекти гігантського магнітоопору в «симетричному» $F_{\alpha_m>1}$ / S/$F_{\alpha_m>1}$ та «несиметричному» $F_{\alpha_{m1}>1}$ / S/$F_{\alpha_{m2}>1}$ сендвічах. Показано, що в областях малих (великих) значень товщини d_{m2} накривного магнітного шару у порівнянні з товщиною d_{m1} базового магнітного шару ефект гігантського магнітоопору мізерно малий. Це обумовлено тим, що у разі виконання нерівностей $d_{m2} << d_{m1}$ $\left(d_{m2} >> d_{m1}\right)$ зазначений ефект малий внаслідок шунтування опору верхнього шару (опору базового шару) опорами базового магнітного шару та немагнітного прошарку (опорами накривного шару і спейсера). У разі відсутності ефекту шунтування, тобто коли товщина накривного магнітного шару збігається з сумарною товщиною прошарку та базового магнітного шару, величина магніторезистивно відношення набуває максимального значення.

Проведені аналітичні розрахунки параметрів електронного спін-поляризованного транспорту як в «симетричному» $F_{\alpha_m>1}$ / S/$F_{\alpha_m>1}$, так і в «несиметричному» $F_{\alpha_m>1}$ / S/$F_{\alpha_m>1}$ магнітовпорядкованих сендвічах з ультратонкими прошарками.

Ключові слова: Магнітовпорядковані системи, Сендвіч, Базовий шар, Ефект шунтування.