Journal of Nano- and Electronic Physics
Vol. **17** No 6, 06037(6pp) (2025)

Журнал нано- та електронної фізики
Том **17** № 6, 06037(6cc) (2025)

REGULAR ARTICLE

Chaotic Laser Mode

Yu.S. Kurskoy, O.S. Hnatenko*, O.V. Afanasieva, D.S. Sherstyuk

Kharkiv National University of Radio Electronics, 61166 Kharkiv, Ukraine

(Received 04 September 2025; revised manuscript received 17 December 2025; published online 19 December 2025)

Based on an analysis of the characteristics of chaotic laser radiation, we introduce the notion and notation of a chaotic laser mode. Within a semiclassical model of single-mode laser generation with modulated injection of an external coherent field we derive conditions for the onset and maintenance of stable chaotic generation in the form of parameter intervals of the external field. For these regimes we analyze measurable radiation characteristics – envelope amplitude and frequency, degree of monochromaticity, coherence time, coherence length and coherence area, beam divergence and transverse intensity distribution – and show that all of them exhibit chaotic temporal dynamics. We discuss the capabilities and limitations of Fourier spectral analysis of broadband chaotic signals and demonstrate that, when combined with fractal analysis of the radiation spectrum, this approach allows one to determine the interval and evolution of the instantaneous frequency and intensity and to distinguish chaotic regimes from stochastic and quasiperiodic ones. On this basis we define a chaotic mode as a globally stable, distinguishable and reproducible type of chaotic oscillation in a nonlinear dynamical system; the results can be used for the design of laser systems with controlled chaotic generation for secure optical communications, sensing and metrology.

Keywords: Chaotic laser generation, Radiation spectrum, Beam parameters, Coherence, Fourier analysis, Fractal dimension, Measurement portrait.

DOI: 10.21272/jnep.17(6).06037 PACS numbers: 05.45.Df, 05.45.Mt, 42.65.Sf

1. INTRODUCTION

The study of chaotic laser dynamics is usually divided into two directions that share the same object – chaotic laser radiation (optical chaos) – but pursue different goals. One direction suppresses stochastic and chaotic regimes to stabilize laser output and improve the quality of coherent sources [1], whereas the other develops methods for controlled generation of chaotic radiation with prescribed characteristics for optics, photonics and information technologies [2]. Controlled chaotic generation in solid-state, semiconductor and gas lasers has been extensively studied [3-5], with emphasis on deterministic chaos, which must be distinguished from stochastic noise, turbulence and regular dynamics [6]. A key application is chaotic optical communication, where information is encoded in deterministic but aperiodic broadband signals and recovered at the receiver by a synchronized chaotic generator [3-5, 7, 8]. Practical use of such systems requires reproducible generation, transmission and transformation of chaotic signals with given parameters and quantitative criteria that separate chaotic, stochastic and quasiperiodic regimes; a spectral–fractal–entropic approach to this problem was developed by Yu.P. Machekhin and co-authors [9-13].

Despite progress in modelling chaotic laser generation and determining parameter ranges for chaotic regimes, there is still no generally accepted notion of a “chaotic mode” analogous to an electromagnetic mode in classical laser theory: chaotic regimes are usually described via parameter sets or attractors, without a compact notation that links the type of electromagnetic mode to measurable characteristics of chaos. The aim of this work is therefore to analyze the characteristics of chaotic laser radiation and, on this basis, to introduce a universal definition and notation for chaotic laser modes using a semiclassical model of single-mode laser generation with modulated injection of an external coherent field. For stable chaotic regimes we study the temporal behavior of measurable radiation characteristics and the radiation spectrum and, using Fourier and fractal analysis, formulate criteria that distinguish chaotic regimes from stochastic and quasiperiodic ones, which allows us to define a chaotic mode as a globally stable, distinguishable and reproducible type of chaotic oscillation and to relate it to the parameters of chaos.

2. CHAOTIC GENERATION

The generation of chaotic radiation in a single-mode laser can be described by the semiclassical equations [9]:

* Correspondence e-mail: oleksandr.hnatenko@nure.ua

Cite this article as: Yu.S. Kurskoy et al., *J. Nano- Electron. Phys.* 17 No 6, 06037 (2025) https://doi.org/10.21272/jnep.17(6).06037

$$\frac{dE}{dt} = -\kappa(E - E_p) - igP, \tag{1}$$

$$\frac{dP}{dt} = -\gamma P + igED, \tag{2}$$

$$\frac{dD}{dt} = \gamma_{||}(D_0 - D) + 2ig(PE^* - P^* E), \tag{3}$$

where E is the electric-field strength of the generated radiation, P is the macroscopic dipole moment of the active medium, D is the population inversion, Ep is the amplitude of the external (pumping) field, D_0 is the total unsaturated inversion, κ is the cavity loss rate, g is the coupling coefficient, and the remaining parameters are the transverse and longitudinal relaxation constants of the dipole moment and the inversion. For a constant value of the external field component E_p, Eqs. (1)–(3) admit a stationary solution E_s corresponding to stationary (continuous-wave) laser generation. When the pumping is modulated, and condition (4) is satisfied:

$$\kappa \ll \gamma_{||} \ll \gamma . \tag{4}$$

The system (1)–(3) can be reduced to a single equation for the field amplitude normalized to the stationary value E_s:

$$\frac{d\hat{E}}{d\tau} = -i\Delta\omega\hat{E} + \left(\frac{g^2 D_0}{\gamma\left(1+\hat{E}^2\right)} - 1\right)\hat{E} + E_p(\tau), \tag{5}$$

where Δ is the normalized frequency detuning, ω is the angular frequency of the cavity mode, is the angular frequency of the external field, and τ is the dimensionless time [9].

In the case where the external field is additionally modulated at a frequency ω_m:

$$E_p(\tau) = E_p + E_m \cos(\omega_p \tau)\,,\ E_p > E_m \ge 0\,, \tag{6}$$

with E_p the constant component and E_m the modulation amplitude, numerical integration of Eq. (5) under condition (4) yields a sequence of dynamical regimes as the control parameters are varied: quasi-periodic oscillations for certain values of $\{E_p, E_m, \omega_m\}$ and stable chaotic generation for others [9]. A further increase in the modulation amplitude leads to a transition back to a periodic regime with the modulation period. Thus the average value and modulation amplitude of the external field serve as control parameters, defining intervals in which chaotic generation occurs. In [9] the conditions for chaotic laser generation were formulated as:

$$E_m \in \{E_m\}\,,\ \omega_p \in \{\omega_p\}\,, \tag{7}$$

Analysis of condition (4) and Eq (5) shows that the laser dynamics are governed mainly by the cavity and by the parameters and temporal profile of the pumping. In practice, chaotic generation can be realized by modulating cavity losses or resonator motion, by modulating the inversion, or by injecting a modulated coherent field into the cavity under condition (7) (Fig. 1). Numerical integration of Eq. (5) with conditions (4) and (7) demonstrates the transition from stationary or periodic to stable chaotic regimes as the modulation amplitude increases: in the chaotic domain the normalized intensity $I(\tau) = |E(\tau)|^2$ shows irregular aperiodic oscillations (Fig. 2(b)), while the trajectory of $E(\tau)$ in the ReE–ImE plane forms a strange attractor (Fig. 3) typical of deterministic chaos.

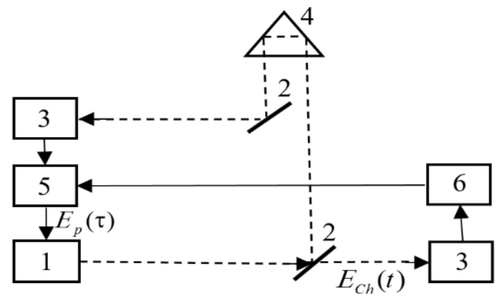


Fig. 1 – Schematic of chaotic laser generation with optoelectronic feedback: 1 – laser; 2 – reflecting plates; 3 – photodetectors; 4 – reflecting prism; 5 – pumping and modulation system; 6 – control and monitoring unit

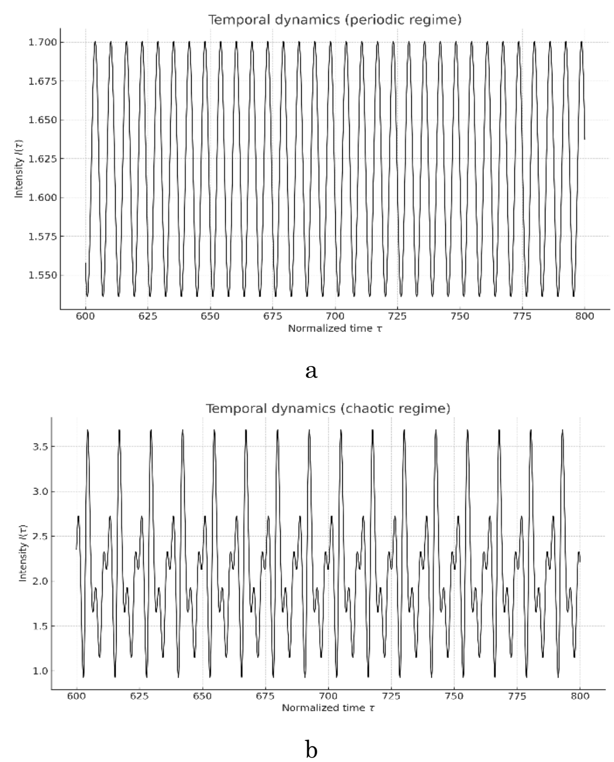


Fig. 2 – Temporal dynamics of the normalized laser intensity *I(τ)* obtained from numerical integration of Eq. (5): (a) periodic regime for weak modulation of the external field; (b) chaotic regime for stronger modulation

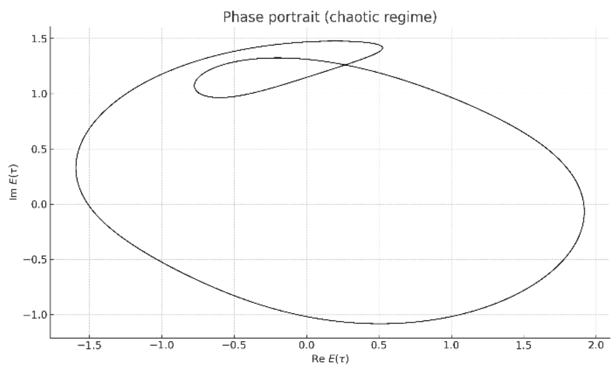


Fig. 3 – Phase portrait of the complex field amplitude *E(τ)* in the *ReE–ImE* plane for the chaotic regime, obtained by numerical integration of Eq. (5); the attractor corresponds to stable chaotic generation

3. CHARACTERISTICS OF RADIATION

The numerical solution of Eq. (5) represents a chaotic signal of laser radiation, which can be written in the form:

$$E_{Ch}(t) = \mathrm{E}_m(t)\exp\left[i\Omega(\omega_i, t)t\right], \qquad (8)$$

where $\mathrm{E}_m(t)$ and $\Omega(\omega_i, t)$ are the amplitude and frequency of the envelope, respectively, and ω_i) is the instantaneous frequency of the *i*-th component of the chaotic radiation.

The chaotic signal (8) is characterized by a broad emission bandwidth. However, for the purposes of classification, reproducible generation and synchronization of chaotic generators it is necessary to study not only the spectral characteristics, but also other parameters of the radiation. It should be noted that all characteristics of stationary laser radiation are determined by the spectral linewidth: the degree of monochromaticity, the coherence time t_c, the coherence length l_c, the coherence area dc and the beam divergence ϑ_c. These quantities are related by the well-known expressions:

$$\delta_m \approx \frac{\Delta\omega}{\bar{\omega}}, t_c \approx \frac{2\pi}{\Delta\omega},\ l_c \approx \frac{2\pi c}{\Delta\omega},\ d_c \approx \frac{\bar{\lambda}^2}{\Delta O},\ \vartheta_c \approx \frac{\bar{\lambda}}{d_c}, \qquad (9)$$

where *c* is the speed of light and ΔO is the solid angle of the source forming the coherence area. Because in the chaotic regime the instantaneous optical frequency becomes a time-dependent quantity, all these parameters also acquire a chaotic temporal behaviour.

Chaotic variation of the field amplitude $E_{Ch}(t)$ and broadening of the emission line make it possible to represent the chaotic signal as a superposition of its components with instantaneous frequencies $\omega_i \in \Delta\omega$ in the form:

$$E_{Ch}(t) = \sum_i E_i(t)\ . \qquad (10)$$

In this case the intensity distribution of the radiation can be described as:

$$I(x,\ y) = \left|\sum_i f_{\varphi_i}(x,\ y)\exp(i\omega_i t)\right|^2, \qquad (11)$$

where $f_{\varphi_i}(x,\ y)$ is a function describing the spatial dependence of the amplitudes and the correlation between the components of the field.

In the particular case of a chaotic Gaussian beam with diameter d_G, the transverse intensity distribution can be approximated by:

$$I(x,\ y) = I_0\exp\left[-\frac{2(x^2+y^2)}{{d_G}^2}\right]Sp(x,y)\ , \qquad (12)$$

where I_0 is the on-axis intensity in the centre of the beam and $Sp(x,y)$ is the normalized distribution of the chaotic speckle structure.

Thus, the chaotic radiation regime manifests itself both in the energy (spectral) characteristics of the signal and in the spatial distribution of the intensity in the beam cross-section. In the limiting case of complete suppression of chaos the distribution (12) reduces to the Gaussian distribution characteristic of a stationary single-mode beam.

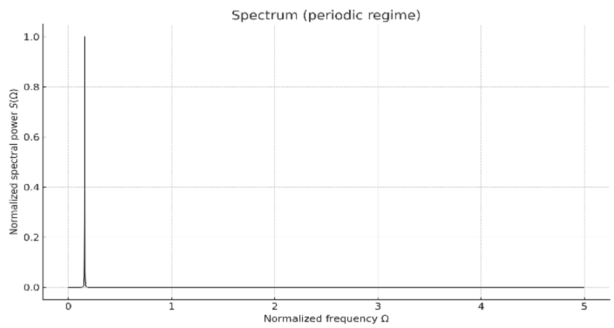
4. SPECTRAL ANALYSIS

To detect chaotic regimes in laser systems, both qualitative (temporal traces, phase portraits, spectra) and quantitative methods are used. Qualitative inspection alone is often insufficient for reliable classification, so statistical, fractal and other mathematical tools are applied. Fig. 4 shows typical intensity spectra for periodic and chaotic generation regimes.

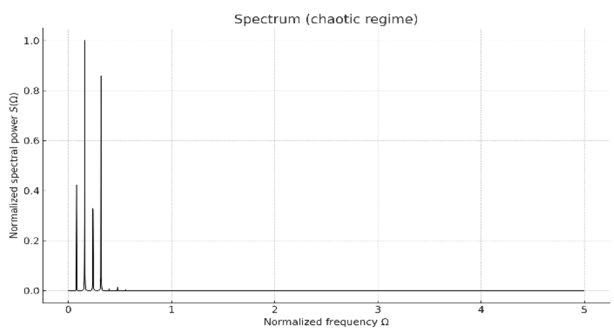
Spectral analysis makes it possible, first, to determine the character of the dynamics (regular, stochastic, quasiperiodic or chaotic) and, second, to obtain integral characteristics of the frequency interval (total width, structure of the spectrum, presence of discrete lines and broadband components). In addition, it reveals the advantages and limitations of the spectral method for analysing chaotic signals. Traditionally, the Fourier transform is used to analyse deterministic signals. For a deterministic process *f(t)* the Fourier transform *F(ω)* exists in the usual sense and is given by [13]:

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt \qquad (13)$$

For the analysis of nondeterministic random processes the Wiener–Khinchin theorem is used, which relates the power spectral density to the Fourier transform of the autocorrelation function. Let us consider a stationary random process *f(t)*.



a



b

Fig. 4 – Spectral distribution of the intensity for the same regimes as in Fig. 2: (a) periodic regime – discrete spectrum with narrow lines at the modulation frequency and its harmonics; (b) chaotic regime – significantly broadened quasi-continuous spectrum

In this case the power spectrum $F(\omega)$ is expressed in terms of the autocorrelation function $R(\tau)$ as:

$$F(\omega) = 2\int_{-\infty}^{+\infty} R(\tau)e^{-i\omega\tau}d\tau, R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{i\omega\tau}d\omega, \quad (14)$$

subject to the limiting condition:

$$\int_{-\infty}^{+\infty}|F(\omega)|d\omega \le A \ , \ \int_{-\infty}^{+\infty}|R(\tau)|d\tau \le B \ , \quad (15)$$

where A and B are constants [14-15]. According to condition (15), the validity of Eq. (14) is restricted to stationary processes for which the correlation decays sufficiently rapidly. In this case the spectrum of the process is a continuous function of frequency.

Thus, the Fourier spectrum of a purely stochastic process is visualized as a continuous curve (an example is stationary noise-like radiation). The question then arises of what the Fourier spectrum of a deterministic chaotic signal looks like. In this case the spectrum has neither the form of a discrete set of sharp lines (as in the periodic regime) nor that of a smooth continuous curve (as in a purely stochastic process). Instead, it exhibits a broadband structure with irregular fine details, which is of great importance for the problem of controlling chaotic dynamics.

Analysis of the spectral pattern of a chaotic signal makes it possible to obtain not only the total bandwidth and structure of the spectrum, but also to determine the fractal properties of the frequency interval. In particular, one can calculate the fractal dimension D_F of the spectrum using the Hurst exponent H. The latter is related to the rescaled range r/s by [16,17]:

$$D_F = 2 - H \ , \quad (16)$$

and is determined from the dependence of r/s on the sample length. The Hurst exponent H is in turn connected with the fractal dimension D_F of the process by:

$$r/s = (n/2)^H \ . \quad (17)$$

The value of H (or D_F) characterizes the dynamics as regular, random or chaotic. If the fractal dimension of the process lies within a certain interval, the process is classified as chaotic [17]. Thus, Fourier analysis provides spectral characteristics of the signal, whereas the additional use of fractal analysis of the spectrum via relations (16) and (17) makes it possible to classify the dynamics and distinguish chaotic regimes from stochastic and quasiperiodic ones.

5. CHAOTIC LASER MODES: DEFINITION AND NOTATION

A laser mode is a type of electromagnetic field that can be established in a cavity under given generation conditions. In the stationary regime the field distribution in a mode is determined by the boundary conditions and the solution of the wave equation, and can be written as [18]:

$$\vec{E}(\vec{r},t) = E_0\vec{u}(\vec{r})\exp[(-t/2\tau_c) + i\omega t] \ , \quad (18)$$

where r is the radius vector, E_0 is a constant, $\vec{u}(\vec{r})$ is a function describing the spatial distribution of the field, ω is the mode frequency and τ is the decay time of the squared electric-field amplitude.

In linear systems with stabilized generation conditions the stationary laser mode is uniquely determined by the cavity geometry and parameters and is stable and reproducible. Modes of stochastic and chaotic radiation, in contrast, exhibit local regions corresponding to intensity maxima and minima, strong temporal fluctuations and complex spatial structure. Such regimes can be characterized by chaotic modes [19-20]. An example of the attractor corresponding to a chaotic laser mode in the complex field plane is shown in Fig. 3. Using the representation of chaotic oscillations (8) and Eq. (18), the field of a chaotic mode can be written as:

$$\vec{E}_{Ch}(\vec{r},t) = \mathrm{E}_m(t)\vec{u}(\vec{r})\exp\left[(-t/2\tau_c) + i\Omega(\omega_i,t)t\right]. \quad (19)$$

A chaotic mode can be defined as a globally stable, distinguishable and reproducible type of chaotic oscillation characteristic of a nonlinear dynamical system.

Global stability means that, under variations of initial conditions and external perturbations within certain limits, the trajectory remains in the basin of attraction and preserves its statistical and fractal characteristics. Distinguishability requires that the mode be separable both from stationary and purely stochastic regimes and from other chaotic modes by a set of measurable parameters (spectral width and structure, coherence properties, fractal dimension, etc.). Reproducibility means the possibility of obtaining the same mode repeatedly by returning the control parameters of the system to the corresponding intervals, which is crucial for applications such as chaotic optical communication based on synchronization.

For the classification of chaotic laser modes we propose an extended notation of transverse electromagnetic waves of the form:

$$\mathrm{TEM}_{mn/XD_F} \tag{20}$$

where m, n are the indices of the classical transverse modes (the numbers of intensity zeros in the corresponding directions in the beam cross-section), X is the measured characteristic of the radiation (for example, intensity, phase, polarization, etc.), and D_F is the fractal dimension of the chaotic dynamics of this characteristic. According to (20), the designation $\mathrm{TEM}_{00/\omega 1.3}$ corresponds, for example, to chaoticization of a Gaussian mode in terms of intensity I with a fractal dimension $D_F=1.3$.

The proposed definition and characteristics of chaotic modes are universal and can be applied not only to laser systems, but also to other physical, biological and climatic systems exhibiting chaotic dynamics.

6. CONCLUSIONS

In this work we analysed the characteristics of chaotic laser radiation and introduced the notion of a chaotic laser mode.

Using a semiclassical model of single-mode laser generation with a modulated external coherent field we identified parameter intervals corresponding to stable chaotic regimes (Eqs. (4), (7)) and illustrated the transition from stationary or periodic generation to chaos by numerical integration of Eq. (5) (Figs. 1–3). For chaotic regimes we examined a set of measurable radiation characteristics – envelope amplitude and frequency, degree of monochromaticity, coherence time, coherence length and coherence area, beam divergence and transverse intensity distribution – and showed that, owing to the time dependence of the instantaneous optical frequency, all these quantities exhibit chaotic temporal behaviour.

We discussed the capabilities and limitations of Fourier spectral analysis for detecting chaotic regimes and showed that the spectrum of a deterministic chaotic signal is broadband with irregular structure, distinct from both periodic and purely stochastic cases (Fig. 4). In combination with fractal analysis based on the Hurst exponent and the corresponding fractal dimension, this provides quantitative criteria for distinguishing chaotic dynamics from stochastic and quasiperiodic regimes.

On this basis we introduced the concept of a chaotic mode as a globally stable, distinguishable and reproducible type of chaotic oscillation characteristic of a nonlinear dynamical system, and proposed an extended notation for chaotic laser modes that links the underlying electromagnetic mode with the measured characteristic and the fractal dimension of its chaotic dynamics. These results are relevant for the design of laser systems with controlled chaotic generation and for applications in secure optical communication, sensing, metrology, medicine and other [21-22].

REFERENCES

1. F. Riehle, *Frequency Standards: Basics and Applications* (John Wiley & Sons:2006).
2. M. Zhang, Y. Wang, *J. Lightwave Technol.* **39**, 3711 (2021).
3. R. Zhang, P. Zhou, Y. Yang, Q. Fang, P. Mu, N. Li, *Opt. Express* **28**, 7197 (2020).
4. L.-J. Qiao, M.F. Jia, Z. Guo, Y.K. Hao, X.-N. Wang, X. Tang, M.-J. Zhang, *Opt. Express* **33**, 10862 (2025).
5. S. Bittner, S. Guazzotti, Y. Zeng, X. Hu, H. Yılmaz, K. Kim, S.S. Oh, QJ. Wang, O. Hess, H. Cao, *Science* **361** No 6408, 1225 (2018).
6. G.H. Schuster, *Deterministic Chaos: An Introductio*, (Wiley-VCH, New York:1995).
7. R.D. Laureano, D.A. Mendes, M.A. Ferreira, arXiv preprint arXiv:2412.20649 (2024).
8. A. Pikovsky, M. Rosenblum, J. Kurths, *Synchronization. A Universal Concept in Nonlinear Sciences* (Cambridge University Press:2001).
9. H. Haken. *Laser Light Dynamics* (Light:1986).
10. Yu.P. Machekhin, Yu.S. Kurskoi, A.S. Gnatenko, *Telecommun. Radio Eng.* **77** No 18, 1631 (2018).
11. Yu. Kurskoy, O. Hnatenko, D. Sherstyuk, V. Zarytskyi, G. Chernovolyk, G. Nussupbekova, L. Pogrebniak, P. Kisała, J. Klimek, K. Muslimov, *Proc. SPIE* **13400**, 134000U (2024).
12. Yu.S. Kurskoy, O.S. Hnatenko, *J. Nano- Electron. Phys.* **15** No 2, 02008 (2023)
13. P.R. Griffiths, J.A. De Haseth, *Fourier Transform Infrared Spectrometry* (John Wiley & Sons: 2007).
14. E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light* (Cambridge University Press: 2007).
15. D.W. Ricker. *Echo Signal Processing* (Kluwer Academic Publishers: 2003).
16. H.-O. Peitgen, H. Jürgens, D. Saupe, *Chaos and Fractals, 2nd* (Springer: 2004).
17. Y.P. Machekhin, Y.S. Kurskoy, *Meas Tech* **57**, 609 (2014).
18. O. Svelto, *Principles of Lasers, 5th Ed.* (Springer: 2010).
19. J. Andreasen, A.A. Asatryan, L.C. Botten, M.A. Byrne, H. Cao, L. Ge, L. Labonté, P. Sebbah, A.D. Stone, H.E. Türeci, C. Vanneste, *Adv. Opt. Photon.***3**, 88 (2011).
20. J. Xia, E. Wang, L. Hu, W. Du, B. Yu, K. Xie, J. Zhang, L. Lu, Z. Hu, *Opt. Lett.* **49**, 4182 (2024).
21. I.M. Lukavenko, *J. Nano- Electron. Phys.* **12** No 1, 01014 (2002).
22. Yu.S. Kurskoy, O.S. Hnatenko, O.V. Afanasieva, *J. Nano-Electron. Phys.* **13** No 4, 04036 (2021).

Хаотична лазерна мода

Ю.С. Курський, О.С. Гнатенко, О.В. Афанасьєва, Д.С. Шерстюк

Харківський національний університет радіоелектроніки, 61166 Харків, Україна

На підставі аналізу характеристик хаотичного лазерного випромінювання запропоновано поняття та систему по-значень хаотичної лазерної моди. У межах напівкласичної моделі одномодової лазерної генерації з модульованим введенням зовнішнього когерентного поля отримано умови виникнення та підтримки стійкої хаотичної генерації у вигляді інтервалів параметрів зовнішнього поля. Для таких режимів проаналізовано експериментально доступні характеристики випромінювання – амплітуду та частоту огинаючої, ступінь монохроматичності, час, довжину й пло-щу когерентності, розбіжність променя та поперечний розподіл інтенсивності – і показано, що всі вони виявляють ха-отичну часову динаміку. Розглянуто можливості й обмеження фур'є-спектрального аналізу широкосмугових хаотич-них сигналів та показано, що його поєднання з фрактальним аналізом спектра випромінювання дає змогу визначати інтервали та еволюцію миттєвої частоти й інтенсивності та розрізняти хаотичні, стохастичні й квазіперіодичні ре-жими. На цій основі хаотичну моду визначено як глобально стійкий, розрізнюваний і відтворюваний тип хаотичних коливань у нелінійній динамічній системі; отримані результати можуть бути використані під час розроблення лазе-рних систем із керованою хаотичною генерацією для задач захищеного оптичного зв'язку, сенсорики та метрології.

Ключові слова: Хаотична лазерна генерація, Спектр випромінювання, Параметри пучка, Когерентність, Фур'є-аналіз, Фрактальна розмірність, Портрет вимірювання.