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REGULAR ARTICLE

Inflationary Cosmological Model of Bianchi Type VI_0 with Exponential Potential

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LRS Bianchi Type VI_0 inflationary spacetime under the impact of the effective potential $V(\phi) = e - \lambda\phi$, $\lambda > 0$, where ϕ Higgs field is denoted by ϕ, has been determined in the current study. It is advised to use the average scale factor $(R^3) = B^2A = e\lambda\phi$ to get the exact solution. Under certain conditions, the model becomes isotropic and shear- free. Also, the model does not show singularity at initially. The effective potential's evolution indicates that the model is consistent with an inflationary scenario in which the universe experiences a rapid expansion in its early stages. Throughout its development, the model displays anisotropic properties because the shear to expansion ratio σ/θ is nonzero. This study indicates the significance of anisotropic inflationary models for solving basic cosmological challenges like the flatness and horizon problems. The deceleration parameter q plays a crucial role in determining whether the universe undergoes accelerated or decelerated expansion. We have used hubble parameter, expansion scalar and shear scalar to discuss the model's kinematic and physical behavior. The results underline how crucial inflationary scenarios are for describing the observed homogeneity and large-scale structure of the current cosmos.

Keywords: Bianchi type, Exponential potential, Cosmological model, Higgs field, Inflationary scenario.

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1. INTRODUCTION

"Inflation" is a word that describes the universe's incredibly rapid expansion. Firstly, the first-order phase transitions were studied in order to build the inflationary Universe scenario. According to Gliner [1], the De Sitter model with positive energy density should be consistent with the universe being ruled by vacuum. Later on, Zeldovich [2] expanded on this concept. According to Linde's [4] framework for gauge theories, the precise assessment of phase transitions was comparable to the current inflationary scenario. Guth [5] initially presented the fundamental theory of inflation while grappling with the issue of why monopolies are no longer in existence. He investigated how an exponential expansion of space is caused by an unreal vacuum with positive energy. Cosmological problems such as the Horizon problem, Isotropy, Homogeneity, and Magnetic Monopole may have an answer in an inflationary universe scenario. Cosmologists have to investigate a wide range of cosmological issues in order to understand the creation of cosmic structures following the big bang.

A mathematical foundation for examining the realistic picture of the current universe is offered by the Bianchi models of cosmology. Specific symmetries and anisotropies in its metric tensor define the Bianchi Type VI_0 model, which is a subset of the larger Bianchi classification [3]. The inflationary condition of the universe was discussed by many authors using different forms of the scalar field (ϕ) in general relativity, such as Benisty et al. [10], Bali and Jain [6], Naidu et al. [8], and Bali [9]. In general relativity, Bali and Poonia [7] have studied the Bianchi type VI_0 inflationary cosmic model. In order to perform a thorough analysis of the accelerating behavior of the physical universe, Sharma et al. [11] have examined the Bianchi type VI inflationary model in the presence of a flat potential. Maheshwari and Poonia [12] have studied an expanding cosmos with a flat potential using a Bianchi-type VI_0 cosmological model in general relativity. Singh and Pillai have examined the stability analysis of anisotropic Bianchi type I cosmological model [13].

In the present work, we examine a scalar potential for the investigation of inflationary scenarios in spatially homogeneous and anisotropic Bianchi Type VI_0 spacetime, which is an exponential function of the scalar field φ, that is, $V = e^{-\lambda\varphi}$, $\lambda > 0$. We try to explain how an inflationary situation can arise from taking a homogeneous and anisotropic metric with exponential potential, and in a special example, how the model isotropizes. Understanding the universe's dynamic behavior, including its expansion, shear, and deceleration

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factors, is possible according to the obtained solutions. The evolution of the model is examined by deriving important geometrical and physical parameters, including the average scale factor, spatial volume, Hubble parameter, and deceleration parameter.

2. THE FIELDS AND METRIC EQUATIONS

The space-time of Bianchi Type VI_0 can be defined by the following equation.

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}dy^2 + C^2e^{-2x}dt^2 \quad (1)$$

The dependent functions A, B, and C are dependent only on time "t."

The scalar field potential V (φ) is accompanied by the gravitational field, which exhibits low coupling with it.

$$L = \int \sqrt{-g}\left(R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - v(\phi)\right)d^4x \quad (2)$$

Following is the Einstein Field Equation:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad (3)$$

For a scalar field, the energy momentum tensor T_{ij} in the presence of viscosity is given by

$$T_{ij} = \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + v(\phi)\right]g_{ij} \quad (4)$$

With,

$$\partial_i\left[\sqrt{-g}\frac{1}{\sqrt{-g}}\partial_i\phi\right] = -\frac{dV}{d\phi} \quad (5)$$

For a given metric (1), the Einstein field Eq. (3) yields

$$\left[\frac{B_4C_4}{BC} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{1}{A^2}\right] = -8\pi\left[\frac{1}{2}\phi_4^2 - v(\phi)\right] \quad (6)$$

$$\left[\frac{A_{44}}{A} + \frac{A_4C_4}{AC} + \frac{C_{44}}{C} - \frac{1}{A^2}\right] = -8\pi\left[\frac{1}{2}\phi_4^2 - v(\phi)\right] \quad (7)$$

$$\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{B_{44}}{B} - \frac{1}{A^2}\right] = -8\pi\left[\frac{1}{2}\phi_4^2 - v(\phi)\right] \quad (8)$$

$$\left[\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{1}{A^2}\right] = 8\pi\left[\frac{1}{2}\phi_4^2 + v(\phi)\right] \quad (9)$$

$$\frac{B_4}{B} - \frac{C_4}{C} = 0 \quad (10)$$

Additionally, the line-element (1)'s scalar field (5) equation is provided by

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = -\frac{dV(\phi)}{d\phi} \quad (11)$$

3. FIELD EQUATION WITH SOLUTION:

Using Eq. (10)

$$B = KC \quad (12)$$

Where K is a constant. Now we take $K = 1$ then

$$B = C \quad (13)$$

Using Eq. (13) in the field equation

$$\left[\frac{B_4^2}{B^2} + 2\frac{B_{44}}{B} + \frac{1}{A^2}\right] = -8\pi\left[\frac{1}{2}\phi_4^2 - v(\phi)\right] \quad (14)$$

$$\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{B_{44}}{B} - \frac{1}{A^2}\right] = -8\pi\left[\frac{1}{2}\phi_4^2 - v(\phi)\right] \quad (15)$$

$$\left[2\frac{A_4B_4}{AB} + \frac{B_4^2}{B^2} - \frac{1}{A^2}\right] = 8\pi\left[\frac{1}{2}\phi_4^2 + v(\phi)\right] \quad (16)$$

We take the average scale factor (R^3) and effective potential $V(\phi)$ for Bianchi type VI_0 space-time into consideration in order to achieve the significant important results.

$$V = e^{-\lambda\phi},\ \lambda > 0 \quad (17)$$

$$R^3 = B^2A = e^{\lambda\phi} \quad (18)$$

Now using Eqs. (10), (17), (18) in Eq. (11),

$$\phi_{44} + \lambda\phi_4^2 = \lambda e^{-\lambda\phi} \quad (19)$$

Eq. (19) can be solved as

$$\frac{d}{d\phi}f^2 + 2\lambda f^2 = 2\lambda e^{-\lambda\phi} \quad (20)$$

Where $\phi_4 = f(\phi)$, $\phi_{44} = ff'$, $f' = \frac{df}{d\phi}$

Now Eq. (20) we have

$$f^2 = 2e^{-\lambda\phi} + le^{-2\lambda\phi} \quad (21)$$

Here, integration constant is l. From Eq. (21), we get

$$e^{\lambda\phi} = pt^2 + qt + r \quad (22)$$

Where $p = \frac{\lambda^2}{2}$, $q = \lambda\sqrt{2}q_1$, $r = q_1^2 - \frac{l}{2}$, q_1 is a constant of integration. Using Eq. (22) in (18), we lead to

$$B^2A = e^{\lambda\phi} = pt^2 + qt + r \quad (23)$$

The following equation gives us

$$\frac{A_4}{A} + 2\frac{B_4}{B} = \lambda\phi_4 = \frac{2pt+q}{pt^2+qt+r} \quad (24)$$

Now using Eqs. (14), (16), (18), we get

$$\frac{B_{44}}{B} + \frac{\lambda\phi_4B_4}{B} - \frac{B_4^2}{B^2} = V(\phi) \quad (25)$$

$$\frac{B_4}{B} = \frac{t+k}{pt^2+qt+r} \quad (26)$$

k is an integration constant, and by resolving Eq. (26) we obtain

$$B = D(pt^2 + qt + r)^{\frac{1}{2p}}\left(\frac{t+s-n}{t+s+n}\right)^{\frac{N}{2np}} \quad (27)$$

Using (23) and (27) we get

$$A = \frac{1}{D^2}(pt^2 + qt + r)^{1-\frac{1}{p}}\left(\frac{t+s+n}{t+s-n}\right)^{\frac{N}{np}} \quad (28)$$

Where, $s = \frac{q}{2p}$, $n^2 = s^2 - \frac{r}{p}$, $N = k - s$, D is a constant of integration.

$$C = \frac{D}{K}(pt^2+qt+r)^{\frac{1}{2p}}\left(\frac{t+s-n}{t+s+n}\right)^{\frac{N}{2np}} \quad (29)$$

Hence the metric (1) takes the form

$$ds^2 = -dt^2 + \frac{1}{D^2}(pt^2+qt+r)^{2\left(1-\frac{1}{p}\right)}\left(\frac{t+s+n}{t+s-n}\right)^{\frac{2N}{np}}dx^2 + D^2(pt^2+qt+r)^{\frac{1}{p}}\left(\frac{t+s-n}{t+s+n}\right)^{\frac{N}{np}}e^{2x}dy^2 + E^2(pt^2+qt+r)^{\frac{1}{p}}\left(\frac{t+s-n}{t+s+n}\right)^{\frac{N}{np}}e^{-2x}dz^2 \quad (30)$$

4. PHYSICAL AND GEOMETRICAL ASPECTS

It is obvious that in the presence of an exponential potential *V*, model (30), together with Eqs. (27, 28, 29), represents the Bianchi type VI_0 metric. Here, we examine the existence of our model by discussing several dynamical parameters. Eq. (17) provides the Higgs Field as,

$$\phi = \frac{1}{\lambda}\log(pt^2+qt+r) \quad (31)$$

Using Eq. (17) in (22), we get the effective potential as

$$V(\phi) = \frac{1}{pt^2+qt+r} \quad (32)$$

The average scale factor R is taken as

$$R = (pt^2+qt+r)^{\frac{1}{3}} \quad (33)$$

And Spatial volume (τ) of the model is

$$\tau = pt^2+qt+r \quad (34)$$

Using (19), where θ is the expansion and H is the Hubble parameter, it can be expressed as

$$H = \frac{1}{3}\left(\frac{A_4}{A}+2\frac{B_4}{B}\right) = \frac{1}{3}\frac{2pt+q}{pt^2+qt+r} \quad (35)$$

$$\theta = 3H = \frac{A_4}{A}+2\frac{B_4}{B} = \frac{2pt+q}{pt^2+qt+r} \quad (36)$$

The shear (σ) is,

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{A_4}{A}-\frac{B_4}{B}\right) = \frac{1}{\sqrt{3}}\left(\frac{(2p-3)t+(q-3k)}{pt^2+qt+r}\right) \quad (37)$$

$$\frac{\sigma}{\theta} = \frac{\mathbf{1}}{\sqrt{\mathbf{3}}}\left(\frac{(2p-3)t+(q-3k)}{2pt+q}\right) \quad (38)$$

The Deceleration Parameter,

$$q = \frac{1}{2}+\frac{3}{2}\left(\frac{q^2-4rp}{(2pt+q)^2}\right) \quad (39)$$

5. CONCLUSION

A scenario of inflation is shown by the rising spatial volume (τ) over time. When the value of time(t) is large, the effective potential (V) reduces to zero. For cosmic time t, negative deceleration parameter $q < 0$ if $(2pt+q)^2+3q^2 < 4rp$, indicates an expanding and accelerating universe. Conversely, deceleration parameter q becomes positive when $(2pt+q)^2+3q^2 > 4rp$, it indicates the universe's decelerating phase. In case deceleration parameter '*q*' became zero if $(2pt+q)^2+3q^2 = 4rp$, than all the galaxies in this situation accelerate at the same speed. The model also shows that at the initial epoch, the average Hubble parameter (*H*), the expansion (θ) and the shear scalar (σ) all attain constant values. We also find out $\frac{\sigma}{\theta} \neq 0$ which shows that general anisotropic model. This model does not show singularity at initially. The resulting solutions provide information on the expansion, shear, and deceleration parameters of the universe's dynamic behavior. We discussed the physical behaviors of the model here with the aid of the dynamical parameters, and we discovered that the cosmos is anisotropic and inflationary, as represented by our model. Since space was stretched during the inflationary epoch to the point where it became a geometrically attracted universe, this inflationary scenario may solve the flatness problem.

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Інфляційна космологічна модель типу Б'янкі VI_0 з експоненціальним потенціалом

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У цьому дослідженні було визначено інфляційний простір-час LRS Bianchi типу VI_0 під впливом ефективного потенціалу $V(\phi) = e - \lambda\phi$, $\lambda > 0$, де ϕ поле Хіггса позначено як ϕ. Для отримання точного розв'язку рекомендується використовувати середній масштабний коефіцієнт$(R^3) = B^2A = e\lambda\phi$. За певних умов модель стає ізотропною та вільною від зсуву. Також модель спочатку не демонструє сингулярності. Еволюція ефективного потенціалу вказує на те, що модель узгоджується з інфляційним сценарієм, в якому Всесвіт зазнає швидкого розширення на ранніх стадіях. Протягом свого розвитку модель проявляє анізотропні властивості, оскільки відношення зсуву до розширення σ/θ не дорівнює нулю. Це дослідження вказує на важливість анізотропних інфляційних моделей для вирішення основних космологічних проблем, таких як проблеми площинності та горизонту. Параметр уповільнення q відіграє вирішальну роль у визначенні того, чи Всесвіт зазнає прискореного чи уповільненого розширення. Ми використали параметр Хаббла, скаляр розширення та скаляр зсуву для обговорення кінематичної та фізичної поведінки моделі. Результати підкреслюють, наскільки важливими є інфляційні сценарії для опису спостережуваної однорідності та великомасштабної структури сучасного космосу.

Ключові слова: Тип Б'янкі, Експоненціальний потенціал, Космологічна модель, Поле Хігса, Інфляційний сценарій.