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REGULAR ARTICLE

Anti Stiff LRS Bianchi Type I String Cosmological Model in General Relativity

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In this work, we have explored anti-stiff LRS Bianchi type-I string cosmological model within the framework of general relativity. The general solution of nonlinear Einstein's field equations for the cosmological model has been obtained by using suitable condition on metric components $A = B^{\frac{1}{n}}$. Assuming an anti-stiff fluid i.e. $p + \rho = 0$, where ρ represents the rest energy density and p denotes the pressure of the fluid. This assumption helps in simplifying the complex nonlinear equations and facilitates the derivation of a viable cosmological solution. We analyse the physical and structural properties of the model, and explain its expansion behaviour, anisotropy and overall dynamical evolution. The study provides insight into the role of string cosmology in the boarder context of general relativity and contributes to the understanding of early universe dynamics under the influence of an anti-stiff fluid. The physical and structural features of the model are discussed in this model.

Keywords: Bianchi, Cosmology, General relativity, LRS model, Ant stiff fluid, String cosmology.

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1. INTRODUCTION

In the last several years, Einstein's hypothesis [1] has gained attention due to its success in finding the early universe's mystery. The inflation word suggests that the Universe has experienced exponential growth since its initial stages of existence and that this expansion is still occurring in the physical universe today. Some conceptual issues in cosmology are not sufficiently addressed by the Big Bang theory when it comes to explaining the inflation phenomenon. Many of the mysteries of contemporary cosmology, including the flatness, homogeneity, and isotropy of the known physical world, are effectively shown by the inflation scenario. Guth [2] initially presented the basic idea of inflation while investigating the absence of monopoles in the current universe. Several cosmologists [3-6] work with the FRW space-time model, which is thought to maintain its isotropic and homogeneity features.

In cosmology, the Bianchi models provide a mathematical basis for studying the realistic image of the current universe. Bianchi Type I space-time becomes a fantastic instrument for studying the early universe's history in a deeper manner than the standard model. A Bianchi Type-I cosmological model with a stiff fluid magnetization has been studied by Bali and Sharma [8]. Bali and Jain [7] and Bali [12] have developed a Bianchi Type I inflationary model for the flat potential in general relativity. Solutions for string cosmological models have been investigated by Sharma A. et al. [14], R. Venkateswarlu et al. [13], Ladke L.S. et al. [17], and Gore et al. [16]. The cosmological model for a perfect fluid with string was examined by Bali R and Pareek U [10]. Bhoyar and Chirde [15] study cosmological models with anti-stiff fluids. The Bianchi Type I cosmological model in the presence of a string in general relativity was examined by Bali R et al. [9],[11] for a barotropic perfect fluid. Bianchi Type-VIII inflationary cosmological models with flat potential and a stiff perfect fluid distribution in general relativity have been studied. Bianchi Type III inflationary universe with flat potential and stiff fluid distribution in general relativity has been investigated by Maheshwari V et al [18]. Poonia and Kumawat [19],[20] have also investigated Bianchi Type I models, though in different circumstances.

In this paper, we have concentrated on the general relativity anti-stiff Bianchi Type I string cosmological model. We apply the condition $A = B^{\frac{1}{n}}$, where n is a non-negative integer and both are dependent functions of time, to get a determinate solution. This model's physical and geometrical behavior are examined.

2. THE METRIC AND FIELDS EQUATIONS

We examine the LRS Bianchi Type I metric, which can be expressed as

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \qquad (1)$$

Where A and B are metric coefficients that are functions of the parameter t alone

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The gravitational field coupled minimally with scalar field $V(\phi)$ is given by

$$L = \int \sqrt{-g}\left(R - \frac{1}{2} g^{ij}\partial_i\phi\partial_j\phi - v(\phi)\right) d^4x \tag{2}$$

The energy-momentum tensor for a string can be written as

$$T_{ij} = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j \tag{3}$$

Where λ is string tension density, p is the pressure, ρ is the density, x_i is the unit space-like vector with the following conditions

$$v_i v^i = -x_i x^i = -1 \text{ and } v^i x_i = 0 \tag{4}$$

The flow vector v^i satisfying

$$g_{ij} v^i v^j = -1 \tag{5}$$

x^i parallel to the x-axis So that $x^i = (A^{-1}, 0,0,0)$

The equation of Einstein's field (gravitational units $8\pi G = c = 1$) is provided by

$$R_i^j - \frac{R}{2} g_i^j = -T_i^j \tag{6}$$

The equation (1) leads to

$$\frac{B_4^2}{B^2} + 2\frac{B_{44}}{B} = \lambda - p \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -p \tag{8}$$

$$\frac{B_4^2}{B^2} + 2\frac{A_4 B_4}{AB} = \rho \tag{9}$$

3. SOLUTION OF FIELD EQUATION

We assume some condition

$$A = B^{\frac{1}{n}} \tag{10}$$

We have the following conditions for anti-stiff fluid

$$p + \rho = 0 \tag{11}$$

Now using eq. (8) and (9), we get

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{B_4^2}{B^2} - \frac{A_4 B_4}{AB} = -(p + \rho) \tag{12}$$

Using condition eq. (11)

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{B_4^2}{B^2} - \frac{A_4 B_4}{AB} = 0 \tag{13}$$

Using the condition

$$\frac{A_4}{A} = \frac{1}{n}\frac{B_4}{B}$$

$$\frac{A_{44}}{A} = \frac{1}{n}\frac{B_{44}}{B} + \frac{1}{n}\left(\frac{1}{n} - 1\right)\frac{B_4^2}{B}$$

Now equation (13) leads to

$$\frac{B_{44}}{B_4} + P\frac{B_4}{B} = 0 \tag{14}$$

Where $P = \frac{(1-2n-n^2)}{n(1+n)}$

Integrating equation (14)

$$B = (P+1)^{\frac{1}{P+1}} (Nt + S)^{\frac{1}{P+1}}$$

Where N and S are constants of integration.

$$B = LT^{\frac{1}{P+1}} \tag{15}$$

Where $L = (P+1)^{\frac{1}{P+1}}, T = (Nt + S)$.

The line element becomes

$$ds^2 = -dt^2 + \left(L^{\frac{1}{n}} T^{\frac{1}{n(P+1)}}\right)^2 dx^2 + L^2 T^{\frac{2}{P+1}} (dy^2 + dz^2) \tag{16}$$

4. SOME PHYSICAL AND GEOMETRICAL FEATURES

We examine the existence of our model by discussing several dynamical parameters.

The deceleration parameter (q) for the model can be expressed as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \tag{17}$$

$$q = \frac{-2n(2+n)}{(1+n)(1+2n)} \tag{18}$$

The scalar of the expansion for a model can be expressed as

$$\theta = 3H = \frac{A_4}{A} + 2\frac{B_4}{B}$$
$$\theta = \left[\frac{(1+n)(1+2n)}{(1-n)}\right]\frac{1}{T} \tag{19}$$

The shear (σ) of the model is

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{A_4}{A} - \frac{B_4}{B}\right]$$
$$\sigma = \frac{(1+n)}{\sqrt{3}}\frac{1}{T} \tag{20}$$

As finite value approaches, the ratio of shear (σ) and expansion scalar (θ)

$$\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\left[\frac{1-n}{1+2n}\right] \quad , \ n = -\frac{1}{2} \tag{21}$$

$$\frac{\sigma}{\theta} = 0, \ n = 1$$

The model isotropies for $n = 1$.

The Hubble parameter (H), pressure (p), density (ρ), and string tension density (λ) for the model can be given by

$$p = -\left(1 + \frac{2}{n}\right) n^2 \frac{(1+n)^2}{(1-n)^2}\frac{1}{T^2} \tag{22}$$

$$\rho = \left(1 + \frac{2}{n}\right) n^2 \frac{(1+n)^2}{(1-n)^2}\frac{1}{T^2} \tag{23}$$

$$\lambda = \frac{2(2+n)n^2(1+n)}{(1-n)^2}\frac{1}{T^2} \tag{24}$$

$$H = \frac{1}{3}\left[\frac{(1+n)(1+2n)}{(1-n)}\right]\frac{1}{T} \tag{25}$$

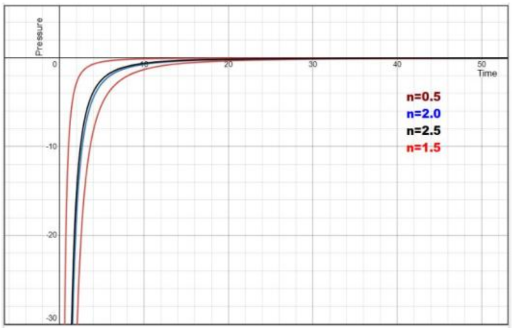

Fig. 1 – Graph plot between Pressure with Time

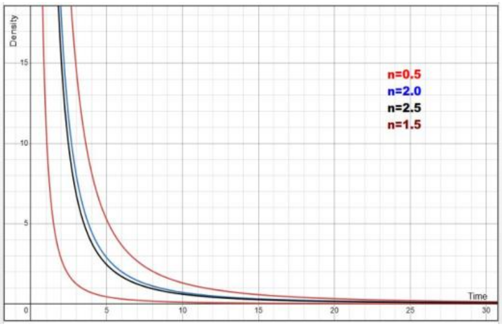

Fig. 2 – Graph plot between Density with Time

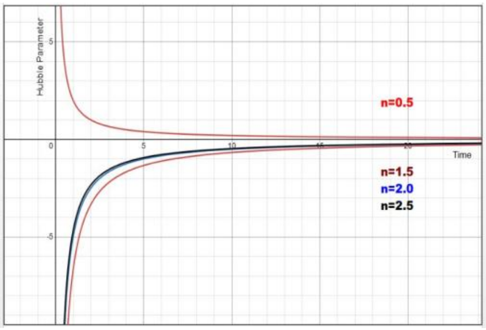

Fig. 3 – Graph plot between String Density with Time

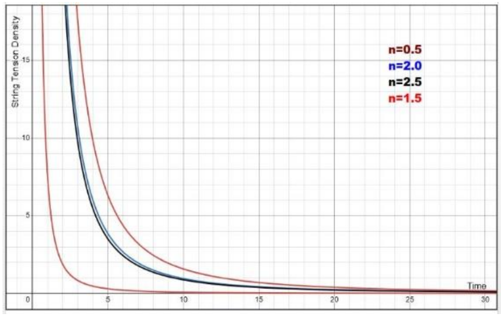

Fig. 4 – Graph plot between Hubble Parameter with Time

5. CONCLUSION

In conclusion, the LRS Bianchi Type I cosmological model gives an attractive framework for researching the evolution of the universe's anisotropic phases. Key cosmological quantities in this model such as the deceleration parameter q, the expansion scalar θ, and the shear scalar σ are derived. Different cosmological phases can be represented by the deceleration parameter q, which depends on n and shows the model's rate of expansion. In this model's expansion (θ) Reduces with increasing time and expansion stops $T \to \infty$. The ratio σ/θ a measure of anisotropy, approaches finite values and becomes zero for $n = 1$, indicating isotropy. Other parameters, such as pressure p, density ρ, string tension density λ scale with time as T, show the power-law behavior of the model throughout the universe's evolution. The Hubble parameter decreases inversely with time. This outcome shows the decelerated expansion of the universe. The anti-stiff fluid condition is a special characteristic of this model and is confirmed by the equation of state $p + \rho = 0$.

The study finds that, depending on the value of n, the model accurately represents a universe changing from anisotropic to isotropic states. The model is completely isotropic for $n = 1$, which corresponds to large-scale universe measurements. The model allows for anisotropic phases for various values, which may be relevant to the early universe. These outcomes are important for investigating anisotropies in the early universe and for using the role of scalar fields and string matter in cosmology. This framework could be expanded in future research to better comprehend the dynamics of the universe by taking into consideration additional matter fields, higher-dimensional spacetimes, or different coupling processes.

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Космологічна модель струн типу I антижорсткого LRS Біанкі у загальній теорії відносності

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У цій роботі ми дослідили антижорстку космологічну модель струн типу I LRS Bianchi в рамках загальної теорії відносності. Загальний розв'язок нелінійних рівнянь поля Ейнштейна для космологічної моделі був отриманий з використанням відповідної умови на метричні компоненти $A = B^{\frac{1}{n}}$. Припускаючи антижорстку рідину, тобто $p + \rho = 0$, де ρ являє собою густину енергії спокою, а p позначає тиск рідини. Це припущення допомагає спростити складні нелінійні рівняння та полегшує виведення життєздатного космологічного розв'язку. Ми аналізуємо фізичні та структурні властивості моделі та пояснюємо її поведінку розширення, анізотропію та загальну динамічну еволюцію. Дослідження дає уявлення про роль струнної космології в прикордонному контексті загальної теорії відносності та сприяє розумінню динаміки раннього Всесвіту під впливом антижорсткої рідини. У цій моделі обговорюються фізичні та структурні особливості моделі.

Ключові слова: Біанкі, Космологія, Загальна теорія відносності, Модель LRS, Космологія струн.