Journal of Nano- and Electronic Physics
Vol. **17** No 5, 05029(3pp) (2025)

Журнал нано- та електронної фізики
Том **17** № 5, 05029(3cc) (2025)

REGULAR ARTICLE

Cloud String Cosmological Model of Bianchi Type-III with Bulk Viscosity in General Relativity

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(Received 10 August 2025; revised manuscript received 22 October 2025; published online 30 October 2025)

In our study of the Bianchi Type-III cosmological model with cloud strings and bulk viscosity in General Relativity, we explore its physical and geometric properties by considering a generalized equation of state $\rho = k\lambda$, ρ is the energy density and λ is the string tension density. The influence of bulk viscosity is examined through the relation $\xi \propto \rho^{\frac{1}{2}}$, which helps understand its effects on the model's evolution. Bulk viscosity plays a significant role in the dynamics of the early universe by modifying the expansion rate and entropy production. The assumed power-law dependence provides insights into dissipative processes affecting cosmic fluid evolution. The study also includes the calculation of the shear scalar and expansion scalar to examine the model's kinematic properties. This cosmological model represents a shearing, non-rotational, and continuously expanding universe, beginning from a big-bang state. However, since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$the model does not become isotropic for large values of T. These quantities provide insight into the anisotropic behaviour and dissipation effects in the universe. The obtained solutions highlight the impact of viscosity on structure formation and cosmic evolution.

Keywords: Bianchi type-III, String, Cosmology, Bulk viscosity.

DOI: 10.21272/jnep.17(5).05029 PACS numbers: 04.20. – q, 98.80. – k, 98.80.Jk

1. INTRODUCTION

String cosmology has attracted a lot of interest lately. Stable topological objects known as cosmic strings may have been created during an early universe symmetry-breaking phase transition [1]. According to grand unified theories, they are thought to have originated during the phase transition following the big bang when the temperature dropped below a particular threshold [2-4] and are quite important for early universe cosmology. It follows that these superstrings may be responsible for the formation of density fluctuations that occur at a time when galaxies are assumed to have been formed [5]. Finally, it is interesting to explore the gravitational impacts of cosmic strings since they have stress-energy and are related to the gravitational field. The Einstein field equations for a cloud of strings with spherical, planar, and cylindrical symmetries are solved by Letelier [6]. He finds cosmological answers in the Bianchi I context in 1983 by solving Einstein's field equations for the cloud of massive strings.

On the other hand, the large-scale distribution of galaxies in our universe is often effectively modelled by a perfect fluid. Nonetheless, a more accurate approach to this problem requires considering material distributions beyond the perfect fluid model. It's well established that after neutrino decoupling, matter behaves as a viscous fluid in the early universe.

Viscous fluid cosmological models for this period have been extensively studied .Bali and Dave [7] proposed a Bianchi type III string cosmological model incorporating bulk viscosity, where they assumed a constant coefficient for bulk viscosity.

However, it's recognized that this coefficient actually decreases as the universe expands. Additionally, Bianchi Type III models have been explored by Poonia and Sharma [8, 9]. Bianchi type III with stiff fluid explore by Maheshwari and Poonia [10]. Bianchi type model explore by [11-14].

In this paper, we have investigated the Bianchi Type-III cosmological model of cloud strings with bulk viscous fluid in general relativity, in which we assumed the equation of state $\rho = k\lambda$. Discussion also covers the physical and geometric characteristics of the model.

2. METRIC AND FIELD EQUATION

We consider general Bianchi – III type space-time metric

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2x} dy^2 + C^2 dz^2 \quad (2.1)$$

In this context, α is treated as a constant. The energy-momentum tensor corresponding to a cloud of strings, incorporating the effects of bulk viscosity, is expressed as follows

$$T_i^j = \rho v_i v^j - \lambda x_i x^j - \xi\theta\left(g_i^j + v_i v_j\right) \quad (2.2)$$

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Cite this article as: V. Maheshwari et al., *J. Nano- Electron. Phys.* 17 No 5, 05029 (2025) https://doi.org/10.21272/jnep.17(5).05029

Here, the rest energy density (ρ) is given by

$$\rho = \rho_p + \lambda \tag{2.3}$$

Where λ represents the tension density of the string cloud. θ represents the scalar of expansion, and ξ is the bulk viscosity coefficient. For the coupled system, the energy density ρ and tension density λ satisfy $\rho + \lambda = 0$, while λ can be either positive or negative. The symbol v^i denotes the velocity, and x^i represents the direction of the string, as given by the standard relation.

$$v_i v^i = -x_i x^i = -1 \quad and \quad x_i v^i = 0 \tag{2.4}$$

The Einstein's field equations is

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \tag{2.5}$$

In a co-moving co-ordinate system, the following holds

$$v^i = (0,0,0,1), \quad x^i = \left(0,0,\frac{1}{c},0\right) \tag{2.6}$$

The system of equations that follows can be derived from the field equations in expression (2.5) together with equation (2.2)

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = \xi\theta \tag{2.7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = \xi\theta \tag{2.8}$$

$$\frac{A_{44}}{A} + \frac{A_4 B_4}{AB} + \frac{B_{44}}{B} - \frac{1}{A^2} = \lambda + \xi\theta \tag{2.9}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = \rho \tag{2.10}$$

$$\left(\frac{B_4}{B} - \frac{A_4}{A}\right) = 0 \tag{2.11}$$

For the variables A, B, and C, the subscript '4' indicates the derivative of the variable with respect to time.

The expansion (θ) and components of shear tensor (σ_{ij}) are given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \tag{2.12}$$

$$\sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} \tag{2.13}$$

$$\sigma^2 = \frac{1}{3}\left[\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2} - \frac{A_4 B_4}{AB} - \frac{B_4 C_4}{BC} - \frac{A_4 C_4}{AC}\right] \tag{2.14}$$

3. SOLUTIONS OF THE FIELD EQUATIONS

From equation (2.11) we obtain

$$A = mB \tag{3.1}$$

Where m is constant.
Equation of state Takabayasi's [12]

$$\rho = k\lambda \tag{3.2}$$

Equations (2.7), (2.9), (2.10), and (3.2) yielded

$$k\frac{A_{44}}{A} + (k-1)\frac{A_4 B_4}{AB} - k\frac{C_{44}}{C} - (k+1)\frac{B_4 C_4}{BC} - \frac{A_4 C_4}{AC} - (k-1)\frac{1}{A^2} = 0 \tag{3.3}$$

Using (3.1) and (3.3)

$$k\frac{B_{44}}{B} + (k-1)\frac{B_4^2}{B^2} - k\frac{C_{44}}{C} - (k+2)\frac{B_4 C_4}{BC} - \frac{(k-1)}{m^2 B^2} = 0 \tag{3.4}$$

In this model, if the expansion θ is proportional to the shear σ, we obtain

$$B = lC^n \tag{3.5}$$

l is constant.

Using (3.5) in (3.4)

$$C_{44} + \frac{[kn(n-1) + (k-1)n^2 - (k+2)n]}{k(n-1)} \frac{C_4^2}{C^2} - \frac{(k-1)}{km^2 l^2} C^{1-2n} = 0 \tag{3.6}$$

To solve this equation $C_4 = f(C)$ then

$$C_{44} = f\frac{df}{dC} \tag{3.7}$$

Using (3.7) equation (3.6) reduce to

$$\frac{df}{dC} + \beta\frac{f}{C} = \frac{(k-1)}{k(n-1)l^2 m^2} C^{1-2n} f^{-1} \tag{3.8}$$

Where

$$\beta = \frac{[kn(n-1) + (k-1)n^2 - (k+2)n]}{k(n-1)} \tag{3.9}$$

$$f = \left\{\frac{(k-1)C^{2-2n}}{k(n-1)(\beta - n + 1)l^2 m^2} + LC^{-2\beta}\right\}^{\frac{1}{2}} \tag{3.10}$$

L is constant.

$$dt = \left\{\frac{(k-1)C^{2-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + LC^{-2\beta}\right\}^{-\frac{1}{2}} dC \tag{3.11}$$

$$\frac{C_4}{C} = \left\{\frac{(k-1)C^{-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + LC^{-2(\beta+1)}\right\}^{\frac{1}{2}} \tag{3.12}$$

As a result, the metric (2.1) becomes

$$ds^2 = -\left\{\frac{(k-1)T^{2-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + LT^{-2\beta}\right\}^{-1} dT^2 + m^2 T^{2n} dX^2 + l^2 T^{2n} e^{2X} dY^2 + T^2 dZ^2 \tag{3.13}$$

Under suitable transformation

$$x = X,\ y = Y,\ z = Z,\ C = T \tag{3.14}$$

4. SOME PHYSICAL AND GEOMETRICAL FEATURES

From equation (2.12)

$$\theta = (2n+1)\frac{C_4}{C} \tag{3.15}$$

$$\theta = (2n+1)\left\{\frac{(k-1)T^{-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + LT^{-2(\beta+1)}\right\}^{\frac{1}{2}} \tag{3.16}$$

From equation (2.14)

$$\sigma^2 = \frac{(n-1)^2}{3}\left\{\frac{(k-1)T^{-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + LT^{-2(\beta+1)}\right\} \tag{3.17}$$

$$\rho = \frac{k(2n+1)T^{-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + n(n+2)LT^{-2(\beta+1)} \tag{3.18}$$

From equation (3.2)

$$\rho_p = \rho - \lambda = \left(1 - \frac{1}{k}\right)\rho \tag{3.19}$$

$$\xi = \frac{1}{\theta}\left[(n+1)\left(\frac{(k-1)T^{-2n}}{[k(n^2-1) - n(n+2)]m^2 l^2} + LT^{-2(\beta+1)}\right)^{\frac{1}{2}} + \right.$$

$$n\left(\frac{(k-1)T^{-2n}}{[k(n^2-1)-n(n+2)]m^2l^2}+LT^{-2(\beta+1)}\right)\Bigg] \quad (3.20)$$

Clearly the energy criteria $\rho \geq 0$ and $p \geq 0$ are met by equations (3.18) and (3.19).

For cases $n > 1$ where and either $k > \frac{n(n+2)}{n^2-1}$ or $k < 0$, it is clear $\beta + 1 > 0$. Consequently, equation (3.16) implies that the expansion scalar θ grows unbounded as $T \to 0$, while it tends towards zero as $T \to \infty$

Additionally, as $T \to 0$ the energy density ρ becomes infinitely large. Conversely, as, $T \to \infty$ ρ diminishes to zero. This behaviour suggests that the cosmological model represents a shearing, non-rotational, and continuously expanding universe, beginning from a big-bang state.

Since $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, the model does not achieve isotropy even as T grows very large.

5. PARTICULAR CASE

We choose $L = 0$ and $k \neq 1$ equation (3.13) reduce to

$$ds^2 = -\frac{[k(n^2-1)-n(n+2)]m^2l^2}{(k-1)T^{2-2n}}dT^2 + m^2T^{2n}dX^2 + l^2T^{2n}e^{2X}dY^2 + T^2dZ^2 \quad (3.21)$$

$$\theta = (2n+1)\left\{\frac{(k-1)T^{-2n}}{[k(n^2-1)-n(n+2)]m^2l^2}\right\}^{\frac{1}{2}} \quad (3.22)$$

$$\sigma^2 = \frac{(n-1)^2}{3}\frac{(k-1)T^{-2n}}{[k(n^2-1)-n(n+2)]m^2l^2} \quad (3.23)$$

$$\rho = \frac{k(2n+1)T^{-2n}}{[k(n^2-1)-n(n+2)]m^2l^2} \quad (3.24)$$

$$\rho_p = \rho - \lambda = \left(1-\frac{1}{k}\right)\rho \quad (3.25)$$

$$\xi = \left[\frac{k-1}{[k(n^2-1)-n(n+2)](2n+1)^2m^2l^2}\right]^{\frac{1}{2}}T^{-n} \quad (3.26)$$

Clearly the energy criteria $\rho \geq 0$ and $p \geq 0$ are met by equations (3.24) and (3.25) respectively.

For $n > 1$, regardless of whether $k > \frac{n(n+2)}{n^2-1}$ *or* $k < 0$ equation (3.22) indicates that the expansion scalar θ diverges to infinity as T approaches zero, while θ approaches zero as tends to infinity. Likewise, the energy density ρ becomes unbounded as T approaches zero and diminishes to zero as T increases toward infinity. This behaviour suggests that the mode represents a continuously expanding, shearing, and non-rotating universe that originates from a big bang. However, since $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$ the model does not become isotropic for large values of T.

CONCLUSIONS

In our investigation of the Bianchi Type-III cosmological model with cloud strings and bulk viscosity within the framework of General Relativity, we assume a more general model by introducing the equation of state $\rho = k\lambda$, where ρ represents the energy density and λ is the string tension density. The model outlines a shearing, non-rotating, and expanding universe with an initial big bang. We also examine particular case Based on equations (3.24) and (3.26), we find that the relationship between the bulk viscosity coefficient ξ and the energy density follows $\xi \propto \rho^{\frac{1}{2}}$.

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Космологічна модель хмарних струн Біанкі III типу з об'ємною в'язкістю у загальній теорії відносності

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У нашому дослідженні космологічної моделі Біанкі III типу з хмарними струнами та об'ємною в'язкістю в загальній теорії відносності ми досліджуємо її фізичні та геометричні властивості, розглядаючи узагальнене рівняння стану $\rho = k\lambda$, де ρ – густина енергії, а λ – густина натягу струни. Вплив об'ємної в'язкості розглядається через співвідношення $\xi \propto \rho^{\frac{1}{2}}$, що допомагає зрозуміти її вплив на еволюцію моделі. Об'ємна в'язкість відіграє значну роль у динаміці раннього Всесвіту, змінюючи швидкість розширення та виробництво ентропії. Передбачувана степенева залежність дає уявлення про дисипативні процеси, що впливають на еволюцію космічної рідини. Дослідження також включає розрахунок скаляра зсуву та скаляра розширення для вивчення кінематичних властивостей моделі. Ця космологічна модель представляє Всесвіт, що змінюється, не обертається та безперервно розширюється, починаючи зі стану Великого вибуху. Однак, оскільки $\lim_{T\to\infty}\frac{\sigma}{\theta} \neq 0$, модель не стає ізотропною для великих значень T. Ці величини дають уявлення про анізотропну поведінку та ефекти дисипації у Всесвіті. Отримані розв'язки підкреслюють вплив в'язкості на формування структури та космічну еволюцію.

Ключові слова: Біанкі типу III, Струна, Космологія, Об'ємна в'язкість.