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REGULAR ARTICLE

Finite-time Stabilization of Discrete-time Linear Systems Subject to Time-varying Delay Using a Novel Summation Inequality

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To improve the performance and reduce the conservatism of developed results over a FT (finite-time) interval, a new analysis and design technique is proposed in this paper. Then, a system is said to be FT stable if, at a certain time interval, its state does not exceed some bounds. Hence, novel delay-dependent finite-time stability (FTS) conditions are provided for discrete-time linear systems subject to time-varying delay. The delay-dependent stability criteria, which take into account the information on the size of time delays, are less conservative than delay-independent ones. Then, the obtained conditions are applied to the design problem of FT controllers. For this reason, a new approximation of the single summation that appears in the Lyapunov-Krasovskii functional (LKF) is studied using an innovative Free-Matrix-Based-Integral (FMBI) inequality. This has led to the proposal of new conditions in the form of LMIs that ensure FTS. Although some results improve the stability criteria, FTS has received little attention, and more results can be obtained to reduce the conservatism. That is the keystone of our research. To illustrate the potential gain of employing this new approach, a detailed numerical example is provided. Finally, a less conservative LMI-based design is proposed and solved with MATLAB showing very good results.

Keywords: Finite-time stability (FTS), Discrete-time systems, Free-Matrix-Based-Integral (FMBI).

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1. INTRODUCTION

Time delay is the major cause of poor performance, oscillation, and even instability for dynamic systems. Over the last half-century, such systems have attracted much attention and great efforts have been devoted to investigating the influence performance, especially in control theory fields [1-3]. Hence, all most works focused on stabilization conditions, that is given over an infinite-time interval. However, in practice, the interest is often concerned with the behavior of the system over an FT interval. The FTS concept dates back to the 1950s [4, 5], when the term FTS was introduced for the first time. Then, important results are obtained for various sorts of systems in the last years [1, 7-8]. Then, many of these results improve the stability criteria, however, FTS has received little attention, and is still room for further research to reduce the conservatism using these methods, which motivates our research.

A new analysis and design technique is given in this paper in order to improve the performance and reduce the conservatism of developed results over a specific time interval. The purpose of the paper is then to guarantee the FT stabilizability of closed-loop delayed systems despite the time-varying delay. The obtained results are based on a new approximation of the single summation that appears in the LKF using an innovative FMBI inequality. Then, results are established in terms of LMIs

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to achieve the desired performance. To illustrate the potential gain of employing this new approach, a detailed numerical example is provided. Finally, a less conservative LMI-based design is proposed and solved with MATLAB showing very good results.

2. PROBLEM FORMULATION

Consider the following system:

$$x(k+1) = Ax(\mathrm{k}) + A_d x(k-d(k)) + Bu(k) \tag{1}$$

where $x(k)\in\Re^n$ is the state vector, $u(k)\in\Re^m$ is the control input vector, $A, A_d \epsilon \Re^{n\times n}, B\epsilon\Re^{n\times m}$ are constant matrices, and $0 \le d_1 \le d(k) \le d_2$ is the system delay.

We define $y(k) = x(k+1) - x(k)$ with:

$$y^T(k)y(k) < \varepsilon \text{ , for } k\,\epsilon \left\{-d_{2,} - d_2 + 1, \ldots, 0\right\}$$

Definition 1. [9]. The system (1) is FT stable according to (c_1, c_2, R, N), $c_1 < c_2$, if $x^T(\mathrm{k})\mathrm{x}(\mathrm{k}) < c_2$, $k \in 1, \ldots, N$, with $sup_{\eta\epsilon\{1,\ldots,\mathrm{N}\}}\, \varphi^T(\eta)\varphi(\eta) < c_1$.

Lemma 1. [7]. For $n, k \ge 1$ the following inequality:

$$\gamma_{n.2k}^{\min} < \gamma_{n.2k} < \gamma_{n.2k}^{\max}$$

holds where $\gamma_{n.2k} = \frac{1}{(n+1)^{2k}} \sum_{i=1}^{n} (n+1-2i)^{2k}$,

$$\gamma_{n.2k}^{\max} = \begin{cases} \dfrac{1}{2k+1}\dfrac{(n+2)^{2k+1}}{(n+)^{2k}} & \text{if } n \text{ is even} \\ \dfrac{1}{2k+1}(\mathrm{n}+1) & \text{if } n \text{ is odd} \end{cases},$$

$$\gamma_{n.2k}^{\min} = \begin{cases} \dfrac{1}{2k+1}\dfrac{(n-2)^{2k+1}}{(n+1)^{2k}} & \text{if } n \text{ is even} \\ \dfrac{1}{2k+1}\dfrac{(n-1)^{2k+1}}{(n+1)^{2k}} & \text{if } n \text{ is odd} \end{cases}$$

Lemma 2. For appropriately dimensioned matrices (ADM) $R > 0, Z_i, L_i, N_i, i = 1,2, Z_3$ with $\begin{pmatrix} Z_1 & Z_2 & N_1 \\ * & Z_4 & N_2 \\ * & * & R \end{pmatrix} \ge 0$, integers a and b satisfying $a < b$, and vectors g_1 and g_2, the following inequalities hold:

1) JBI (Jacobian-Based Inversion) [10]:

$$\mathfrak{S} \ge \tfrac{1}{l} v_1^T R v_1 \tag{2}$$

2) WBI (Wavelet-Based Inversion) [11]:

$$\mathfrak{S} \ge \tfrac{1}{l}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3\left(\frac{l+1}{l-1}\right)R \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3}$$

$$\ge \tfrac{1}{l}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \tag{4}$$

3) FMBI (Frequency Modulation Based Inversion) [12]:

$$\mathfrak{S} \ge 2\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}^T \begin{bmatrix} -N_1 & 0 \\ 0 & N_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - l v_4^T \left(\tfrac{3Z_1+Z_2}{3}\right) v_4 \tag{5}$$

4) GFMBI Generalized Frequency Modulation Based Inversion [13]:

$$\mathfrak{S} \ge 2\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - l \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T \begin{bmatrix} \Omega_1 & 0 \\ 0 & \Omega_3 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{6}$$

$$\ge 2\begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - l \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}^T \begin{bmatrix} \Omega_1 & 0 \\ 0 & \Omega_3 \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \tag{7}$$

where $v_1 = x(b) - x(a), v_2 = x(b) + x(a) - \sum_{i=a}^{b} \frac{2x(i)}{l+1}$,

$$v_3 = \left[x^T(b), x^T(a), \sum_{i=a}^{b} \frac{x(i)}{l+1}\right]^T,$$

$$\Omega_i = L_i R^{-1} L_i^T,\ \bar{\Omega}_3 = \frac{l-1}{3(l+1)}\Omega_2, \Omega_3 = \frac{1}{3}\Omega_2$$

Lemma 3. If there exist ADM $Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, N, M$, an integer $m\epsilon\{n/n = 2s+1; s\,\epsilon\, N\}$, and vector functions $\{y(i), x(i) : i\,\epsilon\,[a, a+n-1]\}$, the following inequalities are true:

$$\Theta = \begin{pmatrix} Z_1 & Z_2 & Z_3 & N \\ * & Z_4 & Z_5 & M \\ * & * & Z_6 & M \\ * & * & * & R \end{pmatrix} \ge 0,$$

$$-\sum_{i=a}^{a+n+1} y^T(i)Ry(i) \leqslant \varpi^T \Omega \varpi,\ y(i) = x(i+1) - x(i),$$

$$\varpi = \left[x^T(a+n) \quad x^T(a) \quad \tfrac{1}{n+1}\sum_{i=a}^{a+n} x^T(i)\right]^T,$$

$$\Omega = (nZ_1 + \gamma_1(n)Z_4 + \gamma_2(n)Z_6) + \mathrm{sym}(N\Pi_1 + M\Pi_2 + \gamma_3(n)Z_5),$$

$$\Pi_1 = [I \quad -I \quad 0], \Pi_2 = [-I \quad -I \quad 2I], \gamma_2(n) = \textstyle\sum_1^n \left(1 - \frac{2i}{n+1}\right)^{2m},$$

$$\gamma_1(n) = \textstyle\sum_{i=1}^n \left(1 - \frac{2i}{n+1}\right)^{2m} + 2\sum_1^n \left(1 - \frac{2i}{n+1}\right)^{m+1} + \sum_1^n \left(1 - \frac{2i}{n+1}\right)^2,$$

$$\gamma_3(n) = -\textstyle\sum_1^n \left(1 - \dfrac{2i}{n+1}\right)^{2m} - \sum_1^n \left(1 - \dfrac{2i}{n+1}\right)^{m+1}$$

Proof. Consider the following functions:

$$f_n(i) = -g_n(i) + \left(1 - \tfrac{2(i+1-a)}{n+1}\right),$$

$$g_n(i) = -\left(1 - \tfrac{2(i+1-a)}{n+1}\right)^m : m \in \{2s+1/\in N\},$$

$$\xi(i) = [\varpi^T, f_n(i)\varpi^T, g_n(i)\varpi^T, y(i)]^T$$

Then, we set: $p = \begin{cases} n-1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$

Thus, we have:

$$\textstyle\sum_{i=a}^{a+n-1} g_n(i) = -\sum_{i=a}^{a+n-1} \left(1 - \frac{2(i+1-a)}{n+1}\right)^m$$
$$= -\textstyle\sum_{i=1}^{n} \left(1 - \frac{2i}{n+1}\right)^m,$$
$$= -\left\{\textstyle\sum_{i=a}^{p/2} \left(1 - \frac{2i}{n+1}\right)^m + \sum_{i=(p/2)+1}^{p/2} \left(1 - \frac{2i}{n+1}\right)^m\right\}$$

It is easy to see that

$$\textstyle\sum_{\mathrm{i=a}}^{\mathrm{p}/2} \left(1 - \frac{2\mathrm{i}}{\mathrm{n}+1}\right)^{\mathrm{m}} = -\sum_{\mathrm{i=(p/2)+1}}^{\mathrm{p}/2} \left(1 - \frac{2\mathrm{i}}{\mathrm{n}+1}\right)^{\mathrm{m}}$$

which make to conclude that $\sum_{\mathrm{i=a}}^{\mathrm{a+n-1}} \mathrm{g_n(i)} = 0$ and

$$\textstyle\sum_{\mathrm{i=a}}^{a+n-1} \mathrm{f_n(i)} = \sum_{\mathrm{i=a}}^{a+n-1} \left\{\left(1 - \frac{2(\mathrm{i}+1-\mathrm{a})}{\mathrm{n}+1}\right)^{\mathrm{m}} + \left(1 - \frac{2(\mathrm{i}+1-\mathrm{a})}{\mathrm{n}+1}\right)\right\}$$
$$= -\textstyle\sum_{i=a}^{a+n-1} g_n(i) + \left\{n - \left(\frac{2}{n+1}\right)\left(\frac{n(n+1)}{2}\right)\right\} = 0,$$

$$\textstyle\sum_{i=a}^{a+n-1} \left(g_n(i)\right)^2 = \sum_{i=a}^{a+n-1} \left(1 - \frac{2(i+1-a)}{n+1}\right)^{2m} \neq 0,$$

$$\textstyle\sum_{i=a}^{a+n-1} (f_n(i))^2 = \sum_{i=a}^{a+n-1} \left(\left(1 - \frac{2(i+1-a)}{n+1}\right)^m + \left(1 - \frac{2(i+1-a)}{n+1}\right)\right)^2$$
$$= \textstyle\sum_{i=a}^{a+n-1} \left\{\left(1 - \frac{2(i+1-a)}{n+1}\right)^{2m} + 2\left(1 - \frac{2(i+1-a)}{n+1}\right)^{m+1} + \left(1 - \frac{2(i+1-a)}{n+1}\right)^2\right\} \neq 0,$$

$$\textstyle\sum_{i=a}^{a+n-1} f_n(i)g_n(i) = -\left(1 - \frac{2(i+1-a)}{n+1}\right)^{2m} - \left(1 - \frac{2(i+1-a)}{n+1}\right)^{m+1} \neq 0,$$

$$\textstyle\sum_{i=a}^{a+n-1} \left(f_n(i) + g_n(i)\right)y(i) = \sum_{i=a}^{a+n-1} \left(1 - \frac{2(i+1-a)}{n+1}\right)y(i)$$
$$= \textstyle\sum_{i=a}^{a+n-1} y(i) - \frac{2}{n+1}\sum_{i=a}^{a+n-1} \sum_{j=i}^{a+n-1} y(j)$$
$$= (x(a+n) - x(a)) - \tfrac{2}{n+1}\textstyle\sum_{i=a}^{a+n-1} (x(a+n) - x(i))$$
$$= -x(a+n) - x(a) - \tfrac{2}{n+1}$$

Summating $\xi^T(i)\Theta\xi(i)$ from a to $a+n-1$, we get:

$$\textstyle\sum_{i=a}^{a+n-1} \xi^T(i)\Theta\xi(i) = \varpi^T(nZ_1 + \gamma_1(n)Z_4 + \gamma_2(n)Z_6)$$
$$+ sym(N\Pi_1 + M\Pi_2 + \gamma_3(n)Z_5)\varpi + \textstyle\sum_{i=a}^{a+n-1} y^T(i)Ry(i)$$

Finally, considering $\xi^T(i)\Theta\xi(i) \ge 0$, we have:

$$-\sum_{i=a}^{a+n-1} y^T(i)Ry(i) \le \varpi^T(nZ_1+\gamma_1(n)Z_4+\gamma_2(n)Z_6) + sym(N\Pi_1+M\Pi_2+\gamma_3(n)Z_5)$$

Proposition 1. [13]. The relationships among (2)-(7) are summarized as follows:

1) Both WBIs (3)-(4) are tighter than JBI (2);
2) FMBI (5) includes WBI (4);
3) GFMBI (6) is tighter than FMBI (4); since the former includes extra positive terms.

Remark 1. The condition (7) is tighter than FMBI with $g_i = v_3,\ i = 1,2$ [13]. For this reason, we prove in the following that the GFMBI is particular case of our lemma. Setting $Z_1 = L_1R^{-1}L_1^T, Z_2 = Z_3 = L_1R^{-1}L_2^T, Z_4 = Z_5 = Z_6 = L_2R^{-1}L_2^T, N = L_1, M = L_2$ and using the Schur complement, the following inequality is verified:

$$\begin{pmatrix} L_1R^{-1}L_1^T & L_1R^{-1}L_2^T & L_1R^{-1}L_2^T & -L_1 \\ * & L_2R^{-1}L_2^T & L_2R^{-1}L_2^T & -L_2 \\ * & * & L_2R^{-1}L_2^T & -L_2 \\ * & * & * & \mathrm{R} \end{pmatrix} \ge 0$$

Then, we have:

$$-\sum_{i=a}^{a+n-1} y^T(i)Ry(i) \le \varpi^T\left(2\begin{bmatrix} -L_1 & 0 \\ 0 & -L_2 \end{bmatrix} + \begin{bmatrix} \Omega_1 & 0 \\ 0 & \bar{\Omega}_3 \end{bmatrix}\right)\varpi$$

3. FINITE TIME STABILIZATION

The effective control signal to be applied to the system (1) is $u(t) = K_1x(t) + K_2x(k-d(k))$.

Theorem 1. (1) is FT stable according to (c_1, c_2, R, N) if there exist SPDM $\bar{P}, \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{R}_1, \bar{R}_2, U_j, j = 1, \dots, 5$, ADM $\bar{Z}_{d1}^2, \bar{Z}_{d1}^4, \bar{Z}_{d1}^6, \bar{Z}_{dk1}^1, \bar{Z}_{dk1}^4, \bar{Z}_{dk1}^6, \bar{Z}_{dk2}^1, \bar{Z}_{dk2}^4, \bar{Z}_{dk2}^6, \bar{Z}_{d1}^2, \bar{Z}_{d1}^3, \bar{Z}_{d1}^5, \bar{Z}_{dk1}^1, \bar{Z}_{dk1}^3, \bar{Z}_{dk1}^5, \bar{Z}_{dk2}^1, \bar{Z}_{dk2}^3, \bar{Z}_{dk2}^5, \bar{N}_{d1}, \bar{M}_{d1}, \bar{N}_{dk1}, \bar{M}_{dk1}, \bar{N}_{dk2}, \bar{M}_{dk2}, \bar{H}_l, l = 1,2,3, \bar{\gamma}_4, W_\mathrm{i}$, scalars $\mu_\mathrm{i} > 0$, odd integer $m \ge 1$, and a positive scalar $\alpha \ge 0$ such that:

$$\bar{Z}_{dki}^5 + \left(\bar{Z}_{dki}^5\right)^T \ge 0,$$

$$\begin{pmatrix} \bar{Z}_{d1}^1 & \bar{Z}_{d1}^2 & \bar{Z}_{d1}^3 & \bar{N}_{d1} \\ * & \bar{Z}_{d1}^4 & \bar{Z}_{d1}^5 & \bar{M}_{d1} \\ * & * & \bar{Z}_{d1}^6 & \bar{M}_{d1} \\ * & * & * & \mu_1\bar{P} \end{pmatrix} \ge 0,$$

$$\begin{pmatrix} \bar{Z}_{dk1}^1 & \bar{Z}_{dk1}^2 & \bar{Z}_{dk1}^3 & \bar{N}_{dk1} \\ * & \bar{Z}_{dk1}^4 & \bar{Z}_{dk1}^5 & \bar{M}_{dk1} \\ * & * & \bar{Z}_{dk1}^6 & \bar{M}_{dk1} \\ * & * & * & \mu_2\bar{P} \end{pmatrix} \ge 0,$$

$$\begin{pmatrix} \bar{Z}_{dk2}^1 & \bar{Z}_{dk2}^2 & \bar{Z}_{dk2}^3 & \bar{N}_{dk2} \\ * & \bar{Z}_{dk2}^4 & \bar{Z}_{dk2}^5 & \bar{N}_{dk2} \\ * & * & \bar{Z}_{dk2}^d & \bar{M}_{dk2} \\ * & * & * & \mu_2\bar{P} \end{pmatrix} \ge 0,$$

$$\bar{\Omega} + \begin{bmatrix} \bar{Y}_1^T H_1 \bar{Y}_1 + \bar{Y}_2^T H_2 \bar{Y}_2 + \bar{Y}_3^T H_3 \bar{Y}_3 - (\alpha-1)\bar{Y}_4 & 0 \\ * & 0 \end{bmatrix} < 0,$$

$$(U_2 + d_1U_3 + d_2U_4 + d_2U_5)c_1 + \left(\frac{d_1(d_1+1)}{2} + \frac{(d_2-d_1)(d_2+d_1+1)}{2}\right)U_2\epsilon < \alpha^{-N}c_2\mathrm{U}_1$$

where

$$\bar{\Omega} = \begin{bmatrix} -(d_2+1)\bar{P}+\bar{Q}_1 & 0 & 0 & 0 & 0 & 0 & 0 & \bar{P}A^T+\bar{P}{W_1}^T & \mu_1d_1\left(\bar{P}A^T+\bar{P}{W_1}^T\right) & \mu_2(d_2-d_1)(\bar{P}A^T+\bar{P}{W_1}^T) \\ * & -\bar{Q}_1+\bar{Q}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\bar{Q}_2+\bar{Q}_3 & 0 & 0 & 0 & 0 & \bar{P}{A_d}^T+\bar{P}{W_2}^T & \mu_1d_1\left(\bar{P}{A_d}^T+\bar{P}{W_2}^T\right) & \mu_2(d_2-d_1)(\bar{P}{A_d}^T+\bar{P}{W_2}^T) \\ * & * & * & -\bar{Q}_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\bar{P} & 0 & 0 \\ * & * & * & * & * & * & * & * & -\mu_1d_1\bar{P} & 0 \\ * & * & * & * & * & * & * & * & * & -\mu_2(d_2-d_1)\bar{P} \end{bmatrix},$$

$$U_1 < \bar{P} < U_2, 0 < U_3 < \bar{Q}_1, 0 < U_4 < \bar{Q}_2, K_1 = W_1\bar{P}^{-1}, K_2 = W_2\bar{P}^{-1}$$

Proof. Theorem 1 can be obtained by pre and post multiplying the inequalities of Corollary 1 [7] by $diag(\bar{P},\bar{P},\bar{P},\bar{P})$ and $diag(\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P})$, respectively, where:

$$\bar{P} = P, \bar{Q}_1 = \bar{P}Q_1\bar{P}, \bar{Q}_2 = \bar{P}Q_2\bar{P}, \bar{Q}_3 = \bar{P}Q_3\bar{P},$$

$$\begin{pmatrix} \bar{Z}_{d1}^1 & \bar{Z}_{d1}^2 & \bar{Z}_{d1}^3 & \bar{N}_{d1} \\ * & \bar{Z}_{d1}^4 & \bar{Z}_{d1}^5 & \bar{M}_{d1} \\ * & * & \bar{Z}_{d1}^6 & \bar{M}_{d1} \\ * & * & * & \bar{P} \end{pmatrix} = \begin{pmatrix} \bar{P}Z_{d1}^1\bar{P} & \bar{P}Z_{d1}^2\bar{P} & \bar{P}Z_{d1}^3\bar{P} & \bar{P}N_{d1}\bar{P} \\ * & \bar{P}Z_{d1}^4\bar{P} & \bar{P}Z_{dd}^5\bar{P} & \bar{P}M_{d1}\bar{P} \\ * & * & \bar{P}Z_{d1}^6\bar{P} & \bar{P}M_{d1}\bar{P} \\ * & * & * & \bar{P} \end{pmatrix},$$

$$\begin{pmatrix} \bar{Z}_{dki}^1 & \bar{Z}_{dki}^2 & \bar{Z}_{dki}^3 & \bar{N}_{dki} \\ * & \bar{Z}_{dki}^4 & \bar{Z}_{dki}^5 & \bar{M}_{dki} \\ * & * & \bar{Z}_{dkj}^6 & \bar{M}_{dkj} \\ * & * & * & \mu_i\bar{P} \end{pmatrix} = \begin{pmatrix} \bar{P}Z_{dki}^1\bar{P} & \bar{P}Z_{dki}^2\bar{P} & \bar{P}Z_{dki}^3\bar{P} & \bar{P}N_{dki}\bar{P} \\ * & \bar{P}Z_{dki}^4\bar{P} & \bar{P}Z_{dki}^5\bar{P} & \bar{P}M_{dki}\bar{P} \\ * & * & \bar{P}Z_{dki}^6\bar{P} & \bar{P}M_{dki}\bar{P} \\ * & * & * & \mu_i\bar{P} \end{pmatrix},$$

$$\bar{H}_l = diag(\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P})H_l diag(\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P}),$$

$$\bar{\gamma}_4 = diag(\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P})\gamma_4 diag(\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P},\bar{P})$$

4. Examples

Example 1. Consider the system (1) where

$$A = \begin{pmatrix} 0.80 & 0.30 \\ 0.80 & 0.70 \end{pmatrix}, A_d = \begin{pmatrix} 0.15 & 0.00 \\ 0.40 & 0.05 \end{pmatrix}, B = \begin{pmatrix} 0.20 \\ 0.10 \end{pmatrix}$$

Using the results of Theorem 1, $K_2 = 0, (c_1, c_2, N) = (18.1, 80, 50), \epsilon = 1.1, d_1 = 2, d_2 = 11, \mu_1 = 0.05, \mu_2 = 0.001$, and $\alpha = 1.00026$, the control gain matrix K_1 is given by:

$$K_1 = (-5.7612\ \ -3.9194)$$

In addition, the initial conditions are:

$$\varphi_{\eta\in\{-11,-10,\dots,0\}}(\eta) = [\varphi(-11), \varphi(-10), \dots, \varphi(0)] = \begin{bmatrix} 2 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 & -3 \end{bmatrix}$$

Moreover, it can be seen that the initial values satisfy the following conditions:

$$\sup_{\eta\in\{-11,-10,\dots,0\}} \varphi^T(\eta)\varphi(\eta) < \alpha = 18.1,$$

$$\sup_{\eta\in\{-11,-10,\dots,0\}} [\varphi^T(\eta+1) - \varphi^T(\eta)][\varphi(\eta+1) - \varphi(\eta)] < \alpha = 1.1$$

Figure 1 and Figure 2 give respectively the evolution of state and norme response trajectories with the controller. Then, it is shown that the states trajectories converge more faster using our obtained controller gain. Also, the considered system is FT stabilizable using our approach, as that the state norm trajectory converges more faster compared with [9]. So, the simulation results show the accuracy and the effectiveness of the proposed approach for which the closed-loop system is stable.

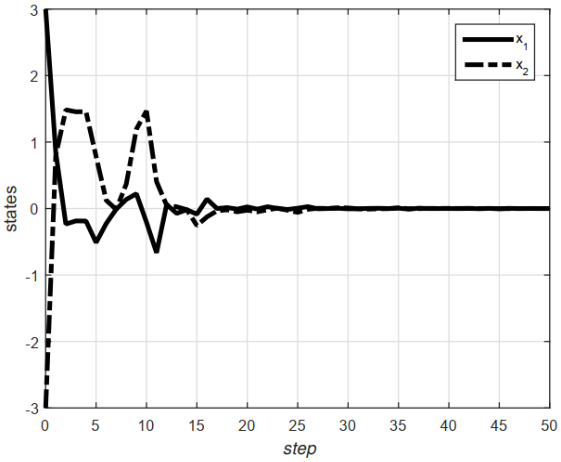

Fig. 1 – State responses of the system

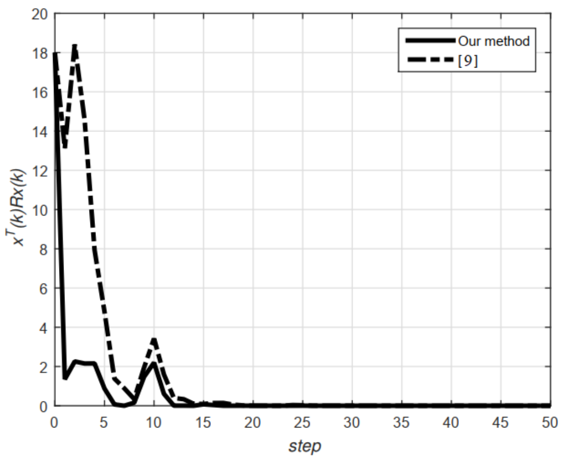

Fig. 2 – State norm responses of the system

Example 2. The satellite system shown in Figure 3 has two rigid bodies joined by a flexible link [14-16].

The dynamic equations of this system are as follows:

$$\begin{aligned} J_1\ddot{\theta}_1(t) + f\big(\dot{\theta}_1(t) - \dot{\theta}_2(t)\big) + \kappa(\theta_1 - \theta_2) &= u(t) \\ J_2\ddot{\theta}_2(t) + f\big(\dot{\theta}_1(t) - \dot{\theta}_2(t)\big) + \kappa(\theta_1 - \theta_2) &= 0 \end{aligned} \tag{8}$$

where $J_i (i = 1,2)$ are the moments of inertia of the two bodies (the main body and the instrumentation module), f is the viscous damping, κ is the torque constant, and θ_1, θ_2 are the yaw angles for the two bodies.

Then, the system (8) can be transformed as the following discrete system:

$$x(k+1) = \begin{pmatrix} 1.0000 & 0.0000 & 0.0100 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 & 0.0100 \\ -0.0090 & 0.0090 & 0.9996 & 0.0004 \\ 0.0090 & -0.0090 & 0.0004 & 0.9996 \end{pmatrix} x(k)$$

$$+ \begin{pmatrix} 0.00 \\ 0.00 \\ 0.01 \\ 0.00 \end{pmatrix} u(k)$$

where $J_i = 1, \kappa = 0.09, f = 0.09$, and the sampling time $T_s = 10\text{ms}$. Also, the state vector is:

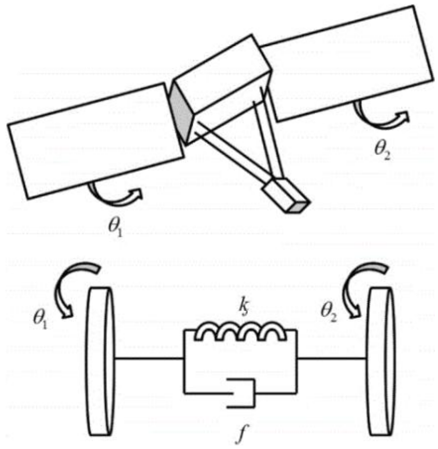

Fig. 3 – Two-body satellite system with a flexible link

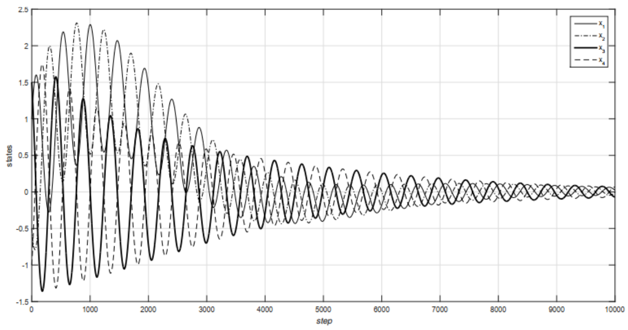

Fig. 4 – State responses of the system

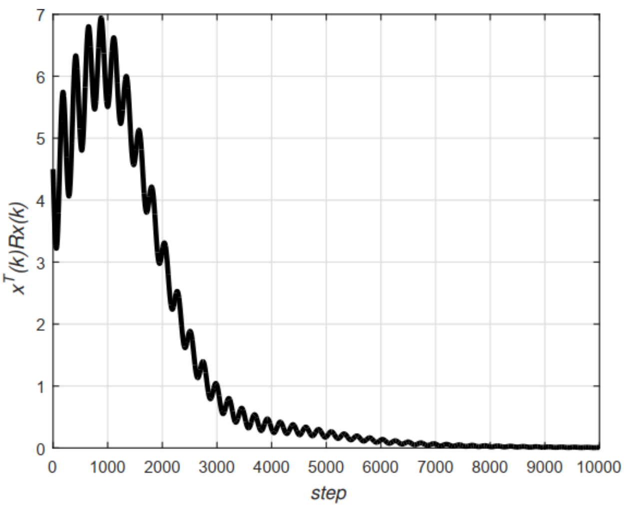

Fig. 5 – State norm responses of the system

$$x(t) = [x_1(t), x_2(t), x_3(t), x_4(t)]^T = \big[\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2\big]^T$$

Now, let $K_1 = 0, (c_1, c_2, N) = (5.5, 30, 10^3), \epsilon = 1.1, d_1 = 2, d_2 = 11, \mu_1 = 0.5, \mu_2 = 0.06, \alpha = 1.00001$. Using Theorem 1, the control gain matrix K_2 is given as follows:

$$K_2 = (-0.0102 \quad -0.0100 \quad -0.1185 \quad -0.1173)$$

In order to check the effectiveness of the obtained results, simulation results by applying our derived controller gain are given in Figure 4 and Figure 5. Then, the closed-loop system is FT stabilizable and the state responses converge to zero when initial values of the states are $x(0) = [1, -0.5, 1.5, -1]^T$.

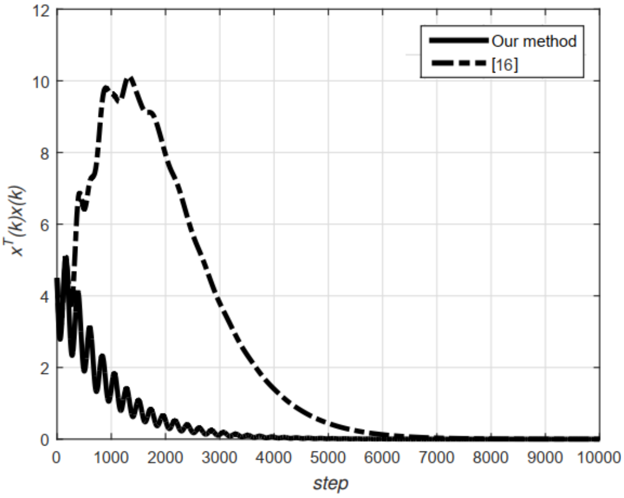

Fig. 6 – State norm responses of the system

Finally, using $\mu_i = 1$ and $\alpha = 1$, the control gain matrix K_2 is given as follows:

$$K_2 = (-0.1954 \quad -0.0739 \quad -0.7507 \quad -0.5859)$$

In Figure 6, it is shown that the system is asymptotically stable and the obtained control gain ensures a faster convergence compared to [16], which proves the efficiency of our results.

4. CONCLUSION

This paper treats FTS for discrete-time systems with time-varying delays. An LKF and a new FMBI inequality have been developed; and, a new derivation of the single summation that appears in the deduction of the LKF has been obtained. Then, novel and sufficient conditions, expressed in terms of LMIs, are given ensuring the FTS. The numerical example clarifies the advantages and effectiveness of the proposed method. These new results are illustrated by a numerical example, and great improvements are obtained compared to the existing results. The conservativeness of the derived results can be further reduced by combining the proposed technique with the delay-decomposition technique.

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Стабілізація дискретних лінійних систем у скінченному часі з урахуванням змінної затримки з використанням нової нерівності підсумовування

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Для покращення продуктивності та зменшення консерватизму розроблених результатів протягом інтервалу скінченного часу (FT) у цій статті пропонується новий метод аналізу та проектування. Тоді система називається FT-стійкою, якщо на певному інтервалі часу її стан не перевищує певних меж. Отже, для дискретних лінійних систем із змінною затримкою (FT) надаються нові умови FT-стійкості (FTS), що залежать від затримки. Критерії стійкості, що залежать від затримки, які враховують інформацію про розмір часових затримок, є менш консервативними, ніж ті, що не залежать від затримки. Потім отримані умови застосовуються до задачі проектування FT-контролерів. З цієї причини досліджується нове наближення одноразового підсумовування, що з'являється у функціоналі Ляпунова-Красовського (LKF), за допомогою інноваційної нерівності на основі вільних матриць (FMBI). Це призвело до пропозиції нових умов у вигляді LMI, які забезпечують FTS. Хоча деякі результати покращують критерії стійкості, FTS (теорія точного розподілу) отримала мало уваги, і можна отримати більше результатів для зменшення консерватизму. Це є ключовим елементом нашого дослідження. Щоб проілюструвати потенційну вигоду від використання цього нового підходу, наведено детальний числовий приклад. Нарешті, запропоновано та вирішено за допомогою MATLAB менш консервативний проект на основі LMI, який показав дуже хороші результати.

Ключові слова: Стійкість у скінченному часі, Дискретні системи, Інтеграл на основі вільних матриць.