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REGULAR ARTICLE

Free Vibration Analysis of a Functionally Graded Nanoplate (Al_2O_3/Al): Parametrically Analyzed

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This research focuses on the parametric analysis of the free vibrations of a functionally graded material (FGM) nanoplate made of (Al_2O_3/Al), the material properties of the FGM nanoplate are graded only in the thickness direction. The four-unknown shear deformation theory incorporated in Eringen's nonlocal elasticity theory is employed to deduce the equations of motion from Hamilton's principle. The solutions of simply supported FGM nanoplates are obtained and the results are com-pared with those available in the literature. Detailed numerical analysis is performed to demonstrate the influences of some parameters like nonlocal parameter, aspect ratio, and side-to-thickness ratio on the behavior of FGM nanoplates.

Keywords: Nonlocal elasticity theory, Parametric analysis, FGM nanoplate, Hamilton's principle

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1. INTRODUCTION

Functionally graded materials (FGMs) plates are of-ten composed of a blend of metal and ceramic components, with the volume fractions of the two materials changed to the appropriate extent between the two sides. The characteristics of the plate differ between interfaces.

Nanostructures, such as nanoplates and nanobeams, are now engineering structures and are used in nanoelectromechanical (NEMS) and microelectronmechanical (MEMS) systems. It is essential to take into consideration the small-scale impact of FGMs utilized in nanodevices. Furthermore, it can be seen from the experimental data that ignoring these effects leads to inaccurate answers, and thus, improper de-signs [1]. Since classical continuum theories lack internal length scales, they are no longer useful for under-standing the behavior of FGM nanodevices. As a result, a large number of theoretical and experimental studies have been conducted. Many nonlocal theories have been developed to overcome this issue by introducing an intrinsic length scale in the constitutive relations and characterizing the size effect in micro, and nanoscale structures. Examples of these theories include the micropolar theory [2] and the nonlocal elasticity theory [3]. Among these theories, surface tension fluids, dislocation mechanics, fracture mechanics, lattice dispersion of elastic waves, wave propagation in composites, and Eringen's nonlocal elasticity theory were all studied [4]. This theory was created to account for the scale effect in elasticity.

Aghbabaei [5] investigated the bending and free vibration of a simply supported rectangular nanoplate analytically using a third-order shear deformation plate theory. Messas [6] discussed the analyzing vibration behavior of Nano FGM (Si3N4/SUS304) plates and the impact of homogenization models Nano parameters. Nami [7] used nonlocal trigonometric shear deformation theory to study the static behavior of rectangular nanoplates. A non-polynomial four-variable refined plate theory was proposed by Meftah [8] for the free vibration of thick, rectangular plates that are functionally graded on an elastic foundation. A new high-order theory for buckling temperature analysis of functionally graded sandwich plates resting on elastic foundations was presented by Chitour [9]. The impact of mechanical and geometric properties on the thermal buckling of functionally graded sandwich plates was investigated by Berkia [10]. Meftah [11] presented the bending and buckling analysis of functionally graded plates using a new shear strain function with reduced unknowns. Belkorissat [12] studied the free vibration properties of functionally graded nanoplate using a new nonlocal refined four-variable model.

In the present parametric analysis, the free vibration characteristics of FG nanoscale plates are studied using a nonlocal hyperbolic plate theory.

2. NONLOCAL ELASTICITY THEORY

In an elastic continuum, the stress field at point x depends not only on the strain field at the point (hyperelastic case) but also on strains at every other point in the body, according to [3, 4]. Eringen credited experimental

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studies of phonon dispersion and the atomic theory of lattice dynamics for this fact. Consequently, we may represent the nonlocal stress tensor components σ at position x as:

$$\sigma = \int_v \alpha\left(\left|x' - x\right|, \tau\right) t(x') dx' \tag{2.1}$$

where the nonlocal modulus is represented by the kernel function $\alpha(|x' - x|, \tau)$, where $|x' - x|$ is the distance (in Euclidean norm) and τ is a material constant that depends on internal and external characteristic lengths (such as the wavelength and lattice spacing, respectively). where $t(x)$ are the components of the classical macroscopic stress tensor at point x. The nonlocal constitutive equation given in the integral form (see Eq. (2.1)) can be expressed in an analogous differential form as demonstrated by Eringen [4].

$$\left(1 - \tau^2 L^2 \nabla^2\right)\sigma = t,\ \tau^2 = \frac{\mu}{L^2} = \left(\frac{e_0 \bar{a}}{L}\right)^2 \tag{2.2}$$

where the internal and external characteristic lengths are denoted by $\bar{\alpha}$ and L, respectively, and $\mu = (e_0\bar{\alpha}$, e_0 is a material constant.

3. MATERIAL PROPERTIES OF FGM

An FGM nanoplate, as seen in Fig. 1, is investigated in this paper. The volume percentage of two substances can be computed using the following formulas.

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^p,\ V_m = 1 - \left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{3.1}$$

where h is the thickness, and p is a component that affects how the volume fraction varies. The following formula is used to calculate the material properties at any position along the nanoplate.

$$P(z) = P_m V_m + P_c V_c \tag{3.2}$$

where P_c and P_m stand for properties of ceramics and metals, respectively, such as Poisson's ratio, mass density, and Young's modulus.

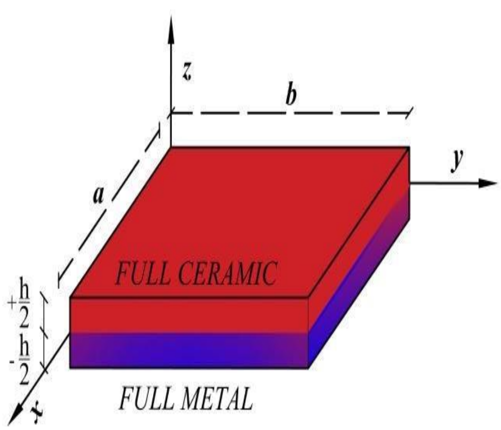

Fig. 1 – Geometric of FG nanoplate model

4. GOVERNING EQUATIONS

4.1 The Displacement Field

The displacement field of the higher-order shear deformation theory (HSDT) can be written as:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial \varphi}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial \varphi}{\partial x} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{4.1}$$

where u_0, v_0 and w_0 are midplane displacements, φ is the rotation of normal to the midplane of the plate. $f(z)$ represents the mode shapes determining the thickness-dependent stress and transverse deformation distributions, written as:

$$f(z) = \sin\left(\frac{\pi}{h}.z\right) + \left(\frac{\pi}{2.h}.z\right), \text{and } g(z) = 1 - \frac{df}{dz}. \tag{4.2}$$

4.2 The Nonlocal Constitutive Relations

For elastic FG nanoplate, the two-dimensional nonlocal constitutive relations can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} - \mu\nabla^2 \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \Upsilon_{yz} \\ \Upsilon_{xz} \\ \Upsilon_{xy} \end{Bmatrix} \tag{4.3}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator in a two-dimensional Cartesian coordinate system. In which $\left(\sigma_{xx}, \sigma_{yy}, \tau_{yz}, \tau_{xz}, \tau_{xy}\right)$and $\left(\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{yz}, \gamma_{xz}, \gamma_{xy}\right)$ are the stresses and strains components, respectively. The stiffness coefficients, C_{ij}, can be expressed as:

$$\begin{aligned} Q_{11}(z) &= Q_{22}(z) = \frac{E(z)}{1 - \left(v(z)\right)^2}, Q_{12}(z) = v(z) Q_{11}(z), \\ Q_{44}(z) &= Q_{55}(z) = Q_{66}(z) = \frac{E(z)}{2\left(1 + v(z)\right)} \end{aligned} \tag{4.4}$$

4.3 Equations of Motion

Hamilton's principle is herein used to derive the equations of motion:

$$\delta \int_0^t \left(U - K\right) dt = 0 \tag{4.5}$$

U: the deformation energy; K: the kinetic energy of the FGM nanoplate.

4.4 Analytical Solution for Simply-Supported FG Nanoplates

Navier's procedure, based on the double Fourier series, is used to solve the partial differential equation under specified boundary conditions, expressing the solution of displacement variables.

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} u_{mn}^0 \cos(\alpha x)\sin(\beta y)e^{i\omega t} \\ v_{mn}^0 \sin(\alpha x)\cos(\beta y)e^{i\omega t} \\ x_{mn}^0 \sin(\alpha x)\sin(\beta y)e^{i\omega t} \\ y_{mn}^0 \sin(\alpha x)\sin(\beta y)e^{i\omega t} \end{Bmatrix}, \begin{cases} 0 \le x \le a \\ 0 \le y \le b \end{cases} \quad (4.6)$$

$$\alpha = m\pi / a \text{ and } \beta = n\pi / b \quad (4.7)$$

Where $\left(u_{mn}^0, v_{mn}^0, x_{mn}^0, y_{mn}^0\right)$ are unknown parameters to be determined and ω is the natural frequency. we get the below eigenvalue equation for any fixed value of m and n, for the free vibration problem:

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{13} & s_{23} & s_{33} & s_{34} \\ s_{14} & s_{24} & s_{34} & s_{44} \end{bmatrix} - \lambda w^2 \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{34} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} u_{mn}^0 \\ v_{mn}^0 \\ x_{mn}^0 \\ y_{mn}^0 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.8)$$

In which:

$$\begin{aligned}
&s_{11} = A_{11}\alpha^2 + A_{66}\beta^2, s_{12} = \left(A_{11} + A_{66}\right)\alpha\beta, \\
&s_{13} = -B_{11}\alpha^3 - \left(B_{12} + 2B_{66}\right)\alpha\beta^2, \\
&s_{14} = -B_{11}^s\alpha^3 - \left(B_{12}^s + 2B_{66}^s\right)\alpha\beta^2, \\
&s_{22} = A_{66}\alpha^2 + A_{22}\beta^2, \\
&s_{23} = -B_{22}\beta^3 - \left(B_{12} + 2B_{66}\right)\alpha^2\beta, \\
&s_{24} = -B_{22}^s\beta^3 - \left(B_{12}^s + 2B_{66}^s\right)\alpha^2\beta, \\
&s_{33} = D_{11}\alpha^4 + 2\left(D_{12} + 2D_{66}\right)\alpha^2\beta^2 + D_{22}\beta^4, \\
&s_{34} = D_{11}^s\alpha^4 + 2\left(D_{12}^s + 2D_{66}^s\right)\alpha^2\beta^2 + D_{22}^s\beta^4, \\
&s_{44} = H_{11}^s\alpha^4 + 2\left(H_{12}^s + 2H_{66}^s\right)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2, \\
&m_{11} = m_{22} = I_0, m_{33} = I_0 + I_2\left(\alpha^2 + \beta^2\right), m_{34} = J_2\left(\alpha^2 + \beta^2\right), \\
&m_{44} = K_2\left(\alpha^2 + \beta^2\right), \lambda = 1 + \mu\left(\alpha^2 + \beta^2\right).
\end{aligned} \quad (4.9)$$

The mass inertias of (I_i, J_i, K_i) are described as follows:

$$\begin{bmatrix} I_0 & I_1 & J_1 \\ I_1 & I_2 & J_2 \\ J_1 & J_2 & K_2 \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)\left[\bar{A}\right]dz, \quad \left[\bar{A}\right] = \begin{bmatrix} 1 \\ z \\ f(z) \end{bmatrix} \begin{bmatrix} 1 & z & f(z) \end{bmatrix}. \quad (4.10)$$

$$\left(A, B, D, B^s, D^s, H^s\right) = \int_{-h/2}^{+h/2} \left(1, z, z^2, f, zf, f^2\right) C(z)\, dz \quad (4.11)$$

$$A_{44}^s = A_{55}^S = \int_{-h/2}^{+h/2} g^2(z) C_{44}(z)\, dz. \quad (4.12)$$

5. RESULTS AND DISCUSSION

In this section, the parametrically analyzed free vibration behavior of a simply supported FG nanoplate is discussed, by supposing the top surface of the plate is ceramic-rich (Al_2O_3) and the bottom surface is metal-rich (Al). The mass density ρ and Young's modulus E are: $\rho_c = 380$ kg/m³, $E_c = 380$ GPa for Al_2O_3 and $\rho_m = 2707$ kg/m³, $E_m = 70$ GPa for Al. Poisson's ratio υ is considered to be constant and taken as 0.3 for the current study. The numerical results are presented in graphical and tabular forms using dimensionless quantities for convenience.

$$\bar{\omega}_1 = \omega.h.\sqrt{\frac{\rho_c}{G_c}}, \; \tilde{\omega} = (\omega.a^2 / \pi).\sqrt{\frac{\rho_c}{D_c}}, \; D_c = \frac{E_c.h^3}{12.(1-\nu^2)}. \quad (5.1)$$

Firstly, to confirm the accuracy of the current model, the nanoplate results are calculated and compared with the dimensionless natural frequency results of Malekzadeh [13]. in Table 1. Simply supported homogeneous FG nanoplates ($p = 0$) with different mode values, non-local parameters, plate thickness, and plate aspect ratio are considered. It can be seen that the numerical results presented are in very good agreement with the results of Malekzadeh [13].

Fig. 2 shows a 3D interaction diagram of the power law index (p), nonlocal parameter (mu), side-to-thickness ratio (a/h), and dimensionless natural frequency. It can be seen in this figure that the dimensionless natural frequencies decrease with the increase in the power law index (p), this is due to the fact that a higher value corresponds to a lower value of the volume fraction of the ceramic phase and thus makes the plates softer. He also observed from this figure that for a given value of (p), the dimensionless natural frequency in-creases if the side-to-thickness ratio (a/h) is decreased. the natural frequency does not change too much if the side-to-thickness ratio (a/h) takes large values like ($a/h = 100$).

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Fig. 3 illustrates the effect of the nonlocal parameter (μ) on the dimensionless frequencies of the square FG nanoplate with the volume fraction exponent ($p = 5$) for different values of the thickness parameter (a/h). It can be observed that the dimensionless frequency decreases as the nonlocal parameter increases. Indeed, an increase in the thickness parameter (a/h) leads to a decrease in the frequency of the FG nanoplate. but for a thickness parameter ($a/h > 20$) the graphs become flat.

Table 1 – Comparison of dimensionless natural frequency $(\tilde{\omega})$ simply supported FG nanoplate (a = 10 nm)

a/b	*a/h*	*m, n*	Method	Nonlocal Parameter (μ)				
				0	1	2	3	4
0.5	10	1, 1	[13]	0.058883	0.055556	0.052736	0.050305	/
			Present	0.058887	0.055560	0.052739	0.050309	0.048186
	20	1,1	[13]	0.014965	0.014119	0.013402	0.012785	/
			Present	0.014965	0.014120	0.013403	0.012785	0.012246
1	10	1, 1	[13]	0.093029	0.085016	0.078771	0.073726	/
			Present	0.093039	0.085025	0.078779	0.073734	0.069549
		2, 2	[13]	0.34064	0.25464	0.21212	0.16704	/
			Present	0.340741	0.254713	0.212171	0.185649	0.167097
		3, 3	[13]	0.64400	0.41049	0.32055	0.27184	/
			Present	0.684266	0.410654	0.320681	0.271979	0.240334
	20	1, 1	[13]	0.023864	0.021808	0.020206	0.018912	/
			Present	0.023864	0.021809	0.020207	0.018913	0.017839

Fig. 4 illustrates the effect of the volume fraction exponent on the dimensionless frequencies of the square FG nanoplate with thickness parameter ($a/h = 10$) for different values of the small-scale parameter. It can be observed that the dimensionless frequency decreases as the volume fraction exponent increases. Indeed, an increase in the volume fraction exponent leads to a decrease in the rigidity of the FG nanoplate. There is an abrupt change in the responses as the volume increases, and we also see that increasing the nonlocal parameter results in a decrease in the nondimensional frequency.

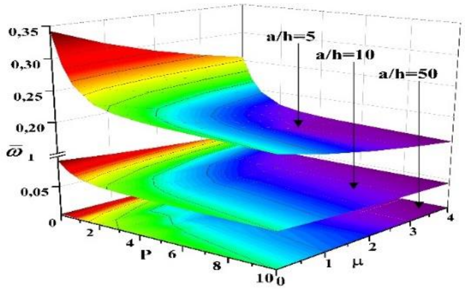

Fig. 2 – Effect of the power-law index (p), nonlocal parameter (μ), and side-to-thickness ratio (a/h) on the non-dimensional natural frequency $(\bar{\omega}_1)$ of simply supported FG nanoplates

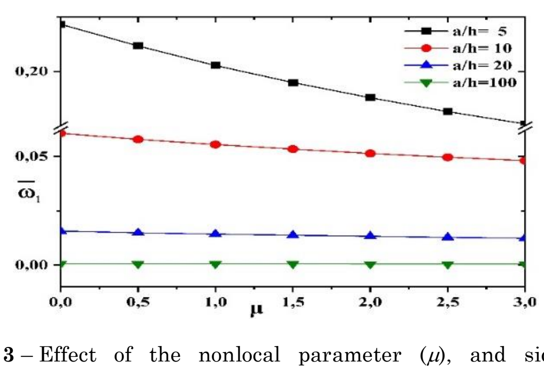

Fig. 3 – Effect of the nonlocal parameter (μ), and side-to-thickness ratio (a/h) on the non-dimensional natural frequency $(\bar{\omega}_1)$ of simply supported FG nanoplates

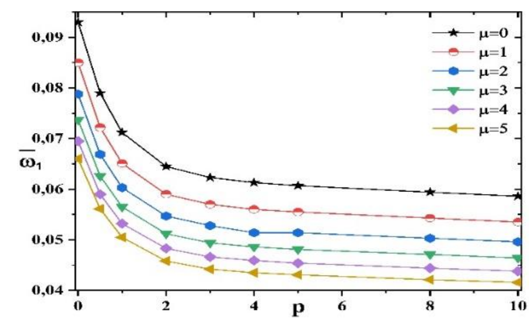

Fig 4 – Effect of the power-law index (p), and nonlocal parameter (μ) on the non-dimensional natural frequency $(\bar{\omega}_1)$ of simply supported FG nanoplates

Fig. 5 illustrates the effect of the nonlocal parameter (μ) on the dimensionless natural frequency ratio, simply supported by a square FG nanoplate for the first three modes with ($a/h = 10$) and volume fraction exponent ($p = 5$), for different values of the parameter on a small scale. It can be observed that the frequency ratio decreases as the nonlocal parameter (μ) increases. and the influence of non-local parameter (μ) is more apparent with the increase in the number of modes, i.e. the frequency ratio is reduced from mode 1 to mode 3.

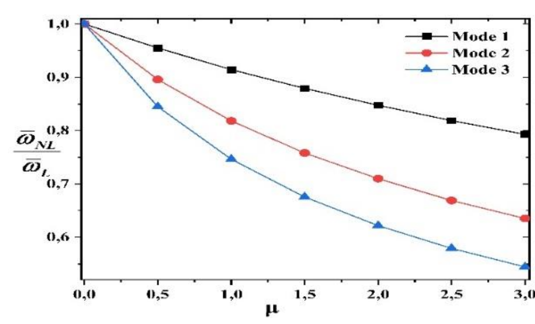

Fig. 5 – Effect of the nonlocal parameter (μ) on dimensionless frequency ratio $\left(\bar{\omega}_{NL}/\bar{\omega}_L\right)$ for a simply supported square FG nanoplate for the first three frequencies with ($a/h = 10$)

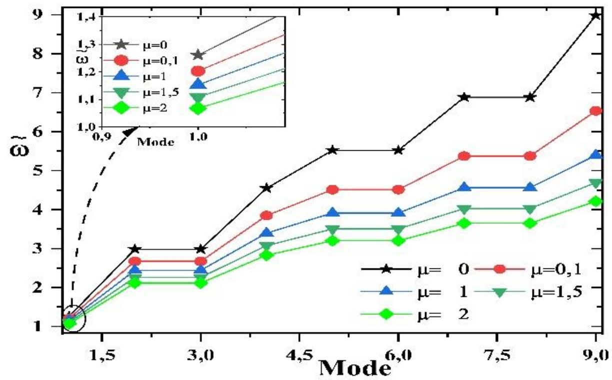

Fig. 6 – The first nine mode shapes of square FG nanoplate (p = 5) and (a/h = 10)

Fig. 6 It is obvious that the variation of the parameter nonlocal parameter (μ) has a significant effect on the higher-order frequencies of the FG nanoplates. Overall, for all cases of nonlocal parameters, the non-dimensional frequencies of the nonlocal FG nanoplates are smaller than those of local ones.

6. CONCLUSION

In this work, the free vibration analysis of FG nanoscale plates is numerically investigated by employing the higher-order hyperbolic shear deformation theory based on the nonlocal differential constitutive relations of Eringen. Hamilton's principle serves as the basis for the motion equations. The natural frequency of FG nanoplates is obtained by analytically solving these equations. The accuracy of the results is investigated by utilizing available data in the literature. It is concluded that various factors, such as the nonlocal scale parameter, the volume fraction exponent, the aspect ratio, and side-to-thickness ratios, play considerable roles in the dynamic response of FG nanoscale plates.

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Аналіз вільної вібрації функціонально градуйованої нанопластини (Al_2O_3/Al): параметричний аналіз

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Це дослідження зосереджено на параметричному аналізі вільних коливань нанопластини з функціонально градуйованого матеріалу (FGM), виготовленої з (Al_2O_3/Al), властивості матеріалу нанопластини FGM градуюються лише в напрямку товщини. Теорія чотирьох невідомих деформацій зсуву, включена в теорію нелокальної пружності Ерінгена, використовується для виведення рівнянь руху з принципу Гамільтона. Отримано розчини нанопластин FGM з простою опорою та порівняно результати з наявними в літературі. Проведено детальний чисельний аналіз, щоб продемонструвати вплив деяких параметрів, таких як нелокальний параметр, співвідношення сторін і відношення сторони до товщини, на поведінку нанопластин FGM.

Ключові слова: Нелокальна теорія пружності, Параметричний аналіз, FGM нанопластинка, Принцип Гамільтона.