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REGULAR ARTICLE

Effects of Nonlocality and Porosity on Frequency Vibration Characteristics of SUS304/Si_3N_4 Functionally Graded Nanoplates

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In this work, the impacts of porosity and nonlocal parameter on the frequency of free vibration behaviors of functionally graded nanoplates are investigated. The porosity distribution is described by the model. Using the higher-order shear deformation theory, which has the four unknowns, the displacement field of the nanoplates is defined. The governing equations of the motion of the functionally graded nanoplates are established by utilizing the nonlocal elasticity theory in conjunction with Hamilton's principle. A thorough investigation is conducted into the effects of several parameters, including porosity and nonlocal parameter, on the free vibration behaviors of functionally graded porous nanoplates. Navier's method is used to determine the closed-form solutions, and the solutions to the eigenvalue problems are then solved to give fundamental frequencies. The outcomes of the current studies are provided and contrasted with those found in the literature.

Keywords: Nonlocal theory, Porous functionally graded nanoplates, Porosity, Navier's method

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1. INTRODUCTION

The domains of mechanical, civil, and aerospace engineering have recently shown a great deal of interest in metal foams, as they are typically porous materials [1]. Pore distributions were the only thing studied in the early stages of metal foams. Thus, increased focus has been placed on the creation of functionally graded (FG) porous composite materials to attain the required me-chanical properties by adjusting the size and density of interior pores in one or more directions [2]. With their exceptional mechanical qualities, including damage tolerance, these materials can be employed extensively in energy absorption systems, heat exchangers, noise absorbers, building materials, etc. [3]

Consequently, it overestimates the behaviors of nanostructures. [4] created the nonlocal elasticity theory, which is based on continuum mechanics theory, and included a length scale parameter to address these drawbacks which consists of small size effects with good accuracy to nanostructures, into the constitutive equations. [5] studied an improved four-variable plate model for thermal buckling characterization of FG nanoplates under uniform temperature distributions. The introduction of the generalized differential quadrature method for the vibration of two-dimensional imperfect functionally graded porous nano-/micro-beams was presented [6]. [7-9] examined the characteristics of advanced smart sandwich plates in terms of thermo-electro-mechanical, free vibration, and buckling, taking into account the impact of porosity. The specific material parameters and porosity are assumed to be the determinants of Young's modulus, mass density, and Poisson's ratio in these examinations. A nonlocal zeroth-order shear deformation theory for the free vibration of functionally graded nanoplates was developed [10]. The buckling behavior of FGM sandwich nanoplates with porosity was examined in [11] using non-local strain gradient theory to examine the impact of heat conduction.

Using a high-order non-local shear deformation theory, the free vibration of a non-local FG nanoplate with porosity is analyzed in this article. Porosity and non-local parameters' impacts on free vibration in FG nanoplates are all examined. It is evident how important the hypothesis is by contrasting the available data with potential fixes.

2. MODELS BOTH MATHEMATICAL AND THEORETICAL

A rectangular FGM nanoplate with porosity is taken into consideration. The plate's dimensions are $a \times b$ and its thickness is h. The center surface of the FGM nanoplates is where the Cartesian coordinates x, y, and z are put, as illustrated in Fig. 1.

In non-local theory, the stress tensor at point x of a physical system is related to the stress tensor in the surrounding environment using an integral equation. A particular type of non-local constitutive link was de-fined by Eringen as follows [4, 12, 13]:

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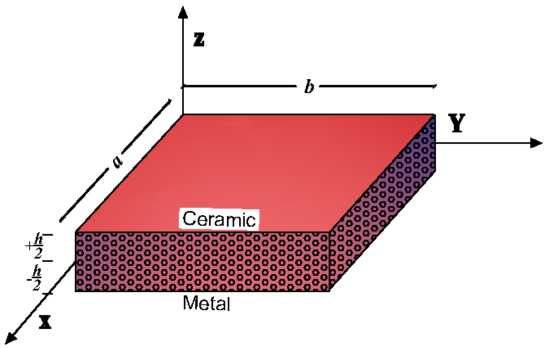

Fig. 1 – The geometric model of FGM porous nanoplates

$$\sigma = \int_v \alpha\left(\left|x' - x\right|, \tau\right) t(x') dx' \qquad (2.1)$$

where the nonlocal modulus is represented by the kernel function $\alpha(|x' - x|, \tau)$, where $|x' - x|$ is the distance (in Euclidean norm) and τ is a material constant that depends on internal and external characteristic lengths (such as the wavelength and lattice spacing, respectively). where $t(x)$ are the components of the classical macroscopic stress tensor at point x. The nonlocal constitutive equation given in the integral form (see Eq. (2.1)) can be expressed in an analogous differential form as demonstrated by Eringen [4].

$$\left(1 - \tau^2 L^2 \nabla^2\right)\sigma = t,\ \tau^2 = \frac{\mu}{L^2} = \left(\frac{e_0 \bar{a}}{L}\right)^2 \qquad (2.2)$$

where the internal and external characteristic lengths are denoted by $\bar{a}$ and L, respectively, and $\mu = (e_0\bar{a})$, e_0 is a material constant.

The Young's modulus $E(z)$ and material density $\rho(z)$ equations of the FG nanoplate with a porosity can be expressed as:

$$E(z) = E_m(1 - ((\frac{1}{2} + \frac{z}{h})^p) - \frac{\zeta}{2}) + E_c(1 - ((\frac{1}{2} + \frac{z}{h})^p) - \frac{\zeta}{2}) \qquad (2.3)$$

$$\rho(z) = \rho_m(1 - ((\frac{1}{2} + \frac{z}{h})^p) - \frac{\zeta}{2}) + \rho_c(1 - ((\frac{1}{2} + \frac{z}{h})^p) - \frac{\zeta}{2}) \qquad (2.4)$$

where $\xi \le 0.5$ is the maximum porosity value[14].

In the present analysis, the shear deformation plate theory is suitable for the displacements:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial \varphi}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial x} - f(z)\frac{\partial \varphi}{\partial x} \\ w(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \qquad (2.5)$$

Where u_0, v_0, and w_0 are mid-plane displacements, φ is the rotation of normal to the midplane of the plate. $f(z)$ represents the mode shapes determining the thick-ness-dependent stress and transverse deformation distributions, written as:

$$f(z) = \sin\left(\frac{\pi}{h}.z\right) + \left(\frac{\pi}{2.h}.z\right), \text{and } g(z) = 1 - \frac{df}{dz}. \qquad (2.6)$$

The strain is calculated by:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} - z\begin{Bmatrix} \frac{\partial^2 w_b}{\partial x^2} \\ \frac{\partial^2 w_b}{\partial y^2} \\ 2\frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} + f(z)\begin{Bmatrix} \frac{\partial^2 w_s}{\partial x^2} \\ \frac{\partial^2 w_s}{\partial y^2} \\ 2\frac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix} \qquad (2.7)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \frac{df(z)}{dz}\begin{Bmatrix} \frac{\partial w_s}{\partial y} \\ \frac{\partial w_s}{\partial x} \end{Bmatrix}, \varepsilon_z = 0$$

For elastic FG nanoplate, the two-dimensional non-local constitutive relations can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} - \mu\nabla^2\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \qquad (2.8)$$

two-dimensional Cartesian coordinate system, in which $\left(\sigma_{xx}, \sigma_{yy}, \tau_{yz}, \tau_{xz}, \tau_{xy}\right)$ and $\left(\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{yz}, \gamma_{xz}, \gamma_{xy}\right)$ are the stresses and strains components, respectively. The stiffness coefficients, C_{ij}, can be expressed as:

$$\begin{aligned} Q_{11}(z) &= Q_{22}(z) = \frac{E(z)}{1 - \left(v(z)\right)^2}, Q_{12}(z) = v(z)Q_{11}(z), \\ Q_{44}(z) &= Q_{55}(z) = Q_{66}(z) = \frac{E(z)}{2\left(1 + v(z)\right)} \end{aligned} \qquad (2.9)$$

To obtain the equations of motion, Hamilton's theory is used herein. The theory can be described as Reddy [15] in empirical terms.

$$\delta\int_0^t (U - K)\, dt = 0 \qquad (2.10)$$

U: the deformation energy; K: the kinetic energy of the FGM nanoplate.

3. ANALYTICAL SOLUTION FOR SIMPLY-SUPPORTED FG NANOPLATES

In order to use the Navier solution process, we assume the following kind of solution (u, v, w_b, w_s).

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} u_{mn}\cos(\alpha x)\sin(\beta y)e^{i\omega t} \\ v_{mn}\sin(\alpha x)\cos(\beta y)e^{i\omega t} \\ w_{bmn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \\ w_{smn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \end{Bmatrix}, \begin{cases} 0 \le x \le a \\ 0 \le y \le b \end{cases} \qquad (3.1)$$

$$\alpha = m\pi / a \text{ and } \beta = n\pi / b \tag{3.2}$$

Where $(u_{mn}, v_{mn}, w_{bmn}, w_{smn})$ are unknown parameters to be determined, and ω is the natural frequency we get the below eigenvalue equation for any fixed value of m and n, for the free vibration problem:

$$\left(\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} - \lambda w^2 \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & M_{34} \\ 0 & 0 & M_{34} & M_{44} \end{bmatrix}\right) \begin{Bmatrix} u_{mn} \\ v_{mn} \\ w_{bmn} \\ w_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{3.3}$$

4. RESULTS AND DISCUSSION

In this section, the parametrically analyzed free vibration behavior of a simply supported FG nanoplate is discussed, by supposing the top surface of the plate is ceramic-rich (Si_3N_4) and the bottom surface is metal-rich (SUS_30_4). The mass density ρ and Young's modulus E are: $\rho_c = 2370$ kg/m^3, $E_c = 348.43$ GPa for Si_3N_4, and $\rho_m = 8166$ kg/m^3, $E_m = 201.04$ GPa for SUS_30_4. Poisson's ratio υ is considered to be constant and taken as 0.3 for the current study. The numerical results are presented in graphical and tabular forms using dimensionless quantities for convenience.

$$\Omega = \omega . h . \sqrt{\frac{\rho_c}{G_c}}, \tag{4.1}$$

In order to check the accuracy and convergence of the proposed method, the numerical results of the free vibration of the nanoplate are compared with the other published results. table 1 shows the first two non-dimensional natural frequencies of a square nanoplate without porosity ($\xi = 0$); $a/h = 10$, with different mode values, nonlocal parameters, and power-law index (p). It can be seen that the numerical results presented are in very good agreement with the results of Phung et al [16].

Table 2 examined the effect of porosity ($\xi = 0.4$) on the free vibration of the square FG nanoplate. The power law index p takes the values 0, 5, and 10, the a/h ratio takes the values 5, 20, and 50 and the nonlocal parameter (μ) takes the values 0 to 4.

Table 1 – The first two non-dimensional natural frequencies Ω of a square nanoplate with $a/h = 10$ and $\xi = 0$

p	**Method**	**Mode1**				**Mode2**			
		μ				**μ**			
		0	**1**	**2**	**4**	**0**	**1**	**2**	**4**
2	[16]	0.0485	0.0443	0.0410	0.0362	0.1154	0.0944	0.0819	0.0669
	Present	0.0488	0.0446	0.0414	0.0365	0.1165	0.0953	0.0827	0.0676
10	[16]	0.0416	0.0380	0.0352	0.0311	0.0990	0.0810	0.0702	0.0574
	Present	0.0419	0.0383	0.0355	0.0313	0.0996	0.0816	0.0707	0.0578

Table 2 – The first natural frequency Ω of a square nanoplate with $\xi = 0.4$

p	**a/h**	**μ**							
		0	**0.5**	**1**	**1.5**	**2.5**	**3**	**3.5**	**4**
0	5	0.8513	0.8123	0.7784	0.7477	0.6968	0.6751	0.6548	0.6365
	20	0.0593	0.0566	0.0542	0.0521	0.0485	0.0470	0.0456	0.0443
	50	0.0096	0.0091	0.0087	0.0084	0.0078	0.0076	0.0074	0.0071
5	5	0.1390	0.1325	0.1270	0.1220	0.1137	0.1101	0.1068	0.1039
	20	0.0097	0.0093	0.0089	0.0086	0.0080	0.0077	0.0075	0.0073
	50	0.0016	0.0015	0.0014	0.0014	0.0013	0.0012	0.0012	0.0012
20	5	0.1209	0.1152	0.1104	0.1062	0.0988	0.0958	0.0930	0.0903
	20	0.0085	0.0081	0.0077	0.0074	0.0069	0.0067	0.0065	0.0063
	50	0.0014	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010

We examine the FG nanoplate, we can say that the increase in the ratio (a/h) remarkably leads to the reduction of the non-dimensional frequency, the displacement of the ceramic phase ($z = h/2$, $p = 0$) towards the metallic phase ($z = -h/2$, $p >> 0$) also causes the reduction of the non-dimensional frequency. we also notice that the increase in the non-local parameter causes the decrease in frequency.

Fig. 2 indicates the distributions of Young's modulus of SUS_30_4/Si_3N_4 across the thickness of the nanoplates. I can observe that Young's modulus without the porosities ($\xi = 0$) is continuous across the upper surface (ceramic-rich) and the lower surface (metal-rich). The porous effect on Young's modulus is also carried out on the curves where the porosity is equal ($\xi = 0.1$, 0.2, 0.3, and 0.4). The shapes of the Young's modulus curves shown in this figure are the same except that a decrease in the magnitude of the Young's modulus also results in a decrease in the stiffness of the plate.

The density patterns of SUS_30_4/Si_3N_4 across the nanoplate thickness are displayed in Fig. 3. I can see that on both the bottom (metal-rich) and upper (ceramic-rich) surfaces, the density without porosities ($\xi = 0$) is continuous. On the curves when porosity is equal ($\xi = 0.1$, 0.2, 0.3, and 0.4), the porosity influence on density also manifests itself, except for a decrease in the porosity's amplitude, the density curve shapes shown in this image are the same.

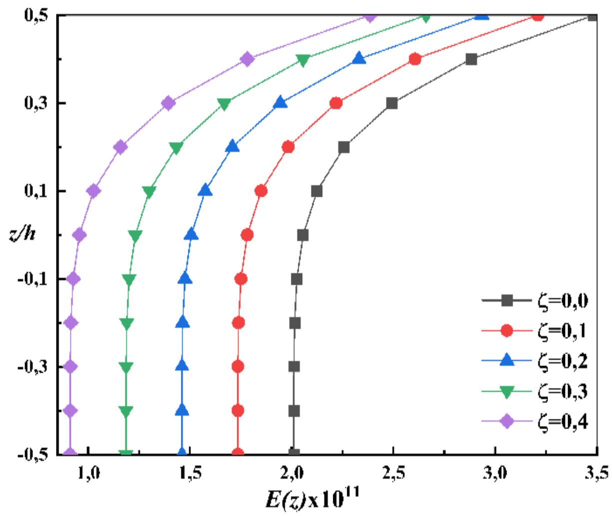

Fig. 2 – Young's modulus of porous SUS_{304}/Si_3N_4, with $a = 10$, $h = a/10$ and $p = 5$

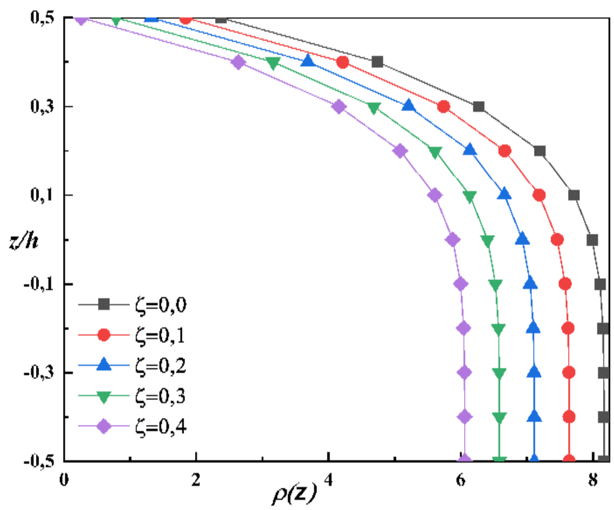

Fig. 3 – Density of porous SUS_{304}/Si_3N_4, with $a = 10$, $h = a/10$ and $p = 5$

Fig. 4 shows the influence of the porosity on the non-dimensional natural frequency of the porous FG nanoplates. the effect of porosity on the nanoplate frequencies is weak, on the other hand, the influence of non-local parameters on the nanoplate is very important, and this is illuminated by the curves in this figure for ($\mu = 0$) that is- i.e. local elasticity, the non-dimensional frequency takes on a large value, and as long as (μ) increases to the value 4, the non-dimensional frequencies will decrease.

Fig. 5 illustrates the effect of the nonlocal parameter (μ) on the dimensionless natural frequency ratio, simply supported by a square FG nanoplate for the first three modes with ($a/h = 10$) and volume fraction exponent ($p = 5$), It is clearly that the non-local parameter has a very important effect on the frequencies of nanoplate, on the other hand, the porosity its influence on the frequencies remains weak.

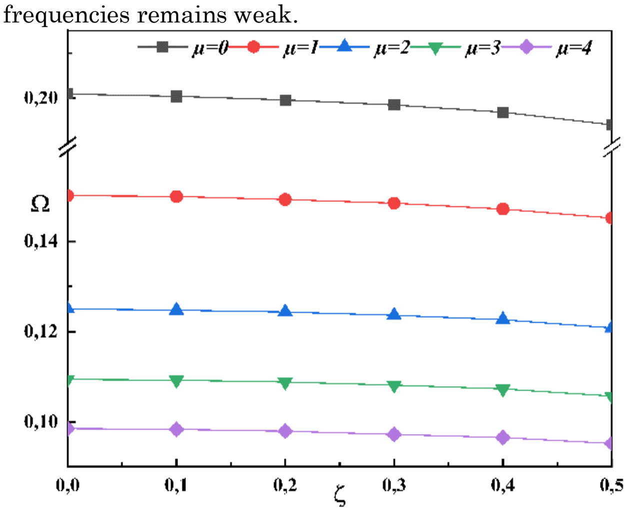

Fig. 4 – The influence of the porosity (ξ) on the non-dimensional natural frequency (Ω) of the porous FG nanoplates

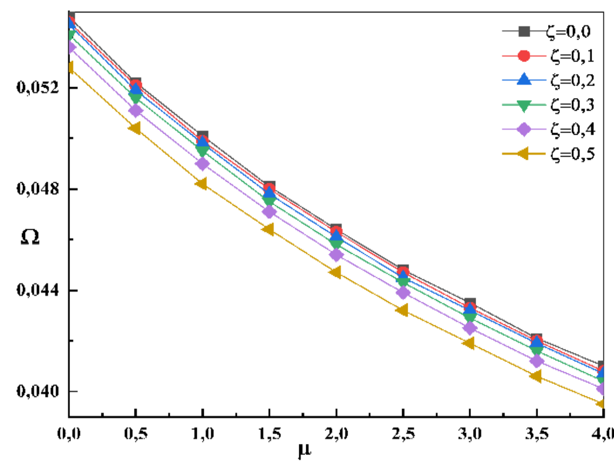

Fig. 5 – The influence of the nonlocal parameter (μ) on the non-dimensional natural frequency (Ω) of the porous FG nanoplates

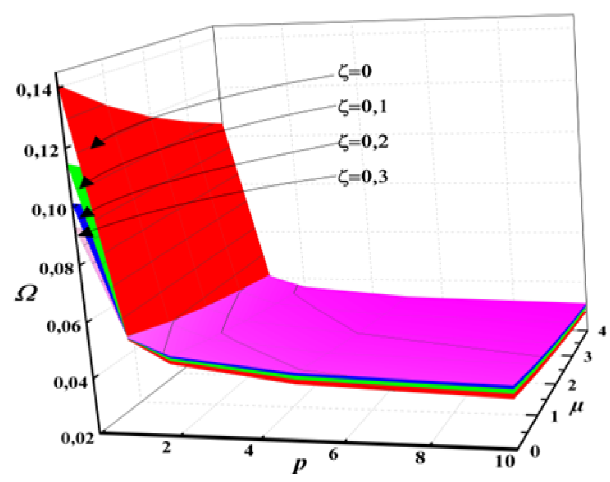

Fig. 6 – Effect of the power-law index (p), the nonlocal parameter (μ) on dimensionless frequency (Ω) for a simply

supported square FG nanoplate

Fig. 6 presents the interaction of the nonlocal parameter, the power index, and the porosity. for a homogeneous plate ($p = 0$), perfect ($\xi = 0$), and a local theory ($\mu = 0$) in this case the non-dimensional frequencies will take a maximum value as indicated in the figure, on the other hand, if one of the parameters takes a value greater than zero, non-dimensional frequencies will take a value less than that cited previously.

As long as the power index tends towards infinity, i.e. the nanoplate becomes rich in metal, the non-dimensional frequency curve becomes flat.

5. CONCLUSION

A comprehensive study on the effects of porosity and non-local parameter on the free vibration of the porous FG nanoplates has been presented. based on high-order shear deformation and non-local theory. The findings obtained from the study of dimensionless frequencies are quite close to those expected in the literature. From the results obtained by the present method, there is remarkable and good agreement with other results.

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Вплив нелокальності та пористості на характеристики частотної вібрації SUS_30_4/Si_3N_4 функціонально градуйованих нанопластин

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У даній роботі досліджено вплив пористості та нелокального параметра на частоту вільних коливань функціонально градуйованих нанопластин. Розподіл пористості описується моделлю. Використовуючи теорію зсуву вищого порядку, яка має чотири невідомі, визначається поле зміщення нанопластин. Керівні рівняння руху функціонально градуйованих нанопластин вста-новлюються шляхом використання теорії нелокальної пружності в поєднанні з принципом Гаміля-тона. Проведено ретельне дослідження впливу кількох параметрів, включаючи пористість і нелокальний параметр, на поведінку вільної вібрації функціонально градуйованих пористих нанопластин. Метод Нав'є використовується для визначення розв'язків закритої форми, а розв'язки задач на власні значення потім вирішуються, щоб отримати основні частоти. Результати поточних досліджень надано та зіставлено з тими, що містяться в літературі.

Ключові слова: Нелокальна теорія, пористі функціонально градуйовані нанопластини, пористість, метод Нав'є.