Journal of Nano- and Electronic Physics
Vol. **17** No 4, 04015(6pp) (2025)

Журнал нано- та електронної фізики
Том **17** № 4, 04015(6cc) (2025)

OPEN ACCESS

REGULAR ARTICLE

Finite-time Stabilization of Delayed Uncertain Systems Using a Novel Integral Inequality Approach

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(Received 18 April 2025; revised manuscript received 22 August 2025; published online 29 August 2025)

Some new delay-dependent finite-time stability (FTS) conditions are provided and applied to the design problem of FT controllers. First, based on a new integral inequality and a simple Lyapunov-Krasovskii Functional (LKF), delay-dependent FTS criteria are proposed by introducing some free-weighting matrices. Thus, a new approximation of the unique integral that appears in the LKF derivative is proposed using an integral inequality, called free-matrix-based integral inequality (FMII). Then, memoryless and memory state-feedback controllers (MSC and MC) are designed to ensure FTS of delay-dependent uncertain systems, which are less conservative than others found in the literature. Although some results improve the stability criteria, FTS has received little attention, and more results can be attained to reduce the conservatism. That is the keystone of our research. The time-varying delays are bounded and differentiable with upper bound of delay derivatives. Also, the sufficient conditions obtained in this paper are established in terms of Linear Matrix Inequalities (LMIs) to achieve the desired performance. To illustrate the potential gain of employing this new approach, a detailed numerical example is provided. Finally, a less conservative LMI-based design is proposed and solved with MATLAB showing very good results.

Keywords: Finite-time stability (FTS), Uncertain systems, Free-matrix-based integral inequality (FMII).

DOI: 10.21272/jnep.17(4).04015 PACS number: 07.05.Kf

1. INTRODUCTION

In order to contribute to the stability of the system and reduce conservatism, a variety of approaches have been reported in the literature [1, 2, 3]. Then, all most studies focused on LMI stability and stabilization conditions, that is defined over an infinite-time interval. However, in practice, the interest is often concerned with the behavior of the system over a specific time interval. The FTS method is then introduced in this case. A system is said to be finite-time stable if, at a certain time interval, its state does not exceed some bounds. This stability concept dates back to the 1950s [4, 5], when the term FTS was introduced for the first time. Then, important results are obtained for various sorts of systems such as linear time-varying systems [6, 7, 8], linear systems with additive time-varying delay [9], discrete-time systems [10], neural network systems [11], T-S Fuzzy systems [12, 13], and impulsive systems [14]. On the other hand, the stability/stabilization criteria can be reduced to the feasibility of a set of LMIs. In general, when the feedback gains have been processed as variable parameters in the LMI feasibility issue, automatic stabilizing control is generated from a set of obtained LMIs. Then, for the performance improvement and the conservative reduction of results over a finite-time interval, a new analysis and

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design technique is proposed in this paper.

Motivated by these observations, a new FT form is provided and applied to the design of MSC and MC. The purpose of the paper is then to guarantee the FTS and FT stabilizability of closed-loop delayed systems despite the uncertainties. These results are based on a simple LKF, an FMII, a new approximation of the unique integral, and some free-weighting matrices. Finally, less conservative LMI-based design conditions are proposed and solved by the LMI Tools of MATLAB to show the effectiveness of the proposed approach.

2. PROBLEM FORMULATION

Consider the following system:

$$\dot{x}(\mathrm{t}) = \bar{A}x(\mathrm{t}) + \bar{A}_d\mathrm{x}(\mathrm{t} - \mathrm{h}(\mathrm{t})) \qquad (1)$$

where $x(t) \in \Re^n$ (state vector), $A, Ad \in \Re^{n\times n}$ (constant matrices), and the delay $0 \le h(t) \le h$, $\dot{h}(t) \leqslant h_D$.

The uncertain matrices are given by:

$$\bar{A} = A + \Delta A(t)), \bar{A}_d = A_d + \Delta A_d(t)$$

where $[\Delta A(t)\ \Delta A_d(t)] = E\Sigma(t)[F\ Fd]$ and $\Sigma(t)$ is an uncertain matrix function that satisfies $\Sigma^T(t)\Sigma(t) \le I_P$.

Definition 1. [8]. The system (1) is said to be FT stable according to (c_1, c_2, T, R), where $0 \le c_1 < c_2$, if

$$x^T(\mathrm{t})\mathrm{x}(\mathrm{t}) < c_2, \forall t \in [0,T]: sup_{\tau\in[-\mathrm{h},0]}\varphi^T(\tau)\varphi(\tau) < c_1$$

Lemma 1. [15]. *Let* $\Theta = \Theta^T, \bar{E}$, and $\bar{F}$ be appropriately dimensioned matrices (ADM). Then, the condition $\Theta + \bar{E}\Sigma(t)\bar{F} + (\bar{E}\Sigma(t)\bar{F})^T < 0$ holds if there exists a scalar $\varepsilon > 0$ such that the inequality $\begin{bmatrix} \Theta & \varepsilon\bar{E} & \bar{F}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} > 0$ is satisfied.

Lemma 2. [1]. For an ADM $R > 0$ and a vector function $x: [a, b] \to \Re^n$, the following inequality holds:

$$\left(\int_a^b x(s)ds\right)^T R \left(\int_a^b x(s)ds\right) \le (b-a)\int_a^b x^T(s)Rx(s)ds$$

Lemma 3. [16]. For ADM $R \in \Re^{n\times n}, Y \in \Re^{2n\times n}, X \in \Re^{2n\times 2n}, \begin{bmatrix} X & Y \\ * & R \end{bmatrix} \ge 0$, and a vector function $x: [a, b] \to \Re^n$, the following inequality holds:

$$-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \le \begin{bmatrix} x(b) \\ x(a) \end{bmatrix}^T [\mathrm{sym}(Y[I - I]) + (b-a)X] \begin{bmatrix} x(b) \\ x(a) \end{bmatrix}$$

Lemma 4. [17]. For an ADM $R > 0$ and a vector function $x{:}[a, b] \to \Re^n$, the following inequality holds:

$$-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \leqslant \frac{1}{b-a}\varpi^T\hat{\Omega}\varpi,$$

$$\hat{\Omega} = \begin{bmatrix} -4R & -2R & 6R \\ * & -4R & 6R \\ * & * & -12R \end{bmatrix}, \varpi = [x^T(b)\ x^T(a)\ \tfrac{1}{b-a}\int_a^b x^T(s)ds]^{\mathrm{T}}$$

Lemma 5. [18]. For symmetric positive definite matrices (SPDM) $R \in \Re^{n\times n}, Z_1, Z_3 \in \Re^{3n\times 3n}$, ADM $Z_2 \in \Re^{3n\times 3n}, N, M \in \Re^{3n\times n}$ satisfying $\begin{bmatrix} Z_1 & Z_2 & N \\ * & Z_3 & M \\ * & * & R \end{bmatrix} \ge 0$, and a vector function $x{:}[a, b] \to \Re^n$, the following inequality holds:

$$-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \le \varpi^T\Omega\varpi,$$

$$\varpi = \left[x^T(b)x^T(a)\tfrac{1}{b-a}\int_a^b x^T(s)ds\right]^T,$$
$$\Omega = (b-a)\left(Z_1 + \tfrac{1}{3}Z_3\right) + \mathrm{sym}(N\Pi_1 + M\Pi_2),$$
$$\Pi_1 = [I \quad -I \quad 0]; \Pi_2 = [-I \quad -I \quad 2I]$$

Lemma 6. [19]. For a SPDM $R \in \Re^{n\times n}$ and ADM L, M, the following inequality holds:

$$-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \le \varpi^T\Omega\varpi,$$

$$\varpi = \left[x^T(b)x^T(a)\tfrac{1}{b-a}\int_a^b x^T(s)ds\right]^T,$$
$$\Omega = (b-a)\left(LR^{-1}L^T + \tfrac{1}{3}MR^{-1}M^T\right) + \mathrm{sym}(N\Pi_1 + M\Pi_2)$$

Lemma 7. [6]. For SPDM $R \in \Re^{n\times n}, Z_1, Z_4, Z_6 \in \Re^{3n\times 3n}$, ADM $Z_2, Z_3, Z_5 \in \Re^{3n\times 3n}, N, M \in \Re^{3n\times n}$ satisfying the condition $\begin{bmatrix} Z_1 & Z_2 & Z_3 & N \\ * & Z_4 & Z_5 & M \\ * & * & Z_6 & M \\ * & * & * & R \end{bmatrix} \ge 0$, and a vector function $x{:}[a, b] \to \Re^n$, the inequality $-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \le \varpi^T\Omega\varpi$ holds for $k \in \{n/n = 2m + 1; m \in \mathbb{Z}^{*+}\}$ where

$$\varpi = \left[x^T(b)x^T(a)\tfrac{1}{b-a}\int_a^b x^T(s)ds\right]^T,$$

$$\Omega = (b-a)(Z_1 + g_1Z_4 + g_2Z_6 + \mathrm{sym}(N\Pi_1 + M\Pi_2 + g_3Z_5),$$

$$g_1 = (b-a)\left(\tfrac{1}{2k+1} + \tfrac{2}{k+2} + \tfrac{1}{3}\right),\ g_2 = \tfrac{b-a}{2k+1},$$

$$g_3 = (b-a)\left(-\tfrac{1}{2k+1} - \tfrac{1}{k+2}\right)$$

Lemma 8. [6]. The system (1) is FT stable according to (c_1, c_2, T, R) if there exist SPDM $Q_1, Q_2 (Q_1 > Q_2), Z_1, Z_4, Z_6 \in \Re^{3n\times 3n}$, ADM $Z_2, Z_3, Z_5 \in \Re^{3n\times 3n}, N, M \in \Re^{3n\times n}$, and positive scalars $\varepsilon, \lambda_i, i = 1, \ldots, 4$, such that:

$$\begin{pmatrix} Z_1 & Z_2 & Z_3 & N \\ * & Z_4 & Z_5 & M \\ * & * & Z_6 & M \\ * & * & * & P \end{pmatrix} \ge 0 \qquad (2)$$

$$\begin{pmatrix} \Lambda & \Omega_{12} & \Omega_{13} + \alpha P & PA_d & hA^TP & \varepsilon PE & F^T \\ * & \Omega_{22} - Q_2 & \Omega_{23} & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33} - \alpha hQ_2 - \alpha P & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{h}_DQ_{12} & hA_d^TP & 0 & F_d^T \\ * & * & * & * & -hP & \varepsilon hPE & 0 \\ * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & -\varepsilon I \end{pmatrix} < 0 \qquad (3)$$

$$\frac{e^{\alpha T}}{\lambda_1}\left(\left(c_1 + \mu\frac{h^2}{2}\right)\lambda_2 + c_1h(\lambda_3 + \lambda_4)\right) < c_2$$

for $k \in \{n/n = 2m + 1; m \in \mathbb{Z}^{+}\}, Q_{12} = Q_1 - Q_2, \bar{h}_D = 1 - h_D$,

$$\begin{pmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13}\\ * & \Omega_{22} & \Omega_{23}\\ * & * & \Omega_{33}\end{pmatrix} = hZ_1 + g_1Z_4 + \mathrm{sym}(N\Pi_1 + M\Pi_2 + g_3Z_5),$$

$$\Lambda = \Omega_{11} + PA + A^TP + Q_1 - 2\alpha P, g_1 = h\left(\tfrac{1}{2k+1} + \tfrac{2}{k+2} + \tfrac{1}{3}\right),$$

$$g_2 = \tfrac{\mathrm{h}}{2k+1}, g_3 = h\left(-\tfrac{1}{2k+1} - \tfrac{1}{k+2}\right), 0 < \lambda_1 I < \tilde{P} < \lambda_2 I,$$

$$0 < \tilde{Q}_1 < \lambda_3 I, 0 < \tilde{Q}_2 < \lambda_4 I, \tilde{P} = R^{\frac{-1}{2}}PR^{\frac{-1}{2}},$$

$$0 < \tilde{Q}_1 < \lambda_3 I, 0 < \tilde{Q}_2 < \lambda_4 I, \tilde{P} = R^{\frac{-1}{2}}PR^{\frac{-1}{2}}$$

Proof. The condition (3) of Lemma 7 is equivalent to the inequality $\Xi + \bar{E}\Sigma(t)\bar{F} + (\bar{E}\Sigma(t)\bar{F})^T < 0$ where

$$\bar{E}^T = [(PE)^T \quad 0 \quad 0 \quad 0 \quad h(PE)^T], \bar{F} = [F \quad 0 \quad 0 \quad F_d \ 0],$$

$$\Xi = \begin{pmatrix}\Lambda & \Omega_{12} & \Omega_{13} + \alpha P & PA_d & hA^TP\\ * & \Omega_{22} - Q_2 & \Omega_{23} & 0 & 0\\ * & * & \Omega_{33} - \alpha hQ_2 - \alpha P & 0 & 0\\ * & * & * & -\bar{h}_D Q_{12} & hA_d^TP\\ * & * & * & * & -hP\end{pmatrix}$$

Using Lemma 1, the condition (3) of [6] is obtained.

Remark 1. In deriving Lemma 7, we supposed that $k \in \{n/n = 2m+1; m \in \mathbb{Z}^{*+}\}$ in order to liberate the matrices Z_2 and Z_3. Then, we can reduce the conservatism and complexity, and have more degree of freedom.

Remark 2. From [19], Lemma 5 and Lemma 6 are equivalent. Also, Lemmas 2, 3, 4 are particular cases of Lemma 5 [18]. Then, it suffices to compare our results with those of Lemma 5 or 6. Therefore, if we choose $Z_1 = NR^{-1}N^T$, $Z_2 = Z_3 = NR^{-1}M^T$, $Z_4 = Z_5 = Z_6 = MR^{-1}M^T$, the inequality below is verified using the Schur complement:

$$\begin{bmatrix}NR^{-1}N^T & NR^{-1}M^T & NR^{-1}M^T & N\\ * & MR^{-1}M^T & MR^{-1}M^T & M\\ * & * & MR^{-1}M^T & M\\ * & * & * & R\end{bmatrix} \geq 0$$

Then, we have:

$$-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \leq \varpi^T\big((b-a)Z_1 + g_1Z_4 + g_2Z_6$$
$$+\mathrm{sym}(N\Pi_1 + M\Pi_2 + g_3Z_5)\big)\varpi = \varpi^T((b-a)NR^{-1}N^T + (g_1 + g_2 + 2g_3)$$
$$\times MR^{-1}M^T + \mathrm{sym}(N\Pi_1 + M\Pi_2))\varpi = \varpi^T\left((b-a)\left(NR^{-1}N^T + \tfrac{1}{3}MR^{-1}M^T\right)\right.$$
$$\left.+\mathrm{sym}(N\Pi_1 + M\Pi_2)\right)\varpi$$

Finally, Lemma 5 and Lemma 6 are particular case of our developed lemma.

3. FINITE TIME STABILIZATION

At this stage, the following system is considered:

$$\dot{x}(t) = \bar{A}x(t) + \bar{A}_d x(t - h(t)) + Bu(t) \qquad (4)$$

where $u(t) \in \Re^m$ is the control input vector and $B \in \Re^{n\times m}$ is a constant matrix. Then, the effective control signal (MSC) to be applied to (4) is $u(t) = Kx(t)$.

Theorem 1. (4) is FT stabilizable according to (c_1, c_2, T, R) if there exist SPDM $\bar{P}, \bar{Q}_1, \bar{Q}_2(\bar{Q}_1 > \bar{Q}_2), \bar{Z}_1, \bar{Z}_4, \bar{Z}_6 \in \Re^{3n\times 3n}$, $H_i \in \Re^n, i = 1, \dots, 4$, ADM $Y \in \Re^{l\times n}, \bar{Z}_2, \bar{Z}_3, \bar{Z}_5 \in \Re^{3n\times 3n}$, $\bar{N}, \bar{M} \in \Re^{3n\times n}$, and a scalar ε such that:

$$\begin{pmatrix}\bar{Z}_1 & \bar{Z}_2 & \bar{Z}_3 & \bar{N}\\ * & \bar{Z}_4 & \bar{Z}_5 & \bar{M}\\ * & * & \bar{Z}_6 & \bar{M}\\ * & * & * & \bar{P}\end{pmatrix} \geq 0 \qquad (5)$$

$$\begin{pmatrix}\Gamma & \bar{\Omega}_{12} & \bar{\Omega}_{13} + \alpha\bar{P} & A_d\bar{P} & h\bar{P}A^T + hY^TB^T & \varepsilon E & \bar{P}F^T\\ * & \bar{\Omega}_{22} - \bar{Q}_2 & \bar{\Omega}_{23} & 0 & 0 & 0 & 0\\ * & * & \bar{\Omega}_{33} - \alpha h\bar{Q}_2 - \alpha P & 0 & 0 & 0 & 0\\ * & * & * & -\bar{h}_D\bar{Q}_{12} & h\bar{P}A_d^T & 0 & \bar{P}F_d^T\\ * & * & * & * & -h\bar{P} & \varepsilon hE & 0\\ * & * & * & * & * & -\varepsilon I & 0\\ * & * & * & * & * & * & -\varepsilon I\end{pmatrix} < 0, \qquad (6)$$

$$\left(\left(c_1 + \mu\frac{h^2}{2}\right)H_2 + c_1h(H_3 + H_4)\right) < c_2e^{-\alpha T}H_1$$

for $k \in \{n/n = 2m+1; m \in \mathbb{Z}^+\}$, $\bar{Q}_{12} = \bar{Q}_1 - \bar{Q}_2$, $Y = KP^{-1}$,

$$\Gamma = \bar{\Omega}_{11} + A\bar{P} + \bar{P}A^T + BY + Y^TB^T + \bar{Q}_1 - 2\alpha\bar{P},\ g_2 = \tfrac{h}{2k+1},$$

$$g_1 = h\left(\tfrac{1}{2k+1} + \tfrac{2}{k+2} + \tfrac{1}{3}\right), g_3 = h\left(-\tfrac{1}{2k+1} - \tfrac{1}{k+2}\right),$$

$$0 < H_1 I < R^{\frac{-1}{2}}\bar{P}R^{\frac{-1}{2}} < H_2 I, 0 < R^{\frac{-1}{2}}\bar{Q}_1R^{\frac{-1}{2}} < H_3 I,$$

$$0 < R^{\frac{-1}{2}}\bar{Q}_2R^{\frac{-1}{2}} < H_4 I, \bar{\Omega}_{jj} = \bar{P}\Omega_{jj}\bar{P}, j = 1,2,3,$$

$$H_i = \bar{P}(\lambda_i I)\bar{P}, i = 1, \dots, 4$$

Proof. The condition (5) is obtained by pre- and post-multiplying (2) by $\mathrm{diag}\{P^{-1}, P^{-1}, P^{-1}, P^{-1}\}$ taking into account that $\bar{Z}_i = \bar{P}Z_l\bar{P}, l = 1, \dots, 6$, $\bar{N} = \bar{P}N\bar{P}$, $\bar{M} = \bar{P}M\bar{P}$. Also, the condition (6) is obtained by pre- and post-multiplying (3) by $diag\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I\}$: $\bar{P} = P^{-1}$, $\bar{Q}_1 = \bar{P}Q_1\bar{P}, \bar{Q}_2 = \bar{P}Q_2\bar{P}, \bar{Q}_3 = \bar{P}Q_3\bar{P}, Y = K\bar{P}, H_i = \bar{P}(\lambda_i I)\bar{P}$.

Now, let the following effective control signal (MC): $u(t) = Kx(t) + K_dx(t - h(t))$. Then, the results can be easily obtained by replacing $A_d\bar{P}$ and $h\bar{P}A_d^T$ by $A_d\bar{P} + BY_d$ and $h\bar{P}A_d^T + hY_d^TB^T$, respectively, where $Y_d = K_dP^{-1}$. On the other hand, the other conditions are the same.

4. NUMERICAL EXAMPLE

Let the uncertain system (4) where

$$A = \begin{bmatrix}0.15 & 0.4\\ 0.1 & 0.3\end{bmatrix}, A_d = \begin{bmatrix}-0.1 & 0\\ 0.15 & 0\end{bmatrix}, B = \begin{bmatrix}0.1\\ 0.2\end{bmatrix},$$
$$E = \begin{bmatrix}0.01\\ 0\end{bmatrix}, F = [1 \quad 0], F_d = [0.5 \quad 0]$$

1. Then, we choose $c_1 = 2.002, T = 1.5, \mu = 0.01, \varepsilon = 1000$, and $h_D = 0$. Applying Theorem 1, Table 1 is given

with $\alpha = 1.005$. Thus, the obtained values of $h_{2\max}$ is larger than those obtained in [8], and then the results are significantly improved.

Table 1 – Comparison of $h_{2\max}$ for different values of c_2

$h_{2\max}$	c_2						
	10	15	20	25	30	40	50
[8] ($h_1 = 0$)	2.7703	3.90344	3.9892	4.0472	4.0852	4.1208	4.1268
Theorem 1 (MSC)	2.8751	7.2232	9.7920	11.6950	13.2045	15.5221	17.2780
Theorem 1 (MC)	6.5002	9.7796	11.3062	12.5805	13.6965	15.6226	17.2880

2. Now, we choose $c_1 = 2.002, c_2 = 50, T = 1.5, \mu = 0.01, \varepsilon = 1000$, and $h_D = 0$ and apply Theorem 1. Then, Table 2 is given with $\alpha = 1.005$. Therefore, it can be concluded that increasing k reduces conservatism even if this reduction is small, and this proves that the idea of not specifying the value of k in Lemma 4 is a good one.

Table 2 – Comparison of $h_{2\max}$ for different values of k

$h_{2\max}$	k								
	1	11	21	31	41	51	91	101	1001
Theorem 1 (MSC)	17.2780	17.2787	17.2789	17.2790	17.2791	17.2791	17.2791	17.2792	17.2792

3. Also, we choose $c_1 = 2.002, c_2 = 20, T = 1.5, \mu = 0.01$, and $\varepsilon = 1000$ and apply Theorem 1. Then, we have:

Table 3 – Comparison of h for different values of h_D

h	h_D					
	0	0.1	0.3	0.5	0.7	0.9
Theorem 1 (MSC)	9.7886	9.3914	8.4568	7.2965	5.7704	3.3905

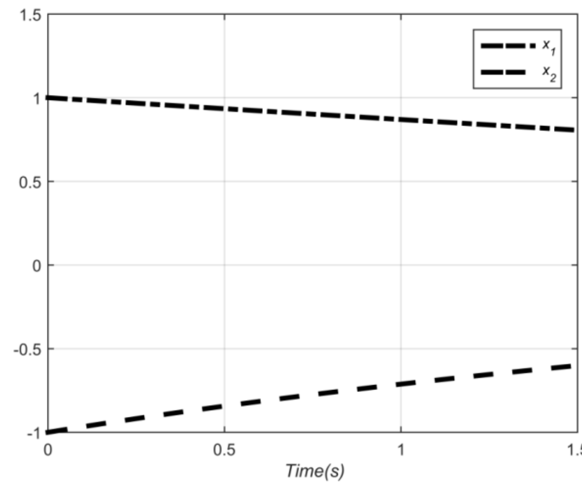

Fig.1 – Evolution of the state variables

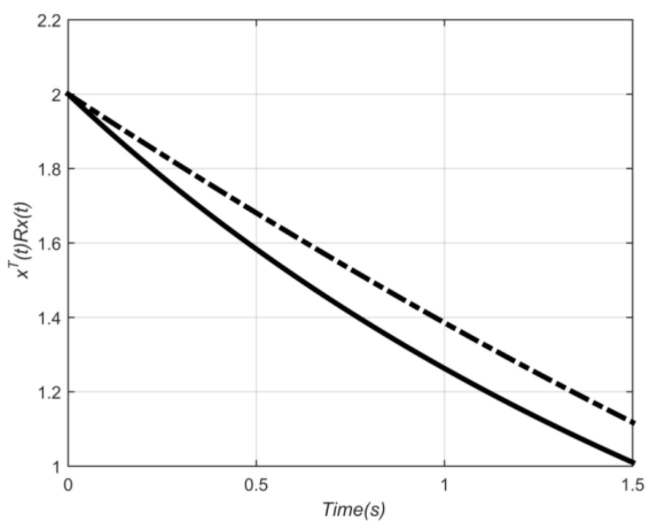

Fig. 2 – Evolution of the state variables norm (—— : Theorem 1, – – – : [8])

Then, it can be seen that increasing the upper bound of the delay time derivative causes a conservation defect.

4. In the end, we choose $c_1 = 2.002, T = 1.5, \mu = 0.01, \varepsilon = 1000$, and $h_D = 0$. Applying Theorem 1, Figure 1 and Figure 2 are given with $\alpha = 1.005, h = 3$, and $c_2 = 12$. Then, it is can be seen from Figure 1 that the state responses converge to the equilibrium point as it reaches the desired tracking performance.

Also, the considered system is FT stabilizable with respect to $(2.002, 12, 1.5, I)$ using our approach, as that the state norm trajectory (see Figure 2) converges more faster compared to [8], which proves the efficiency of our results. Finally, the simulation results show the accuracy and the effectiveness of the proposed approach for which the closed-loop system is stable.

5. CONCLUSION

The lemma developed in this article generalizes some existing ones, and provides a new insight into the study of systems with time delay taking into account uncertainties. Then, based on this lemma and using a new approximation of the single integral [6] $\int_{t-h}^{t} \dot{x}^T(s) R \dot{x}(s) ds$ sufficient conditions are proposed to ensure the FTS and FT stabilizability of the studied system. These new results are expressed in terms of LMIs and illustrated by a numerical example, and great improvements are obtained compared to the existing results. The conservativeness of the derived results can be further reduced by combining the FMII with the delay-decomposition technique. Thus, the adopted methodology opens up new topics for research: that is why we aim to extend the proposed approach to other systems closely related to what we are studying using approaches of delay-partitioning, additive time-delay, etc.

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Стабілізація невизначених систем із затримкою у скінченному часі з використанням нового підходу інтегральної нерівності

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Наведено деякі нові умови стійкості за скінченним часом (СТС), що залежать від затримки, та застосовані до задачі проектування контролерів СТ. Спочатку, на основі нової інтегральної нерівності та простого функціоналу Ляпунова-Красовського (ЛКФ), запропоновано критерії СТС, що залежать від затримки, шляхом введення деяких матриць з вільним зважуванням. Таким чином, запропоновано нове наближення унікального інтеграла, який з'являється в похідній ЛКФ, з використанням інтегральної нерівності, яка називається інтегральною нерівністю на основі вільної матриці (ФНВМ). Потім, для забезпечення СТС невизначених систем, що залежать від затримки, розроблено контролери зі зворотним зв'язком по стану та без пам'яті (MSC та MC), які є менш консервативними, ніж інші, що зустрічаються в літературі. Хоча деякі результати покращують критерії стійкості, СТС отримала мало уваги, і можна отримати більше результатів для зменшення консерватизму. Це є ключовим елементом нашого дослідження. Затримки, що змінюються в часі, обмежені та диференційовані з верхньою межею похідних затримки. Крім того, достатні умови, отримані в цій статті, встановлені в термінах лінійних матричних нерівностей (ЛМН) для досягнення бажаної продуктивності. Для ілюстрації потенційної вигоди від використання цього нового підходу наведено детальний числовий приклад. Нарешті, запропоновано менш консервативний проект на основі LMI, який вирішено за допомогою MATLAB, що показує дуже хороші результати.

Ключові слова: Стійкість у скінченному часі, Невизначені системи, Інтегральна нерівність на основі вільної матриці.