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REGULAR ARTICLE

Recovering Data for Free Induction Decay Signal for MRI Reconstruction with Interpolation

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The problem of magnetic resonance tomographical reconstruction for irregular filling data for free-induction decay signal and the measurements of points in the gradient K-space are considered. The procedure of measuring signals with the irregularity of magnitude, which depends on the distance from the center of the K-space area, is realized. Interpolation methods have been proposed for completing the missed measurements. The reconstructed tomograms for tomographic experiment special conditions are analyzed. Examples of reconstructed images for different degrees of signal irregularity are given. The operation of the measuring system response signal was simulated for this purpose. A real tomographic image was chosen as the object for calculating the signals. Quantitative characteristics of the reconstruction quality dependence for data irregularity different fractions were determined for several types of images. In particular weighted by spin density, spin-spin relaxation time and spin-lattice relaxation time tomograms were considered. A mutual comparison of the signals set characteristics sensitivity for different pulse sequences was carried out which determine the reconstruction type. Histograms were constructed to compare the images generalized characteristics. It is shown that the processing method application a previously measured signal before reconstruction allows partial compensation of the signals set non-ideality and restoration of higher quality medical tomographic images. The conditions are found for high quality visualization for signal data incomplete set for real tomograms. The scope of the application for measured signal preprocessing methods array are proposed in particular for tomographic equipment with a smaller magnetic field constant component.

Keywords: MRI, Interpolation method, Gradient, Reconstruction, Recovering.

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1. INTRODUCTION

In medicine, a large number of instrumental research methods are used, some of which are universal, which allows to diagnose many pathologies from various classification groups of human diseases. Invasive research methods have a large informative base, but non-invasive research methods are of increasing interest. The latter is also true for the MRI technique, which allows visualization of body tissues without using radiation [1-5].

The modern stage of the development of medicine is characterized by the widespread introduction of ever more complex diagnostic methods into the clinical practice, which not only facilitates and focuses on the visualization of the pathological cell, but also in some cases alters our perceptions about the features of a particular disease.

MRI produces contrasting of the soft tissues of the body, which is especially useful in visualizing brain, muscle, heart and cancer tumors, in comparison with other technologies of medical imaging such as computed tomography (CT) or radiography. Unlike CT or conventional X-rays, ionizing radiation is not used in MRI.

MRI scanners use strong magnetic fields, electric gradient fields and radio waves to obtain an image. Magnetic resonance imaging is based on the phenomenon of nuclear magnetic resonance. After measuring the NMR signal, there is a stage of reconstruction of the image using the Fourier method [6]. Reconstruction can be performed with varying degrees of incompleteness of data [6, 7].

This work is focused on numerical simulations of the NMR image reconstruction. Estimates obtained as a result of numerical simulation are widely used in modern research [8, 9]. Also, numerical simulation in combination with physical experiment is used in medicine for better diagnostics [10, 11]. Spin-spin and spin-lattice interactions play an important role in MRI, and they are also used in other techniques [12, 13].

2. RESEARCH METHODSB

The construction of images takes place using Fourier transforms. MRI image is essentially a calculated map or image of the radio frequency of signals produced by the human body. The signal is the simultaneous reception of the magnetization components Mx and My, as a function of time, and is recorded using two separate channels of the sensor, which provide information about the components of the signal (amplitude, phase, frequency). In this phase-sensitive method, a complex number which holds the modulated signal, is divided into real and imaginary components, shifted by 90° relative to each other. The signal of both channels is combined

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into one set of quadrature: real and imaginary spectra, and then processed using the Fourier transform.

Each point of the matrix of raw data (K space) contains some information about the image and does not correspond to the point of the image matrix. K space is equivalent to the space defined by the phase and frequency coding directions, each data line of which corresponds to a digital MR signal with a unique phase coding level. The outer rows of the matrix of raw data give information about the boundaries and contours of the image or individual structures, which determine the resolution of small details.

The trajectory of the K-space is a track traced in the spatial frequency domain when data is collected, and is determined by the applied gradients. K space can be filled in rows or spiral, depending on the applied gradients and the selected algorithms for data collection.

The intensity of each element of the MR image (pixel) is proportional to the intensity of the signal from the corresponding element of the 3D space (voxel) for the given thickness of the cut.

An obligatory condition for the application of the Fourier-Zeimatography methods is the uniformity of the grid of values or the regularity of the grid in the K-space. Under these conditions, the signal of the free induction recession can be inverse Fourier transformed to restore the distribution of spin density. Regularity of the grid in the K-space is a significant limitation of the usage of tomographic methods of reconstruction. The problem of visualization is to restore and interpretation the data. Triangulation methods can be used to restore the signal and interpolation methods can be modified for usage in the K-space [6].

In this work we consider the algorithm of signal restoration by the method of interpolation modified for application in the K-space. By using the methods of signal processing before the reconstruction of tomograms it is possible to increase the informative value of the diagnostic method as a whole [14, 15]. To modulate the data recovery process of the free-induction decline signal, a real tomographic image of the cut of a person's head is used. The trajectory of data collection is in rows.

A real tomographic image of the human brain was taken for the investigation. The irregularity of the data was artificially created in the image. Two cases are considered: the uniform irregularity of the data (zero signal values were inserted evenly into the matrix of the image signal) and the irregularity with the magnitude depending on the distance from the center of the K-space region. Zero values of the signal correspond to the situation of the measurement of the free-induction decline signal during the tomographic examination.

The restoration of the signal value was carried out according to the following method:

Free induction decay signal (FIDS) is described by the following expression

$$S\left(k_i,k_j\right)=S_0\sum_{n,m}\rho\left(x_n,y_m\right)\cdot e^{i\left(k_{xi}x_n+k_{yj}y_m\right)}$$

The values of the free-induction decline signal in three points of the K-space are S_1, S_2, S_3.

These 3 points in the K-space, which are depicted in red were selected as shown in Fig. 1 [6]. A dot with a zero value is depicted in black. An interpolation method is used to calculate the value at this point.

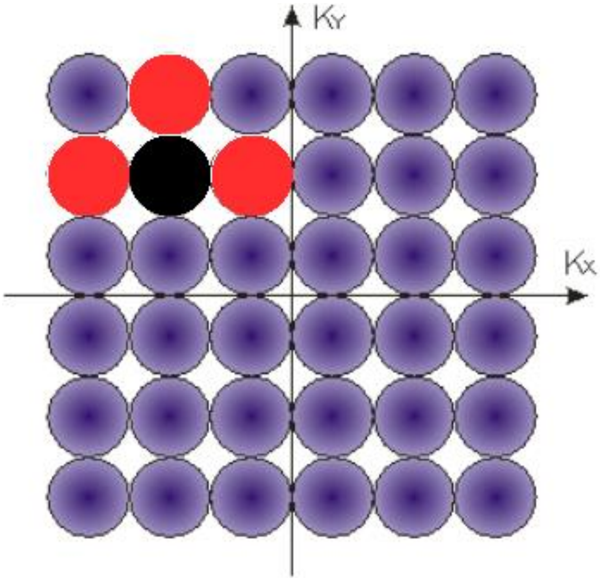

Fig. 1 – Selecting the points of the K-space to calculate the value of the signal

$S_1\left(k_{x1},k_{y1}\right)=\mathrm{Re}\,S_1+i\,\mathrm{Im}\,S_1=\left|S_1\right|e^{i\phi_1}$ where the absolute value of FIDS is:

$$\left|S_1\right|=\sqrt{\left(\mathrm{Re}\,S_1\right)^2+\left(\mathrm{Im}\,S_1\right)^2}$$ and its phase is:

$$\phi_1=arctg\left(\frac{\mathrm{Im}\,S_1}{\mathrm{Re}\,S_1}\right)$$

Similarly

$$\left|S_2\right|=\sqrt{\left(\mathrm{Re}\,S_2\right)^2+\left(\mathrm{Im}\,S_2\right)^2}\qquad \phi_2=arctg\left(\frac{\mathrm{Im}\,S_2}{\mathrm{Re}\,S_2}\right)$$

$$S_3\left(k_{x3},k_{y3}\right)=\mathrm{Re}\,S_3+i\,\mathrm{Im}\,S_3=\left|S_3\right|e^{i\phi_3}$$

$$\left|S_3\right|=\sqrt{\left(\mathrm{Re}\,S_3\right)^2+\left(\mathrm{Im}\,S_3\right)^2}\qquad \phi_3=arctg\left(\frac{\mathrm{Im}\,S_3}{\mathrm{Re}\,S_3}\right)$$

It is possible, using the method of interpolation of data in the K-space, to calculate the value FIDS $S_0\left(k_{x0},k_{y0}\right)$ in some new point of the K-space with coordinates $\left(k_{x0},k_{y0}\right)$.

Distances between readings:

$$a_1=\sqrt{\left(k_{x1}-k_{x0}\right)^2+\left(k_{y1}-k_{y0}\right)^2}$$

$$a_2=\sqrt{\left(k_{x2}-k_{x0}\right)^2+\left(k_{y2}-k_{y0}\right)^2}$$

$$a_3=\sqrt{\left(k_{x3}-k_{x0}\right)^2+\left(k_{y3}-k_{y0}\right)^2}$$

Evaluation of absolute value FIDS $S_0(k_{x0}, k_{y0})$

Evaluation of the phase of the FIDS ϕ_0 :

$$\phi_0 = \frac{a_1\phi_1 + a_2\phi_2 + a_3\phi_3}{\sum\limits_{w=1}^{3} a_w}$$

$$\left|S_0\left(k_{x0}, k_{y0}\right)\right| = \frac{\sum\limits_{n=1}^{3} a_n \left|S\left(k_{xn}, k_{yn}\right)\right|}{\sum\limits_{w=1}^{3} a_w}$$

Total value of the FIDS at the point of assessment:

$$S_0\left(k_{x0}, k_{y0}\right) = \left|S_0\right| e^{i\phi_0} = \left|S_0\right| \cos\left(\phi_0\right) + \\ + i\left|S_0\right| \sin\left(\phi_0\right) = \operatorname{Re} S_0 + i \operatorname{Im} S_0$$

In this work 4 cases of uniform heterogeneity of the data were considered, as well as the case of heterogeneity with a dependency on the distance from the center of the K-space according to the graph in Fig. 2.

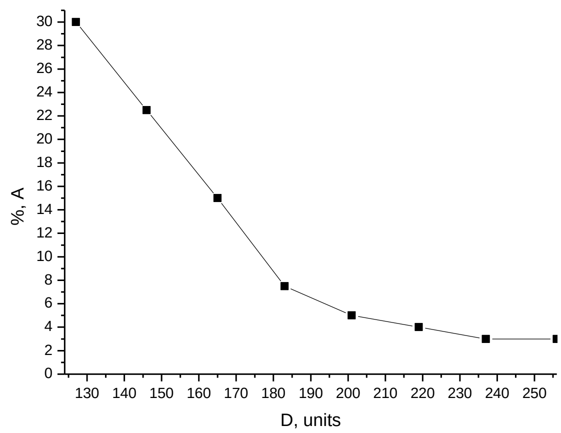

Fig. 2 – Modeling of irregularities, depending on the distance from the center of the K-space

In Fig. 2, A is the percentage of zeros, D – distance from the center of the space K.

As can be seen from Figure 2, the irregularity decreased with increasing distance from the center of the K-space. The whole K image space is a matrix of 256 × 256 pixels. It was divided into 7 zones. The zones are symmetrically located relative to the center of the K-space, as shown in Fig. 3.

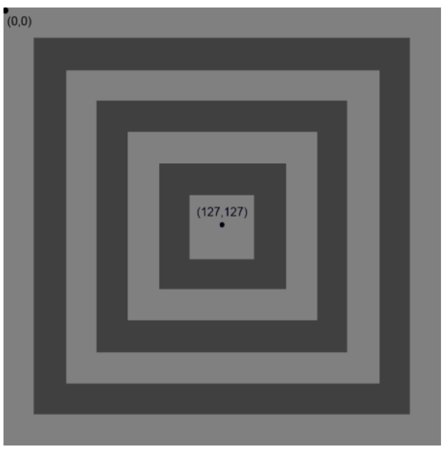

Fig. 3 – Separation of the k-space matrix into zones

The comparison of recovered reconstructed images is estimated by the correlation coefficient.

The formula for the correlation coefficient k is the following:

$$k = \frac{\sum_{i=0}^{255}\sum_{j=0}^{255}(x_{i,j} - \overline{x}) \times (y_{i,j} - \overline{y})}{\sqrt{\sum_{i=0}^{255}\sum_{j=0}^{255}(x_{i,j} - \overline{x})^2 \times \sum_{i=0}^{255}\sum_{j=0}^{255}(y_{i,j} - \overline{y})^2}},$$

$x_{i,j}$ – the brightness of each pixel in the initial image,

$y_{i,j}$ – the brightness of each pixel in the reconstructed image,

$\overline{y}$ – the average brightness of the pixels in the reconstructed image

$\overline{x}$ – the average brightness of the pixels in the original image

3. RESULTS

Special software was written to automate the research of the reconstruction of the free-induced downturn signal for irregular data. It implements the functions of restoring irregular data by the method described above, reconstructing an image using the Fourier transform. The correlation coefficient is used to compare the corresponding restored reconstructed images.

A real medical image of the human head was taken for the simulation of Fig. 4.

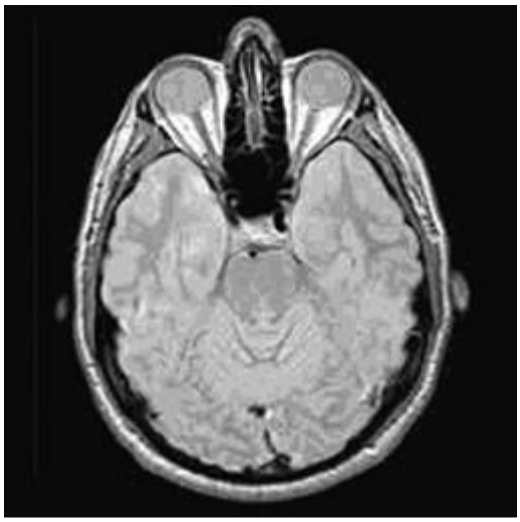

Fig. 4 – Original image.

In the array of values of the signal of the free-induction decline of this image, three cases of uniform heterogeneity of the data were simulated: 10 %, 20 % and 30 % of all points of the image data were plotted. After data recovery, the following image was obtained by the method described above (Fig. 5).

Simulation of the situation of uneven heterogeneity of data is shown in Fig. 6.

In the situation of uniform heterogeneity of data at the level of 7.7 % (for such a percentage the number of points with uniform heterogeneity coincides with the case of uneven heterogeneity), the result is presented in Fig. 7.

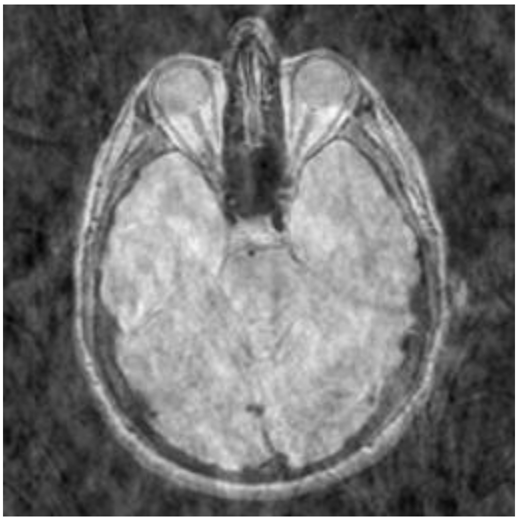

Fig. 5 – Uniform distribution of heterogeneity with 30 % of data restored

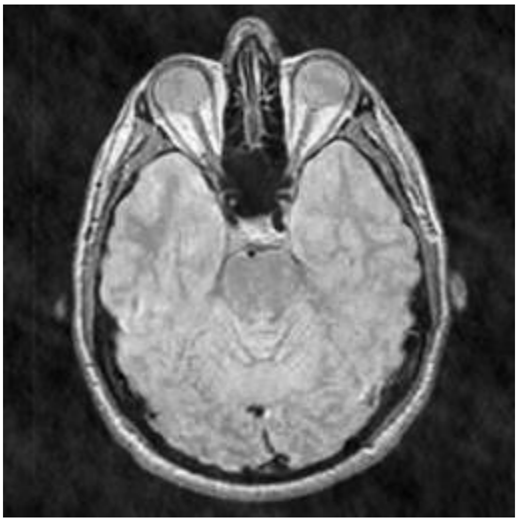

Fig. 6 – Non-uniform heterogeneity of data

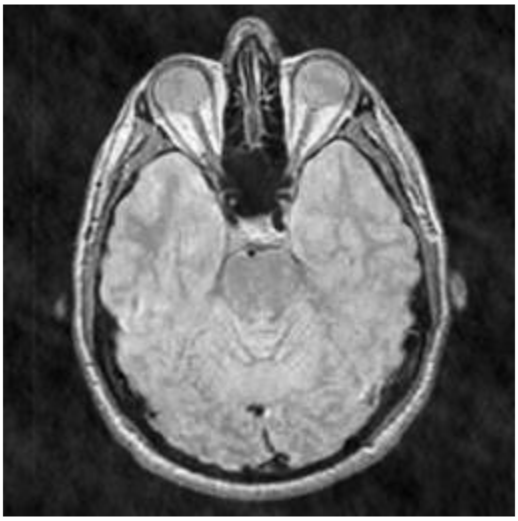

Fig. 7 – Equivalent heterogeneity at the level of 7.7 % of data was restored

Table 1 – Dependence of the correlation coefficient K on recovered data percentage.

Percent of recovered data (%)	K
7.7	0.99066.
10	0.98977
20	0.97581
30	0.96045
7.7 (non-uniform heterogeneity)	0.99452.

For better visualization of the results, histograms for T_1 weighted images were constructed (Fig. 8-10).

Table 2 – Dependence of the correlation coefficient K on recovered data percentage and image type

Percent of recovered data (%)	K 30 %	K 20 %	K 10 %	K 7.7 %	K 7.7(nonuniform heterogeneity)
T_1 weighted	0.9152	0.9490	0.9756	0.9804	0.9937
T_2 weighted	0.9067	0.9448	0.9736	0.9778	0.9901
ρ weighted	0.9443	0.9692	0.9849	0.9865	0.9946

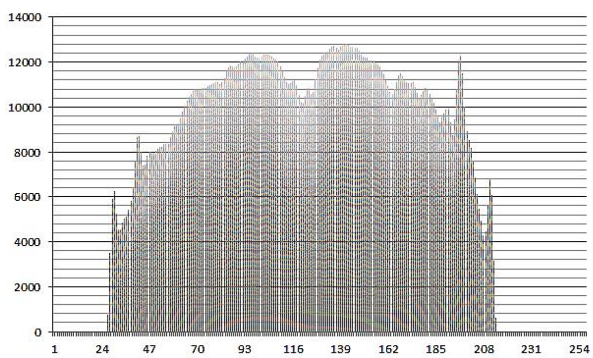

Fig. 8 – Original histogram for T_1 weighted image

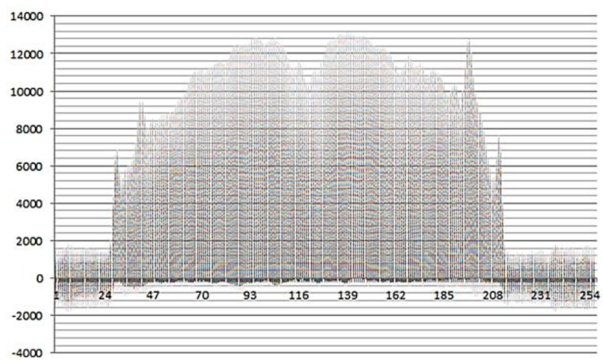

Fig. 9 – Histogram T_1 of weighted image at 30 % of recovered data

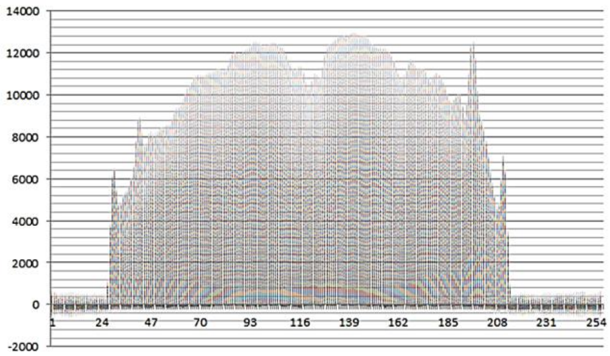

Fig. 10 – Histogram T_1 weighted image with uneven distribution of the restored data in the signal array

4. CONCLUSIONS

As can be seen from the presented results, even given the loss of 30 % of the image data, it can be restored by the proposed method with a fairly high accuracy (the correlation was 0.96045). The image loses quality only for visual comparison. Increasing the number of points taken into account when restoring the signal should improve the visual quality of the image.

We decided to test this signal recovery algorithm for various types of tomographic images. The algorithm was applied to T_1, T_2 and ρ – weighted images. The results are presented in Table 2.

From these histograms it is noticeable that there is an additional noise in the entire image.

From the practical point of view, the results obtained in this work can be used to reduce the time of signaling in the MR tomography. This will increase the convenience of the patients whose tomograms are being made, as they should lie motionless for a few minutes during the examination. Their movements cause the appearance of artifacts in the image.

What poses interest for further research is to compare the results for images obtained when 7,7 % of the uniform data heterogeneity is restored and uneven heterogeneity is present in each of 7 zones, yielding correlation coefficients of 0.99066 and 0.99452 respectively.

The number of restored points in both cases is the same, but the image obtained using uneven heterogeneity is much better visually perceived and has a higher correlation coefficient. Finding image areas where data loss is insignificant is important for the quality of the image and remains open, requiring further research.

The use of interpolation methods for free induction decay signal processing makes it possible to expand the magnetic resonance tomographic capabilities scope.

An example is the prospect of using mobile equipment for which it is more expedient to use a smaller constant component of the magnetic field.

The magnetic field generation systems of smaller size and mass can be used for such conditions. Similar possibilities of simplified technical solutions become available for magnetic gradient field generation systems in the process of phase-frequency encoding of signals.

Under such conditions it becomes more difficult to ensure full regularity of the received signals in the gradient K space. That is why the approach of using compensation methods is justified.

REFERENCES

1. K.S. Nayak, Y. Lim, A.E. Campbell-Washburn, J. Steeden, *J. Magn. Res. Imag.* **55** No 1, 81 (2020).
2. S. Gassenmaier, T. Kustner, D. Nickel, J. Herrmann, R. Hoffmann, H. Almansour, S. Afat, K. Nikolaou, A.E. Othman, *Diagnostics* **11** No 12, 2181 (2021).
3. M. Hori, A. Hagiwara, M. Goto, A. Wada, S. Aoki, *Investigative Radiology* **56** No 11, 669 (2021).
4. M. Martucci, R. Russo, F. Schimperna, G. D'Apolito, M. Panfili, A. Grimaldi, A. Perna, A.M. Ferranti, G. Varcasia, C. Giordano, S. Gaudino, *Biomedicines* **11**, 364 (2023).
5. S. Saito, J. Ueda, *Radiolog. Phys. Technol.* **17**, 47 (2024).
6. A. Netreba, A. Komarov, Y. Kyiashko, I. Pershukov, *Mol. Cryst. Liquid Cryst.* **673**, 97 (2018).
7. X. Yang, W. Xu, R. Luo, X. Zheng, K. Liu, *Magn. Reson. Imag.* **57**, 165 (2019).
8. R.I. Shakirzyanov, V.A. Astakhov, A.T. Morchenko, P.A. Ryapolov, *J. Nano- Electron. Phys.* **8** No 3, 03040 (2016).
9. H. Touati, B. Amiri, A.C. Sebbak, A. Benameur, H. Aït-kaci, *J. Nano- Electron. Phys.* **8** No 2, 02008 (2016).
10. S.N. Danilchenko, A.S. Stanislavov, V.N. Kuznetsov, et al., *J. Nano- Electron. Phys.* 8 No 1, 01031 (2016).
11. A.N. Rybyanets, I.A. Shvetsov, M.A. Lugovaya, E.I. Petrova, N.A. Shvetsova *J. Nano- Electron. Phys.* **10** No 2, 02005 (2018).
12. A.P. Samila, G.I. Lastivka, V.O. Khandozhko, *J. Nano-Electron. Phys.* **8** No 4(2), 04081 (2016)
13. I.V. Boyko, O.A. Bagrii-Zayats, H.B. Tsupryk, Y.M. Stoianov, *J. Nano- Electron. Phys.* **10** No 6, 06001 (2018).
14. M.V. Kononov, O.A. Nagulyak, A.V. Netreba, *Radioelectron. Commun. Syst.* **51**, 163 (2008).
15. O.A. Lebed, A.Yu. Ovcharenko, *J. Nano- Electron. Phys.* **16** No 2, 02021 (2024).

Відновлення даних сигналу спаду вільної індукції для реконструкції у МРТ з інтерполяцією

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Розглянута задача відновлення магнітно-резонансних томограм для нерегулярного заповнення даними сигналу спаду вільної індукції точок вимірювань у градієнтному К-просторі. Реалізована процедура вимірювання сигналів з величиною нерегулярності яка залежить від віддаленості від центру області К-простору. Запропоновані інтерполяційні методи доповнення пропущених вимірювань. Проаналізовані відновлені томограми для спеціальних умов проведення томографічного експерименту. Наведено приклади відновлених медичних томографічних зображень для різної міри нерегулярності сигналів. Для цього проведено моделювання роботи вимірювальної системи сигналу відгуку. В якості об'єкта для розрахунку сигналів обрано реальне томографічне зображення. Визначено кількісні характеристики залежності якості томографічної реконструкції для різної частки нерегулярності даних для декількох типів томографічних зображень, зокрема зважених за спіновою густиною, часом спін-спінової та часом спін-граткової релаксації. Проведене взаємне порівняння чутливості до характеристик набору сигналів різних імпульсних послідовностей, які визначають особливості зваженості реконструкції за спіновими характеристиками. Для порівняння узагальнених характеристик зображень побудовані гістограми. Показано, що застосування методу обробки раніше виміряного сигналу до проведення реконструкції дозволяє провести часткову компенсацію неідеальності набору сигналів та відновити медичні томографічні зображення більш високої якості. Для реальних томограм визначені умови отримання високої якості візуалізації для неповного набору даних сигналу. Запропоновано сфери застосування методів попередньої обробки виміряного масиву сигналу відгуку, зокрема для томографічного обладнання з меншою величиною постійної компоненти магнітного поля.

Ключові слова: МРТ, Інтерполяційні методи, Градієнт, Реконструкція, Відновлення.