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REGULAR ARTICLE

Cubic Nonlinear Theory of Superheterodyne Parametric H-ubitron FEL with Section for Amplifying Longitudinal Space Charge Waves

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In this paper, an analysis of the wave dynamics of a parametric superheterodyne H-ubitron free-electron laser with a longitudinal wave amplification section is carried out within the cubic nonlinear approximation. A key feature of the studied free-electron laser is the implementation of two interconnected three-wave parametric resonances. The first parametric resonance occurs between the electromagnetic signal and the magnetic field of the H-ubitron pump. The second parametric resonance occurs between the longitudinal field of the electrostatic undulator and the fast and slow space-charge waves. The second parametric resonance pro-vides additional amplification of the slow space-charge wave. The studied model considers the effect of electrostatic pumping field generation, an analysis of the influence of a quasi-electrostatic support field is carried out, and the lengths and saturation levels of such devices are determined. It is demonstrated that, due to the additional amplification of the slow space charge wave in the longitudinal wave amplification section, as well as taking into account the effect of electrostatic pumping field generation, the saturation length of the electromagnetic signal wave is reduced by 18%. To increase the output electromagnetic signal's saturation level, we propose using a quasi-electrostatic support field, in which the electrostatic field strength in-creases linearly starting from a specific coordinate. It is demonstrated that an optimal quasi-electrostatic support field allows us to increase the output saturation level of the electromagnetic signal by more than 130%. Thus, the use of a parametric superheterodyne H-ubitron free-electron laser with a longitudinal space charge wave amplification section and a quasi-electrostatic support field has several advantages compared to a traditional H-ubitron free-electron laser, namely, a reduction in the longitudinal dimensions of the en-tire device and an increase in the power of the output electromagnetic signal.

Keywords: Superheterodyne Free-Electron Lasers, Space Charge Waves, Three-wave parametric resonance, Electro-static Undulator

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1. INTRODUCTION

Terahertz free-electron lasers (FELs), including the FEL discussed in this paper, are devices that are actively being developed today and have great potential [1 2]. Developments are underway to increase power, efficiency, compactness and expand the scope of application [1-10]. Progress in accelerator, undulator, and laser technologies [1-2] will, in the future, allow us to create more compact, efficient, and powerful terahertz FELs. This, in turn, will open new opportunities for fundamental research and practical applications of terahertz FELs in various fields of science, technology, and medicine [1-2].

The presented work is focused on the theoretical study of the transverse electromagnetic signal wave dynamics in a parametric superheterodyne FEL with an electrostatic undulator [11-15]. In this device, the electrostatic undulator provides additional amplification of the electromagnetic signal. Because of this, the growth increment of the signal wave becomes larger, and the saturation length becomes shorter, which in turn leads to a decrease in the longitudinal length of the FEL. Using an electrostatic undulator allows us to create more compact devices.

Previous studies of parametric superheterodyne FELs with an electrostatic undulator have focused mainly on studying the dynamics of longitudinal space charge waves (SCWs) in the amplification section [11 14]. We found that in the amplification section, the effect of electrostatic pumping field generation is realized, which increases the total electrostatic pumping field by more than 30%. This increases the growth increment of longitudinal SCWs and allows us to re-duce the length of the FEL. At the same time, this effect reduces the saturation level of the electromagnetic signal, that is, its output power. We proposed using a quasi-electrostatic support field to increase the output signal level, which has proven effective.

Thus, in works [11-14], we analyzed the influence of the generated pump electric field and the influence of a quasi-electrostatic support field on the dynamics of longitudinal space charge waves in the amplification section with an

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electrostatic undulator. At the same time, the influence of such fields on the dynamics of the transverse electromagnetic signal wave remained un-studied. The presented article is focused on eliminating this shortcoming. As shown by the studies presented in this article, such fields allow us to reduce the longitudinal size of parametric superheterodyne FELs by 18% and to increase the output signal amplitude by more than 130%.

2. MODEL

Fig. 1 shows a superheterodyne parametric H-ubitron FEL schematic with a longitudinal space charge wave amplification section. The main components are relativistic electron beam 1; H-ubitron undulator 2, which generates a magnetic pumping field $\boldsymbol{B}_2$ in the working region; and longitudinal electrostatic undulator 3 [11-14]. The electrostatic undulator consists of a set of electrodes arranged in two rows on either side of the electron beam. Electric potentials are applied to these electrodes such that their signs alternate along the direction of beam movement, and each pair of electrodes in the transverse cross-section of the amplification section has the same potential. This creates a longitudinal undulator pump electrostatic field, whose strength $\boldsymbol{E}_{20}$ is collinear with the beam's direction of motion (Fig. 1).

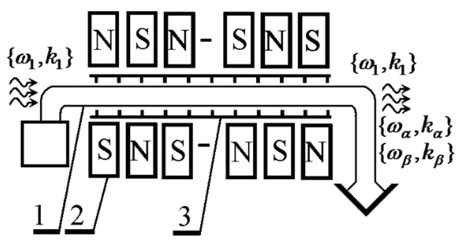

Fig. 1 – Simplified scheme of a superheterodyne parametric FEL: 1 – relativistic electron beam; 2 – H-ubitron undulator; 3 – electrostatic undulator

Due to the presence of two undulator systems (H-ubitron and electrostatic) in the studied FEL, two different three-wave parametric resonances occur. The first three-wave resonance, between the transverse electromagnetic signal wave, the transverse magnetic field of the H-ubitron pump, and the slow space charge wave, is traditional for terahertz FELs and FELs based on Raman scattering. Parametric instability amplifies both the electromagnetic wave and the slow SCW. The second three-wave parametric resonance, between the longitudinal slow and fast SCWs and the longitudinal electrostatic undulator pump field, is designed for additional amplifying of the slow SCW. Thus, the studied system exhibits two three-wave parametric resonances linked by the slow SCW. The additional amplification of the slow SCW due to the second parametric resonance leads to an increased growth increment of the electromagnetic signal wave. It provides additional tools that influence the amplification properties of the superheterodyne parametric FEL.

Let's consider the case where a linearly polarized electromagnetic signal wave enters the FEL. Due to the first parametric resonance between the electro-magnetic signal wave (electric field strength E1 and magnetic field induction B_1) and the pump magnetic field with induction $\boldsymbol{B}_2$, a slow SCW $\boldsymbol{E}_\alpha$ is generated in the working region. After the slow SCW appears, the second parametric resonance generates a fast space charge wave $\boldsymbol{E}_\beta$.

[12-13] demonstrate that the relativistic electron beam (REB) modulates with longitudinal waves and, as a result, creates its own electrostatic field, whose spatial phase coincides with the phase of the electrostatic field of the undulator $\boldsymbol{E}_{20}$. This field is the generated pump electric field $\boldsymbol{E}_2^g$. Thus, the electrons in the beam experience a force from the resulting field $\boldsymbol{E}_{20} + \boldsymbol{E}_2^g$.

To maintain the conditions for three-wave parametric resonances, which depend on the constant component of the relativistic electron beam's velocity, we introduce a quasi-electrostatic support field $E_0(z)$ into the studied system [14].

Assuming that the fields propagating in a superheterodyne FEL are monochromatic, we present them as follows:

$$\mathbf{E}_1 = \left[E_1 \exp(ip_1) + c.c.\right]\mathbf{e}_x \tag{1}$$

$$\mathbf{B}_1 = \left[B_1 \exp(ip_1) + c.c.\right]\mathbf{e}_y \tag{2}$$

$$\mathbf{B}_2 = \left[B_2 \exp(ip_{2,B}) + c.c.\right]\mathbf{e}_y \tag{3}$$

$$\mathbf{E}_{20} = [E_{20} \exp(ip_{2,E}) + c.c.]\mathbf{e}_z \tag{4}$$

$$\mathbf{E}_2^g = \left[E_2^g \exp\left(ip_{2,E}\right) + c.c.\right]\mathbf{e}_z \tag{5}$$

$$\mathbf{E}_2 = \mathbf{E}_{20} + \mathbf{E}_2^g = \left[E_2 \exp\left(ip_{2,E}\right) + c.c.\right]\mathbf{e}_z$$

$$\mathbf{E}_\alpha = \left[E_\alpha \exp\left(ip_\alpha\right) + c.c.\right]\mathbf{e}_z \tag{6}$$

$$\mathbf{E}_\beta = \left[E_\beta \exp\left(ip_\beta\right) + c.c.\right]\mathbf{e}_z \tag{7}$$

$$E_0(z) = \begin{cases} 0, z < z_{E_0} \\ \tau_{E_0}(z - z_{E_0}), z \ge z_{E_0} \end{cases} \tag{8}$$

In these equations E_1, B_1, B_2, E_{20}, E_2^g, E_2, E_α, E_β are the complex amplitudes of the corresponding fields, p_1, $p_{2,B}$, $p_{2,E}$, p_α, p_β, are their phases, $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$ are unit vectors on the corresponding axis, z_{E0} is the beginning of the slow SCW saturation region, τ_{E0} is the gradient of the increase in quasi-electrostatic support strength in the saturation region [14].

We present the phases of each component as:

$$\begin{aligned} p_1 &= \omega_1 - k_1 z \ , \\ p_{2,E} &= -k_{2,E} z \ , \\ p_{2,B} &= -k_{2,B} z \ , \\ p_\alpha &= \omega_\alpha - k_\alpha z \ , \\ p_\beta &= \omega_\beta - k_\beta z \end{aligned} \tag{9}$$

Wave numbers of the input signal k_1, slow k_α and fast k_β SCWs are determined by the dispersion condition.

$$k_1 = \sqrt{\omega_1^2 - \omega_p^2 / \gamma_0^2} / c ,$$
$$k_\alpha = \omega_\alpha / \upsilon_{z0} + \omega_p / \left(\gamma_0^{3/2} \upsilon_{z0}\right) , \tag{10}$$
$$k_\beta = \omega_\beta / \upsilon_{z0} - \omega_p / \left(\gamma_0^{3/2} \upsilon_{z0}\right)$$

where w_α is the cyclic frequency of the slow SCW, w_β is the cyclic frequency of the fast SCW, w_1 is the cyclic frequency of the electromagnetic signal wave, w_p is the Langmuir frequency, u_{z0} is the averaged electron beam velocity, g_0 is REB's Lorentz factor, c is the speed of light in vacuum.

For three-wave parametric resonances to be realized, their conditions must be met. The condition for three-wave resonance between the electromagnetic signal wave, the pump magnetic field, and the slow SCW has the form:

$$p_{2,B} = p_\alpha - p_1 \tag{11}$$

or putting $w_\alpha = w_1$:

$$k_{2,B} = k_\alpha - k_1 \tag{12}$$

The condition of three-wave resonance between the slow, fast SCWs and the pump electrostatic field can be expressed as follows:

$$p_{2,E} = p_\alpha - p_\beta \tag{13}$$

or putting $w_\alpha = w_\beta$:

$$k_{2,E} = k_\alpha - k_\beta \tag{14}$$

Using (7), (8), and (12) we get the wave number $k_{2,E}$ of the electrostatic undulator pump field:

$$k_{2,E} = 2\omega_p / \left(\gamma_0^{3/2} \upsilon_{z0}\right)$$

From the last equation we easily obtain the undulation period Λ of the electrostatic undulator:

$$\Lambda = \pi \gamma_0^{3/2} \upsilon_{z0} / \omega_p \tag{15}$$

3. BASIC EQUATIONS

To investigate the physical processes in the superheterodyne parametric FEL within the cubic nonlinear approximation, we use a system of truncated equations for the amplitudes of the waves propagating in the studied system. These equations are based on the relativistic quasi-hydrodynamic equation, Maxwell's equations, and the continuity equation. To construct the truncated equations for the wave amplitudes, we apply an asymptotic hierarchical approach to the theory of oscillations and waves [15]. The truncated equations for the amplitudes of the electromagnetic signal's electric field, the electric field of the slow and fast space charge waves, and the generated pumping field are as follows:

$$C_{2,1} \frac{d^2 E_1}{dz^2} + C_{1,1} \frac{dE_1}{dz} + D_1 E_1 = C_{3,1}^I E_\alpha B_2^* \tag{16}$$

$$C_{2,\alpha} \frac{d^2 E_\alpha}{dz^2} + C_{1,\alpha} \frac{dE_\alpha}{dz} + D_\alpha E_\alpha = C_{3,\alpha}^I E_1 B_2 + C_{3,\alpha}^{II} E_\beta E_2 + F_\alpha \tag{17}$$

$$C_{2,\beta} \frac{d^2 E_\beta}{dz^2} + C_{1,\beta} \frac{dE_\beta}{dz} + D_\beta E_\beta = C_{3,\beta}^{II} E_\alpha E_2^* + F_\beta \tag{18}$$

$$C_{2,2} \frac{d^2 E_2^g}{dz^2} + C_{1,2} \frac{dE_2^g}{dz} + D_2 E_2^g = C_0 E_{20} + C_{3,2} E_\alpha^* E_\beta + F_2 \tag{19}$$

The equations for the constant components of the velocity and continuity of the beam can be written as:

$$\frac{d\upsilon_0}{dz} = \frac{eE_0(z)}{m_e \upsilon_0 \gamma_0^3} + F_\upsilon , \quad \frac{dn_0}{dz} = -\frac{n_0}{\upsilon_0} \frac{d\upsilon_0}{dz} + F_n \tag{20}$$

In these equations $F_\chi = F_\chi(\boldsymbol{E_0}, \boldsymbol{E_\alpha}, \boldsymbol{E_\beta}, \boldsymbol{E_2})$, $\chi \in \{1, \alpha, \beta, 2\}$, $F_v = F_v(\boldsymbol{E_0}, \boldsymbol{E_\alpha}, \boldsymbol{E_\beta}, \boldsymbol{E_2})$, $F_n = F_n(\boldsymbol{E_0}, \boldsymbol{E_\alpha}, \boldsymbol{E_\beta}, \boldsymbol{E_2})$ are cubic nonlinear terms depending on the amplitudes of the fields of the system under study.

The constructed system of truncated equations allows us to take into account the effect of electrostatic pumping field generation and to analyze the wave dynamics in the presence of a quasi-electrostatic support field.

We also note that the quasi-electrostatic support field $E_0(z)$ influences the change in the constant com-ponent of the longitudinal velocity υ_{z0} with the longitudinal coordinate z [14]. Thus, this field primarily allows us to control the constant component of the beam's longitudinal velocity. We can select the shape and parameters of the quasi-electrostatic support field to maintain the conditions for three-wave resonant interactions through the REB's constant longitudinal velocity, thereby significantly increasing the output level of the electromagnetic signal.

The coefficients of the resulting system of differential equations look like this:

$$C_{1,\chi} = \partial D_\chi / \partial(-\mathrm{i}k_\chi)$$
$$C_{2,\chi} = \partial^2 D_\chi / \partial(-\mathrm{i}k_\chi)^2 / 2$$
$$D_1 \equiv \left[k_1^2 - \frac{\omega_1^2}{c^2} - \frac{\omega_p^2}{\gamma_0 c^2} \right] = 0$$
$$D_\alpha = -\mathrm{i}k_\alpha \left(1 - \frac{\omega_p^2}{(\omega_\alpha - k_\alpha \upsilon_0)^2 \gamma_0^3} \right) = 0$$
$$D_\beta = -\mathrm{i}k_\beta \left(1 - \frac{\omega_p^2}{(\omega_\beta - k_\beta \upsilon_0)^2 \gamma_0^3} \right) = 0$$
$$D_2 = -\mathrm{i}k_2 \left(1 - \frac{\omega_p^2}{(-k_2 \upsilon_0)^2 \gamma_0^3} \right) \tag{21}$$
$$C_{3,\alpha}^I = \frac{\omega_p^2 e k_\alpha}{\upsilon_{z0}^2 \Omega_\alpha^2 m_e \gamma_0^2 c^2 k_2} \left(\frac{\upsilon_{z0}}{c} - \frac{k_\alpha c}{\omega_\alpha} \right)$$

$$C_{3,\alpha}^{II} = \frac{-k_\alpha \omega_p^2 e / m_e}{\Omega_\alpha \Omega_\beta \Omega_2 \gamma_0^6 \upsilon_{z0}^3} \times \left(\frac{k_\alpha}{\Omega_\alpha} + \frac{k_\beta}{\Omega_\beta} + \frac{k_2}{\Omega_2} - \frac{3\upsilon_{z0}^2 \gamma_0^2}{c^2} \right)$$

$$C_{3,\beta}^{II} = -k_\beta C_{3,\alpha}^{II} / k_\alpha$$

$$C_{3,2}^{II} = -k_2 C_{3,\alpha}^{II} / k_\alpha$$

$$C_0 = -\mathrm{i} k_2 \omega_p^2 / (\Omega_2^2 \gamma_0^2)$$

$$\omega_p^2 = 4\pi n_0 e^2 / m_e$$

$$\gamma_0 = 1 \Big/ \sqrt{1 - (\upsilon_{z0} / c)^2}$$

$$\Omega_\chi = \omega_\chi - k_\chi \upsilon_0$$

In these equations $\omega_\alpha = \omega_\beta = \omega_1$, $\omega_2 = 0$, e and m_e are values of the charge and mass of the electron.

We should note that in these equations the dispersion functions for electromagnetic wave D_1, slow D_α and fast D_β SCWs are zero because they are the natural waves; but $D_2(k_2) \neq 0$.

4. ANALYSIS

To carry out numeric analysis, let us consider the SCW amplification section with the following parameters: initial plasma frequency of the electron beam $\omega_p = 3.0 \cdot 10^{11}$ s^{-1}, initial constant term of the relativistic factor $\gamma_0 = 3.5$. According to (15), the period of the pump electric field is $\Lambda = 1.97$ cm. The period of the magnetic undulator is $\Lambda_B = 1.96$ cm. The magnitude of the electrostatic undulator pump field strength is $|E_{20}| = 28$ kV/cm; the magnitude of the magnetic undulator pump field induction is $|B_2| = 0.03$ T. The initial magnitude of the electric strength of the input electromagnetic signal wave is $E_1 = 1.29 \cdot 10^{-2}$ kV/cm. The cyclic frequency of the input signal is $\omega_1 = 0.628 \cdot 10^{12}$ s^{-1}.

Fig. 2 shows the dependence of the electric field strength magnitude of the electromagnetic signal $|E_1|$ on the coordinate z. Graph 1 corresponds to a standard H-ubitron FEL without an electrostatic wave amplification section, graph 2 corresponds to the undulator pump electric field without considering the effect of electrostatic pumping field generation, and graph 3 takes into account both the action of the undulator pump electric field and the effect of additional electro-static pumping field generation. [13] describes the influence of the electrostatic pumping field generation effect on the dynamics of longitudinal SCWs in detail. In this work, unlike [13], we analyze the influence of this effect on the dynamics of ***the transverse electro-magnetic signal wave***.

From Fig. 2, we see that the additional amplification of the longitudinal wave field in the amplification section increases the growth increment and reduces the saturation length of the electromagnetic signal by 11.8% (compare graphs 1 and 2). The effect of electrostatic pumping field generation further increases the growth increment and reduces the saturation length by 6.5% (compare graphs 3 and 2). Overall, the combined action of the amplification section and the effect of electrostatic pumping field generation leads to a reduction in the saturation length by 18.3% (compare graphs 1 and 3), and therefore also reduces the longitudinal dimensions of the FEL.

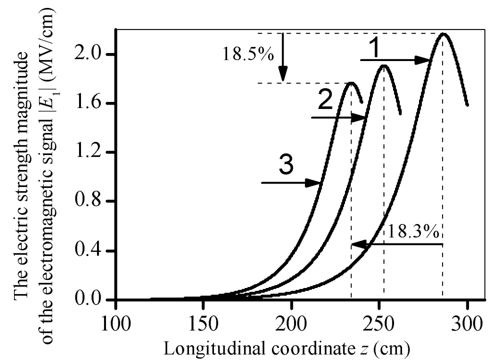

Fig. 2 – The dependence of the electric strength magnitude of the electromagnetic signal $|E_1|$ on the longitudinal coordinate z under three conditions: without any pump electric field (graph 1); with the undulator pump electric field but without the effect of electrostatic pumping field generation (graph 2); with the undulator pump electric field and the effect of electro-static pumping field generation (graph 3)

At the same time, it should be noted that the saturation level in the FEL with the amplification section and considering the effect of electrostatic pumping field generation is 18.5% lower than in the FEL without the amplification section (Fig. 2). To compensate for this drawback, we introduce a quasi-electrostatic support field into the studied system. Fig. 3 shows the dependence of the magnitude of the transverse electro-magnetic signal wave amplitude on the coordinate z in the presence of a quasi-electrostatic support field with the parameters $z_{E0} = 200.0$ cm and $\tau_{E0} = 0.0$ V/cm^2 (graph 1), $\tau_{E0} = -42.0$ V/cm^2 (graph 2), $\tau_{E0} = -112.0$ V/cm^2 (graph 3), $\tau_{E0} = -224.0$ V/cm^2 (graph 4).

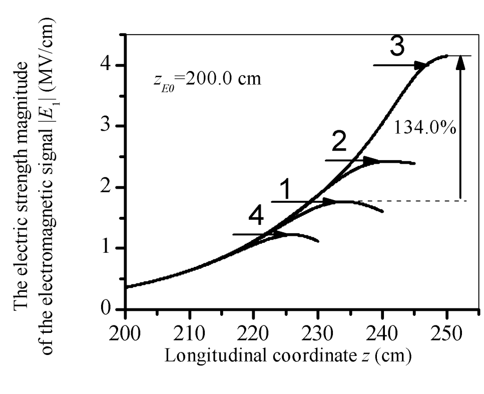

Fig. 3 – Dependences of the electric strength magnitude of the electromagnetic signal $|E_1|$ on the longitudinal coordinate z at different values of the parameter τ_{E0}. Graph 1 corresponds to $\tau_{E0} = 0.0$ V/cm^2, 2 – $\tau_{E0} = -42.0$ V/cm^2, 3 – $\tau_{E0} = \tau_{E0\mathrm{opt}} = 112.0$ V/cm^2, 4 – $\tau_{E0} = -224.0$ V/cm^2. The parameter $z_{E0} = z_{E0\mathrm{opt}} = 200$ cm is the same for all dependencies

Fig. 3 demonstrates that the saturation level of the electromagnetic signal wave significantly depends on the quasi-electrostatic support field, namely on its parameters z_{E0} and τ_{E0}. Fig. 3 shows four graphs corresponding to the same $z_{E0} = 200.0$ cm, but different τ_{E0}. This figure shows that there is an optimal value of the parameter $\tau_{E0} = \tau_{E0opt} = -112.0$ V/cm^2, at which the maximum saturation level of the electromagnetic signal wave is achieved (graph 3). The saturation level of the electromagnetic wave, which corresponds to the optimal quasi-electrostatic support field, increases by 134.0% compared to the case when such a field is absent.

5. CONCLUSION

Thus, in this work, we have, for the first time, developed a cubic nonlinear theory of a parametric superheterodyne H-ubitron type FEL with a longitudinal wave amplification section. We have determined the lengths and saturation levels of such devices. The studied model considers the effect of electrostatic pumping field generation and the influence of a quasi-electrostatic support field.

We have found that a longitudinal wave amplification section allows us to reduce the saturation length of the H-ubitron FEL by 11.8%. Considering the effect of the electrostatic pumping field generation we can reduce the FEL saturation length by another 6.5%. In total, the presence of the amplification section and consideration of the generated pump electric field reduces the saturation length by 18%.

We have investigated the influence of a quasi-electrostatic support field (8) on the dynamics of the electromagnetic signal wave of the parametric superheterodyne FEL. We have found the optimal parameters of such a quasi-electrostatic support field, for which the saturation level of the electromagnetic wave increases by 134%.

Therefore, using a parametric superheterodyne H-ubitron type FEL with a longitudinal SCW amplification section has several advantages compared to a traditional H-ubitron FEL. These advantages include a reduction in the longitudinal dimensions of the entire FEL by 18% and an increase in the saturation level of the electromagnetic signal (by 134%). It should also be considered that this article analyzes the simplest form of the quasi-electrostatic support field (linear). We believe that other forms of support fields are possible and may be more effective than the one studied.

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Кубічно-нелінійна теорія супергетеродинного параметричного Н-убітронного ЛВЕ з секцією підсилення повздовжніх хвиль просторового заряду

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В роботі проведено аналіз хвильової динаміки параметричного супергетеродинного Н-убітронного лазера на вільних електронах (ЛВЕ) з секцією підсилення поздовжньої хвилі в рамках кубічного нелінійного наближення. Ключовою особливістю досліджуваного лазера на вільних електронах є реалізація двох взаємопов'язаних трихвильових параметричних резонансів. Перший параметричний резонанс виникає між електромагнітним сигналом і магнітним полем Н-убітронної накачки. Другий – між поздовжнім полем електростатичного ондулятора, швидкими та повільними хвилями просторового заряду. Другий параметричний резонанс забезпечує додаткове підсилення повільної хвилі просторового заряду. У досліджуваній моделі враховано ефект генерації електростатичного поля накачки, проведе-но аналіз впливу квазіелектростатичного поля підпору, визначено довжини та рівні насичення ЛВЕ. Продемонстровано, що за рахунок додаткового підсилення повільної хвилі просторового заряду у секції підсилення поздовжньої хвилі, а також з урахуванням ефекту генерації електростатичного поля накачки, довжина насичення електромагнітної хвилі сигналу зменшується на 18%. Для підвищення рівня насичення вихідного електромагнітного сигналу використано квазіелектростатичне поле підпо-ру, в якому напруженість електростатичного поля лінійно зростає, починаючи з певної координати. Показано, що оптимальне квазіелектростатичне поле підпору дозволяє підвищити вихідний рівень насичення електромагнітного сигналу більш ніж на 130%. Таким чином, використання параметрично-го супергетеродинного Н-убітронного лазера на вільних електронах з поздовжньою секцією підсилення хвиль просторового заряду та квазіелектростатичним полем підпору має низку переваг порівняно з традиційним Н-убітронним лазером на вільних електронах, а саме зменшення поздовжніх розмірів всього пристрою та збільшення потужності вихідного електромагнітного сигналу.

Ключові слова: Супергетеродинний лазер на вільних електронах, Хвилі просторового заряду, Трихвильовий параметричний резонанс, Інкремент зростання, Електростатичний ондулятор