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REGULAR ARTICLE

Dynamics of Anisotropic Universes in LRS Bianchi Type-*V* Cosmological Model with Perfect Fluid in Generic Viable Non-Minimally Coupled *f*(*R*, *T*) Gravity

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The LRS Bianchi Type-*V* cosmological model has been extensively studied within the framework of general relativity and other modified gravity theories. In this paper, we investigate the dynamics of the LRS Bianchi Type-*V* cosmological model in the presence of a perfect fluid and non-minimally coupled *f*(*R*, *T*) gravity. We derive the field equations for the general non-minimally coupled $f\left(R,T\right)=\alpha_1 R^m T^n+\alpha_2 T\left(1+\alpha_3 T^p R^q\right)$ gravity and study their implications on the evolution of the universe. We consider a perfect fluid as the matter source and explore its influence on the dynamics of the cosmological model. We analyze the behavior of the scale factor, energy density, and pressure in the early and late-time regimes. Furthermore, we investigate the stability of the solutions and their compatibility with observational data. Our findings shed light on the role of the non-minimal coupling and perfect fluid in shaping the cosmological evolution in the LRS Bianchi Type-*V* model. This research contributes to our understanding of the interplay between modified gravity theories, cosmology and the nature of dark energy.

Keywords: LRS Bianchi Type-*V*, Cosmological Model, *f*(*R*, *T*) gravity, Perfect fluid

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1. INTRODUCTION

The study of cosmology aims to understand the origin, evolution, and large-scale structure of the universe. General relativity, Einstein's theory of gravity, has provided a remarkable framework for describing the gravitational interaction on cosmic scales. However, in recent years, the limitations of general relativity in explaining certain observed phenomena, such as the accelerated expansion of the universe, have led to the exploration of modified gravity theories

Cosmological research has undergone significant advancements in the past three decades, driven by observational cosmology studies. Observational evidence strongly suggests that the expansion of the universe is accelerating [1-10]. The findings from experiments such as the Planck collaboration [11], Baryon Oscillation Spectroscopic Survey (BOSS) [12], and Atacama Cosmology Telescope Polarimeter (ACTPol) Collaboration [13] have provided crucial experimental confirmation of the universe's accelerated expansion. Additionally, high-redshift supernova experiments (HRSSE) [14-15], cosmic microwave background (CMB) fluctuations [6, 8] and large-scale structure observations [4] have offered indirect evidence supporting cosmic acceleration.

To explain the observed accelerated expansion, two primary approaches have been pursued. The first involves introducing a dark energy component to the universe and investigating its dynamic behavior. The second strategy focuses on modifying the theory of general relativity itself. Both approaches present novel features along with significant theoretical challenges. In this paper, we specifically explore modified gravity theories. General relativity has undergone various modifications in recent decades to account for the observed phenomena.

Astrophysical observations indicate that the accelerated expansion is driven by an exotic form of energy with large negative pressure, commonly referred to as dark energy. Despite extensive observational evidence, the nature of dark energy remains a challenging problem in modern cosmology. Modified theories of gravitation have been proposed to shed light on this mysterious component. Researchers have sought to study late-time acceleration and investigate dark energy by modifying general relativity, specifically by modifying the geometric part of the Einstein-Hilbert action [16]. This approach provides an effective means to explore dark energy, considering the negative pressure it creates, which leads to faster-than-usual expansion of the universe. Experimental data, such as those from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite, suggest that approximately 73 % of the universe is composed of dark energy, with non-baryonic dark matter accounting for 23 %, and ordinary baryonic matter and radiation constituting the remaining 4 %. Among the modifications to the Einstein-Hilbert action, the *f*(*R*) modified theory of gravity proposed by Nojiri and Odintsov [17] has garnered significant attention. Recently, Harko *et al.* [18] introduced the *f*(*R*, *T*) modified theory of gravity, where the gravitational Lagrangian is an arbitrary function of the Ricci scalar *R* and the trace *T* of the stress-energy

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tensor. In addition to the equations of motion for test particles, they derived the gravitational field equations within the metric formalism, considering the covariant divergence of the stress-energy tensor.

By considering the metric-dependent Lagrangian density, the relevant field equation for $f(R, T)$ gravity is derived from the Hilbert-Einstein variation principle, given by the action:

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R,T) + L_m\right) d^4x \,, \quad (1)$$

Here, $f(R, T)$ represents an arbitrary function of the Ricci scalar R and the trace T of the energy–momentum tensor T_{ij} of the matter source, L_m is the matter Lagrangian density, g is the determinant of the metric tensor, and G is the gravitational constant. The energy-momentum tensor T_{ij} arises from the matter Lagrangian and is defined as:

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}} \,, \quad (2)$$

with $T = g^{ij}T_{ij}$ .Here, we have assumed that the matter Lagrangian L_m depends only on the metric tensor component g_{ij} rather than its derivatives. Hence, we obtain

$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}} \,. \quad (3)$$

The $f(R, T)$ gravity field equations are obtained by varying the action S with respect to metric tensor ($g_{\mu\nu}$).

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} + [g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j] f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \quad (4)$$

Where

$$f_R = \frac{\delta f(R,T)}{\delta R} \,, f_T = \frac{\delta f(R,T)}{\delta T} \,, \Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}} \,.$$

Here ∇ is the covariant derivative and T_{ij} is usual matter energy-momentum tensor derived from the Lagrangian L_m It is mentioned here that these field equations depend on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible; However, Harko *et al.*[18] gave three classes of these models $f(R,T) = f_1(R) + f_2(T)$.

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R) f_3(T) \end{cases} \quad (5)$$

The individual field equation for $f(R,T)$ gravity is given as

1. $f(R,T) = R + 2f(T)$

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \quad (6).$$

2. $f(R,T) = f_1(R) + f_2(T)$

$$f_1'(R)R_{ij} - \frac{1}{2} f_1(R)g_{ij} + [g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j] f_1'(R) = 8\pi T_{ij} - f_2'(T)T_{ij} - f_2'(T)\theta_{ij} + \frac{1}{2} f_2(T)g_{ij} \,. \quad (7)$$

If $L_m = p$ then $\theta_{ij} = -2T_{ij} - pg_{ij}$ (8)

The field equations depend on the specific matter field properties. So we need to choose a viable $f(R, T)$ model in order to represent our results in a meaningful way. Harko *et al.* [18] proposed three classes of $f(R, T)$ models, and we focus on one specific model proposed by Sharif and Zubair [19]

$$f(R,T) = \alpha_1 R^m T^n + \alpha_2 T(1 + \alpha_3 T^p R^q) \quad (9)$$

where α_i' *s* are positive real numbers , whereas m, n, p, q assumes some value greater than or equal to 1. We will analyze our results considering different cases of above mentioned model and we will precede our further discussion under following three cases

1. $f(R,T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2$ for

$m = 1, n = 0,\ \alpha_4 = \alpha_1\alpha_3, p = 1, q = 0$ (10)

2. $f(R,T) = R + \alpha_2 T$ for $\alpha_1 = 1,\ m = 1, n = 0,\ \alpha_3 = 0$ (11)

3. $f(R,T) = \alpha_1 R + \alpha_2 T(1 + \alpha_3 T\ \ R^2)$ (12)

Using equations (6), (7) and (8) along with equations (10), (11) and (12), we have

Model-I:- $f(R,T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2$ for $m = 1, n = 0,\ \alpha_4 = \alpha_1\alpha_3, p = 1, q = 0$ is

$$R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi}{\alpha_1} T_{ij} + \left[\frac{\alpha_2}{\alpha_1} + 2\alpha_3 T\right] \times \times [T_{ij} + pg_{ij}] + \frac{1}{2}\left[\frac{\alpha_2}{\alpha_1} T + \alpha_3 T^2\right] g_{ij} \quad (13)$$

Model-II:- $f(R,T) = R + \alpha_2 T$ for $\alpha_1 = 1,\ m = 1, n = 0,\ \alpha_3 = 0$

$$R_{ij} - \frac{1}{2} R g_{ij} = [8\pi + \alpha_2] T_{ij} + \left[p\alpha_2 + \frac{1}{2}\alpha_2 T\right] g_{ij} \quad (14)$$

In this paper, we investigate the dynamics of anisotropic universes within the LRS Bianchi Type-V cosmological model, incorporating the presence of a perfect fluid as the matter source. We analyze the specific model in the Generic Viable Non-Minimally Coupled $f(R, T)$ Gravity theory proposed by Sharif and Zubair [19]. To achieve this, we derive the field equations for the specific mo del of Generic Viable Non-Minimally Coupled $f(R, T)$ Gravity theory coupled with a perfect fluid in the LRS Bianchi Type-*V* cosmological model. So many researcher studied on $f(R, T)$ [20-26]. We analyze the resulting equations to explore the dynamics of the anisotropic universe, examining the evolution of the scale factor, energy density, pressure, and other relevant cosmological quantities. We also investigate the stability of the solutions and discuss their compatibility with observational data. Researcher also studied on cosmological model and behaviors of universe [27-31].

The outcomes of this research will deepen our understanding of the interplay between modified gravity theories, such as $f(R, T)$ gravity, and the dynamics of anisotropic universes. Additionally, this study will contribute valuable insights into the role of non-minimal coupling and the behavior of perfect fluids in shaping the evolution and structure of the universe.

This paper is organized as follows: In Section 2 we derive the field equations and analyze the resulting equations to explore the dynamics of the anisotropic universe Subsection 2.1 and 2.2 presents a detailed description of the LRS Bianchi Type-*V* cosmological model and the inclusion of a perfect fluid and pressure for different model of Generic Viable Non-Minimally Coupled $f(R, T)$ gravity. Section 3 Discussed Physical parameters as Hubble's parameter H, expansion scalar and shear scalar, anisotropy parameters In Subsection 3.1, we calculate Jerk Parameter which is one of the parameter used to describing dynamics of the anisotropic universe. Subsection 3.2 analyze behavior of state-finder diagnostic $\{r, s\}$ and observed behavior or r vs s. In Section 4 discusses their implications for our understanding of anisotropic universes in $f(R, T)$ gravity and analyze the result and finally Section 5 is summary and conclusion about all results.

Throughout the paper, we refer to relevant literature and cite previous studies that have contributed to the understanding of general relativity, modified gravity theories, $f(R, T)$ gravity, LRS Bianchi Type-*V* cosmological model, and related topics

2. METRIC, FIELD EQUATIONS AND SOLUTIONS

Bianchi type-*V* cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe has a greater generality than isotropic models. The simplicity of the field equations and relative ease of solutions made Bianchi space times useful in constructing models of spatially homogeneous and anisotropic cosmologies.

The LRS Bianchi type-V line element is

$$ds^2 = dt^2 - A^2dx^2 - e^{-2x}B^2(dy^2 + dz^2), \qquad (15)$$

where A and B are the scale factors and function of cosmic time t only.

We have assumed the stress energy tensor of matter as

$$T_{ij} = (p+\rho)u_i u_j - pg_{ij}, \qquad (16)$$

where $u_i = (0,0,0,1)$ is the four-velocity vector in co-moving coordinate system satisfying $u_i u_j = -1$.

2.1 Model-I: $f(R,T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2$ for $m=1, n=0,\ \alpha_4 = \alpha_1\alpha_3, p=1, q=0$

From (14) field equation with (11) for the metric (15) obtained as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = X_1 p + X_2 p^2 + X_3 p\rho + X_4\rho + X_5\rho^2 \qquad (17)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = X_1 p + X_2 p^2 + X_3 p\rho + X_4\rho + X_5\rho^2 \qquad (18)$$

$$\frac{\dot{B}^2}{B^2} - \frac{2\dot{A}\dot{B}}{AB} + \frac{3}{A^2} = Y_1 p + Y_2 p^2 + Y_3 p\rho + Y_4\rho + Y_5\rho^2 \qquad (19)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \qquad (20)$$

Where dot (.) indicate the derivative with to t and

$$X_1 = \frac{8\pi}{\alpha_1} + \frac{7\alpha_2}{2\alpha_1},\ X_2 = \frac{33\alpha_3}{2},\ X_3 = -7\alpha_3,\ X_4 = -\frac{\alpha_2}{2\alpha_1},$$

$$X_5 = \frac{\alpha_3}{2}\ (21)\ Y_1 = \frac{5\alpha_2}{2\alpha_1},\ Y_2 = \frac{21\alpha_3}{2},\ Y_3 = -11\alpha_3$$

$$Y_4 = -\frac{8\pi}{\alpha_1} - \frac{3\alpha_2}{2\alpha_1}\quad Y_5 = \frac{5\alpha_3}{2} \qquad (22)$$

From equation (20) we have

$$A = nB \qquad (23)$$

From equation (17) to (19) and (22) we have

$$\frac{3\ddot{B}}{B} + \frac{5\dot{B}^2}{B^2} - \frac{5}{n^2B^2} = (2X_1+Y_1)p + (2X_2+Y_2)p^2 + (2X_3+Y_3)p\rho + (2X_4+Y_4)\rho + (2X_5+Y_5)\rho \qquad (24)$$

The deceleration parameter is

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \qquad (25)$$

Where a is the average scale factor. We have three equation (17)-(19) involving four parameters as A, B, p, ρ

In order to solve these equations, we assume the time varying deceleration parameter as

$$q = -1 + \frac{\beta}{1+a^{\beta}} \qquad (26)$$

Where $\beta > 0$ is a constant

The mean Hubble parameter H for LRS Bianchi type-I is

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \tag{27}$$

The directional Hubble parameter H for above cosmological model is

$$H_x = \frac{\dot{A}}{A} \text{ and } H_y = H_z = \frac{\dot{B}}{B} \tag{28}$$

Setting $a(t) = \dfrac{1}{(1+z)}$, (29)

where z is red shift, leads to relation

$$q = \frac{\beta}{\left(\frac{1}{z+1}\right)^{\beta} + 1} - 1 \tag{30}$$

The volume is defined

$$V = a^3 = AB^2 \tag{31}.$$

Using (26) in (31), the values of metric potential A, B are obtained as

$$a = \left[e^{\beta t} - 1\right]^{1/\beta} \tag{32}$$

$$A = \left[e^{\beta t} - 1\right]^{2/\beta},\ B = \left[e^{\beta t} - 1\right]^{1/2\beta} \tag{33}$$

Accordingly, metric (15) takes the form

$$ds^2 = dt^2 - \left[e^{\beta t} - 1\right]^{4/\beta} dx^2 - e^{-2x}\left[e^{\beta t} - 1\right]^{1/\beta}(dy^2 + dz^2) \tag{34}$$

To solve above field equation now we consider relation

$$p = \omega\rho \tag{35}$$

From (17) to (19) and from we get

$$\frac{3\ddot{B}}{B} + \frac{5\dot{B}^2}{B^2} - \frac{5}{n^2B^2} = T_1\rho^2 + T_2\rho \tag{36}$$

Where

$T_1 = \dfrac{87}{2}\alpha_3\omega^2 - 25\alpha_3\omega + 7\dfrac{\alpha_3}{2}$, and

$$T_2 = \left[\frac{16\pi}{\alpha_1} + \frac{19\alpha_2}{2\alpha_1}\right]\omega - \left[\frac{8\pi}{\alpha_1} + \frac{5\alpha_2}{2\alpha_1}\right] \tag{37}$$

Now from value of A and B equation (37) we have

$$T_1\rho^2 + T_2\rho - \left[\frac{-3\beta}{4} + \frac{3\beta}{2}e^{-\beta t} - \frac{5}{4}\right]\left[e^{\beta t} - 1\right]^{-2} e^{2\beta t} + \frac{5}{n^2}[e^{\beta t} - 1]^{-1/\beta} = 0 \tag{38}$$

Equation obtained in (38) is quadratic equation in ρ then

$$\rho = \frac{-T_2(1+z)^{-2\beta} \pm \left[T_2^2(1+z)^{-2\beta} + 4T_1\left(\frac{-3\beta-5}{4}\right)\left[(1+z)^{-2\beta} + 2(1+z)^{-\beta} + 1\right] + \frac{3\beta}{2}[(1+z)^{-\beta} + 1] + \frac{5}{n^2}(1+z)^{1-2\beta}\right]^{1/2}}{2T_1(1+z)^{-\beta}} \tag{39}$$

$$p = \omega\frac{-T_2(1+z)^{-2\beta} \pm \left[T_2^2(1+z)^{-2\beta} + 4T_1\left(\frac{-3\beta-5}{4}\right)\left[(1+z)^{-2\beta} + 2(1+z)^{-\beta} + 1\right] + \frac{3\beta}{2}[(1+z)^{-\beta} + 1] + \frac{5}{n^2}(1+z)^{1-2\beta}\right]^{1/2}}{2T_1(1+z)^{-\beta}} \tag{40}$$

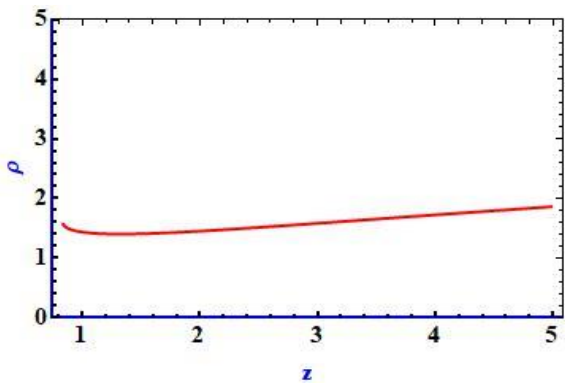

Fig. 1 – Variation of energy density against redshift z

2.2 Model-II: $f(R,T) = R + \alpha_2 T$ for $\alpha_1 = 1, m = 1, n = 0, \alpha_3 = 0$

From (14) field equation with (11) for the metric (15) obtained as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = [8\pi + \frac{7}{2}\alpha_2]p - \frac{1}{2}\alpha_2\rho \tag{41}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = [8\pi + \frac{7}{2}\alpha_2]p - \frac{1}{2}\alpha_2\rho \tag{42}$$

$$\frac{\dot{B}^2}{B^2} - \frac{2\dot{A}\dot{B}}{AB} + \frac{3}{A^2} = \frac{5}{2}\alpha_2 p - [8\pi + \frac{3}{2}\alpha_2]\rho \tag{43}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{44}$$

Dot represents derivatives with respect to time.
From equation (41) to (44)

$$\frac{3\ddot{B}}{B} + \frac{5\dot{B}^2}{B^2} - \frac{5}{n^2B^2} = L_1 p + L_2\rho \tag{45}$$

Where

$$L_1 = 16\pi + \frac{9\alpha_2}{2},\ L_2 = -8\pi - \frac{5\alpha_2}{2} \tag{46}$$

From equations (45), (46) and (33) we have

$$\left(\frac{-3\beta}{4} + \frac{3\beta}{2}e^{-\beta t} - \frac{5}{4}\right)\left(e^{\beta t} - 1\right)^{-2} e^{2\beta t} + \frac{5}{n^2}(e^{\beta t} - 1)^{-1/\beta} = L_1 p + L_2\rho \tag{47}$$

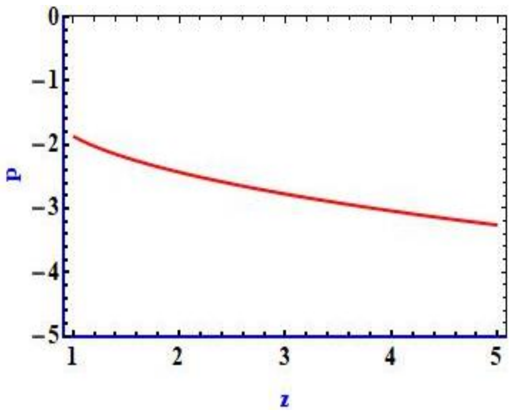

Fig. 2 – Variation of Pressure against redshift z

From equation (30) and (47) energy density and pressure are

$$\rho = \frac{\left(\frac{3\beta}{4} + \frac{3\beta}{2}(1+z)^{-\beta} - \frac{5}{4}\right)(1+z)^{2\beta} + \frac{5}{n^2}(1+z)}{\left(16\pi + \frac{9\alpha_2}{2}\right)\omega + \left(-8\pi - \frac{5}{2}\alpha_2\right)} \tag{48}$$

$$p = \omega \frac{\left[\frac{3\beta}{4} + \frac{3\beta}{2}(1+z)^{-\beta} - \frac{5}{4}\right](1+z)^{2\beta} + \frac{5}{n^2}(1+z)}{\left(16\pi + \frac{9\alpha_2}{2}\right)\omega + \left(-8\pi - \frac{5}{2}\alpha_2\right)} \tag{49}$$

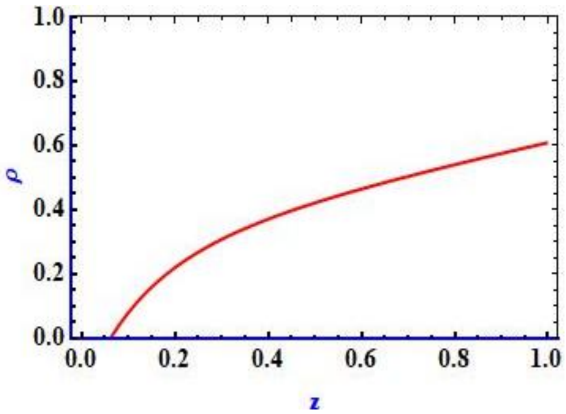

Fig. 3 – Variation of energy density against redshift z

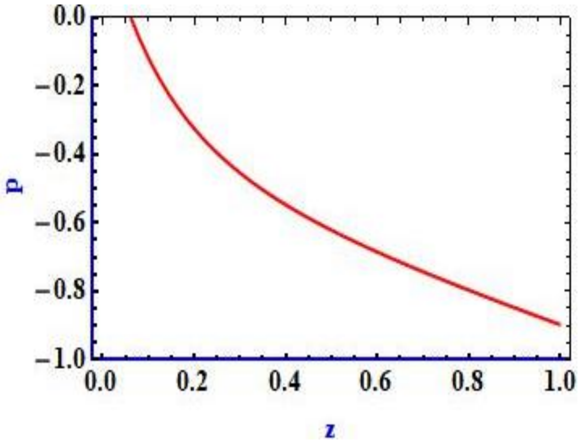

Fig. 4 – Variation of pressure against redshift z

3. PYSICAL PROPERTIES

The spatial volume is given by

$$V = a^3 = AB^2 \tag{50}$$

The Hubble's parameter H, expansion scalar and shear scalar are

$$H = \frac{1}{3}(H_x + H_y + H\ ) = \frac{e^{\beta t}}{\left[e^{\beta t} - 1\right]^{-1}} \tag{51}$$

$$\theta = 3H = 3e^{\beta t}\frac{1}{[e^{\beta t} - 1]} \tag{52}$$

$$\sigma^2 = \frac{1}{2}\left(H_x^2 + H_y^2 + H_z^2 - \frac{\theta^2}{3}\right) = \frac{3}{4}e^{2\beta t}\frac{1}{[e^{\beta t} - 1]^2} \tag{53}$$

The anisotropy parameter

$$\Delta = \frac{1}{3}\sum_{x=1}^{3}\left(\frac{H_x - H}{H}\right)^2 = 6\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{2} \tag{54}$$

3.1 Jerk Parameter

The jerk parameter is considered as one of the important quantities for describing the dynamics of the universe. Jerk parameter is dimensionless third derivative of scale factor a with respect to cosmic time t and is defined as

$$j = q + 2q^2 - \frac{\dot{q}}{H}\,. \tag{55}$$

From equation (55) and (30), we have

$$\Rightarrow j = 1 - \frac{3\beta}{(1+z)^{-\beta} + 1} + \frac{\beta^2}{(1+z)^{-\beta} + 1} + \frac{\beta^2}{[(1+z)^{-\beta} + 1]^2} \tag{56}$$

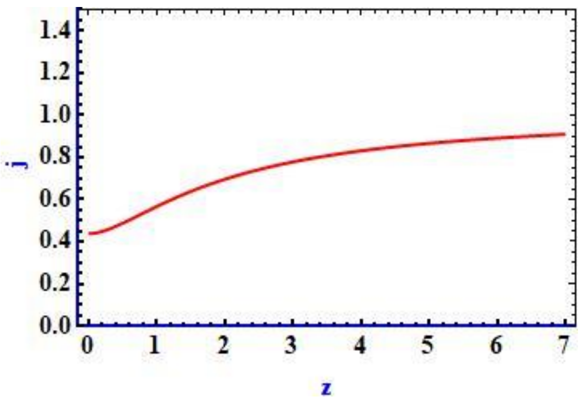

Fig. 5 – Behavior of jerk parameter j versus z with $\beta = 1.5$

3.2 Statefinder Diagnostic

The state-finder pair $\{r, s\}$ is defined as

$$r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3}\,, \quad s = \frac{r - 1}{3(q - \frac{1}{2})} \tag{57}$$

The state-finder pair is a geometrical diagnostic parameter, which is constructed from a space-time metric directly and it is more universal compared to physical variable, which dependents on the properties of physical fields describing DE, since physical variable are model dependent.

The values of the state-finder parameter for our model are from equation (52) and (25) we have

$$r = 1 - \frac{3\beta}{(1+z)^{-\beta}+1} + \frac{\beta^2[(1+z)^{-\beta}+2]}{[(1+z)^{-\beta}+1]^2} \quad (58)$$

$$s = \frac{1}{6\beta - 9[(1+z)^{-\beta}+1]}\{2\beta^2((1+z)^{-\beta}+2) - 6\beta[(1+z)^{-\beta}+1]\} \quad (59)$$

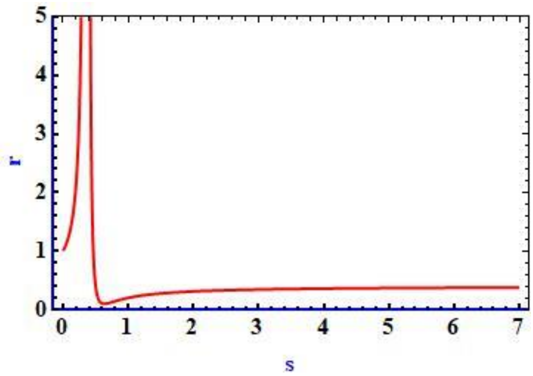

Fig. 6 – Behavior of r vs s

4. DISCUSSION AND ANALYSIS OF THE RESULT

Figure 1 illustrates that in the case of Model-I, the energy density of the Universe exhibits a increasing trend over redshift z and eventually as $z \to \infty$ energy density in increasing order. When it comes to pressure (see Fig. 2), it is a decreasing function of z and takes the negative values throughout cosmic evolution. Energy density and pressure defined for all value except value $z = -1$. The negative pressure is evidence of an accelerating phase of the Universe as shown by recent observations, and thus the validity of our model.

For Model –II Figure 3 shows that, A increasing function of z a describes the energy density of the universe and from figure 4 pressure is negative and decreasing function of z and $p \to$ -1 as $z \to$2 and takes the negative values throughout cosmic evolution

Figure 5 is behavior of jerk parameter over redshift z initially it is increasing and after $z \to 8$ and $j \to 0.9$ i.e. it attends some constant value and takes the positive values over cosmic evaluation and relation between statefinder diagnostic r over s is shown in Figure 6, here r attends high value as $s \to 0.2$ and it will have value 0 at $s \to 0.7$ and after $s = 2.7$ value of r act as constant

5. CONCLUDING REMARK

In conclusion, this research significantly contributes to the understanding of anisotropic universes and their evolution in the context of the LRS Bianchi Type-V cosmological model. By exploring the dynamics within the non-minimally *coupled* $f(R, T)$ gravity framework, the study bridges the gap between modified gravity theories and observed cosmic phenomena. The findings shed light on the intricate interplay between non-minimal coupling, perfect fluids, and anisotropic expansion.

The meticulous derivations, equations, and analyses presented in this paper offer a solid foundation for further exploration of modified gravity theories and their implications for cosmology. The integration of mathematical rigor with physical concepts demonstrates a robust methodology. Moreover, the research's emphasis on compatibility with observational data underscores its relevance in the context of real-world cosmological observations.

In essence, this study advances our understanding of how modified gravity theories, particularly the non-minimally coupled $f(R, T)$ gravity, influence the evolution and structure of anisotropic universes. The research contributes to the broader field of cosmology and offers potential avenues for future investigations into the nature of dark energy, cosmic expansion, and the underlying principles governing the universe's dynamics.

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Динаміка анізотропних всесвітів у космологічній моделі LRS Б'янкі типу-*V* з ідеальною рідиною в загальній життездатній не мінімально пов'язаній *f*(*R*, *T*) гравітації

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Космологічна модель LRS Б'янкі типу-*V* була широко вивчена в рамках загальної теорії відносності та інших модифікованих теорій гравітації. У цій статті ми досліджуємо динаміку космологічної моделі LRS Б'янкі типу-*V* за наявності ідеальної рідини та немінімально пов'язаної *f*(*R*, *T*) гравітації. Стаття присвячена розрахунку рівняння поля для загальної немінімально пов'язаної гравітації $f\left(R,T\right)=\alpha_1 R^m T^n+\alpha_2 T\left(1+\alpha_3 T^p R^q\right)$ та вивчаємо їх вплив на еволюцію Всесвіту. Розглядають ідеальну рідину як джерело матерії та досліджують її вплив на динаміку космологічної моделі. Проаналізована поведінка масштабного фактора, щільності енергії та тиску в режимах раннього та пізнього часу. Крім того, досліджена стабільність рішень та їх сумісність із даними спостережень. Висновки відображають роль немінімального зв'язку та ідеальної рідини у формуванні космологічної еволюції в моделі LRS Б'янкі типу-*V*. Це дослідження дає розуміння взаємодії між модифікованими теоріями гравітації, космологією та природою темної енергії.

Ключові слова: LRS Б'янкі типу-*V*, Космологічна модель, *f*(*R, T*) гравітація, Ідеальна рідина.