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REGULAR ARTICLE

Analysis of Buckling Behavior of Functionally Graded Plates Under Mechanical Loading

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In this research paper a refined shear and deformation theory for buckling of functionally graded FGM plates under mechanical loading is presented. The shear stress variation through the thickness in a parabolic form is accounted for in this theory, and satisfies the condition of the transversal shear stress null on the upper and bottom edges of the plate without using shear correction coefficients. Unlike the conventional other shear stress and deformation theories, only four unknowns involved in the proposed refined theory, which has many resemblances to traditional plate theory. boundary conditions, equilibrium equations and stress expressions. The properties of the functionally graded plate are varying following a power law distribution of the fraction's volume of the constituents. Equilibrium equations are derived from the virtual works principle. The solution of simply supported functionally graded plates is deduced and their results are compared against those of first-order theory and higher-order theories. It can be said that the suggested theory is effective and accurate in determining the buckling behavior of FGM plates.

Keywords: FGM plate, Refined plates theory, Mechanical buckling, Navier's solution, Virtual work principle.

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1. INTRODUCTION

Thanks to their mechanical and thermal features, functionally graded materials have become increasingly used as components in the innovative structures during the last years. Furthermore, FGM structures have a high thermal and mechanical resistance then its peers in conventional materials which resulting in the improvement of structural efficiency and total weight decrease. functionally graded plates are every so often used in advanced industries and pioneering high-tech fields for their ability to resist in very hostile environment which requires the best in terms thermal insulation and mechanical endurance (Jha et al., 2013).

In this research manuscript we will present an analysis of the buckling of FGM plates under mechanical loading, where we will consider the transverse shear effect. We proposed an analytical solution, then, we deduced the mathematical formulation of critical buckling load using a refined four-variables high shear order theory that presents the shear stresses distribution in a parabolic shape and ensuring that these stresses are null on the boundaries (upper and lower) of the FGM plate, then we will compare the obtained results with those issued from the various existing theories specifically: The first-order shear theory (FSDT) (Adim et al., 2016) which takes into account the transverse shear effect however it require shear correction factors in order to ensure the nullity of these shear stresses in the superior and inferior faces of the FGM plate and the high order shear theories (HSDT) (Reddy, 1984) that ensure the nullity of the shear stresses at the superior and inferior edges of the FGM plate (Swaminathan et al, 2015).

2. PROBLEM FORMULATION

In this study, we consider a rectangular plate of total thickness "h" as shown in Fig. 1. The material properties of the FGM plate are varying gradually throw the plate's thickness.

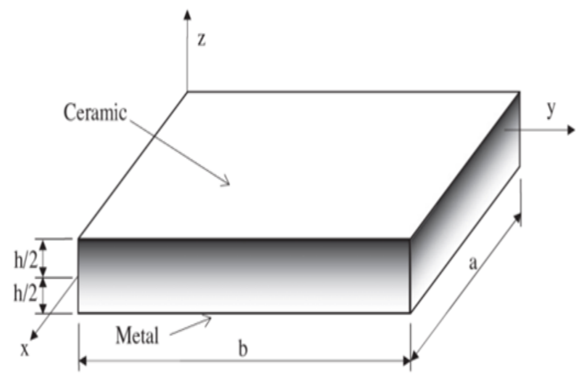

Fig. 1 – Geometry and coordinates of a rectangular FGM plate

2.1 Kinematics

The displacements of a material point located at (x, y, z) of the plate can be written as:

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$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_b}{\partial x} - \left(z - \left(h\arctan\left(\frac{z}{h}\right) - \frac{16z^3}{15h^2}\right)\right)\frac{\partial w_s}{\partial x}$$
$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_b}{\partial y} - \left(z - \left(h\arctan\left(\frac{z}{h}\right) - \frac{16z^3}{15h^2}\right)\right)\frac{\partial w_s}{\partial y} \quad (2.1)$$
$$w(x,y,z) = w_b(x,y) + w_s(x,y)$$

Where u_0 and v_0 are the median plane displacements of the plate in the x and y direction respectively, w_b and w_s are the bending and transverse displacement shear components, respectively, while $f(z)$ represents the form of the functions for determining the distribution of the transverse stresses along the thickness and is given by (Nguyen et al., 2014):

$$f(z) = z - \left(h\arctan\left(\frac{z}{h}\right) - \frac{16z^3}{15h^2}\right) \quad (2.2)$$

The constituent equations for a rectangular plate consisting of a material varying gradually throw the thickness (FGM) can be given as:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}, \begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix}Q_{44} & 0\\ 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \quad (2.3)$$

2.2 Equilibrium Equations

The equations of equilibrium are obtained using virtual work's principle, which can be written for the FGM plate as (Adim and Daouadji 2022):

$$\int_V (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz})dV$$
$$+\int_A \left(N_x^0 \frac{\partial^2(w_b + w_s)}{\partial x^2} + N_y^0 \frac{\partial^2(w_b + w_s)}{\partial y^2}\right)dA = 0 \quad (2.4)$$

2.3 The Exact Solution of Simply Supported FGM Plates

The mechanical behavior of FGM plates is studied according to the type of boundary conditions. We are concerned here with the exact solution of an FGM plate with a simple boundary condition (Daouadji et al., 2016).

To study the buckling problem, Navier assumed the displacements in the form of a double trigonometric series

$$\begin{Bmatrix}u_0\\ v_0\\ w_b\\ w_s\end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}\cos(\alpha\, x)\sin(\beta\, y)\\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y)\\ W_{bmn}\sin(\alpha\, x)\sin(\beta\, y)\\ W_{smn}\sin(\alpha\, x)\sin(\beta\, y)\end{Bmatrix} \quad (2.5)$$

3. NUMERICAL RESULTS

In this research, a new theory of shear deformation of functionally graded plates is considered, and the validation is given by comparing the present results against the solutions obtained using other theories available in the literature.

From now, the materials properties used in this study are (Lanhe, 2004):

Metal (Aluminum) $E_m = 70\,\text{GPa}, \nu = 0.3$
Ceramic (Alumina) $E_c = 380\,\text{GPa}, \nu = 0.3$

The dimensionless critical mechanical buckling load is given by

$$\bar{N} = \frac{N_{cr}\,a^2}{E_m\,h^3} \quad (3.1)$$

Table 1 shows the variation of the buckling loads of a functionally graded simply supported rectangular plates subjected to axial compression, biaxial compression and compression and tension at the same time using different shear deformation theories, where the variation of volume fraction of the components is linear ($P = 1$).

It is clearly shown that the results found by the present refined theory is in excellent convergence with those obtained by other theories (First order FSDT and the high order HSDT).

In the light of this comparison, we can see that the buckling load increases by increasing the thickness ratio a/h, in other words, from the thin plate to thick one.

The buckling load variation is presented in Tables 2 and 3 also Fig. 2(a) and 2(b) for simply supported plates under various types of mechanical loading (axial or biaxial). In each table we took two different dimensions of FGM plates (square and rectangular), the material properties decrease gradually from ceramic to metal.

From the previous results, we can observe undoubtedly that the buckling load decreases with the increase of the material property "P", which is logical since we passed from the rigid material (in this case is the ceramic) to the least rigid material (which is the metal).

Moreover, the buckling load increases with the increase of the side to thickness ratio a/h and the aspect ratio a/b for all load cases.

4. CONCLUSION

In this study, a new refined theory of deformation and shear stress was proposed to analyse the mechanical behaviour of the FGM plates subjected to mechanical loads. The analysis of the buckling of the FGM plate was presented, the results were compared to other preceding theories of shear-stress effect. The elaborated theory gives a parabolic variation of the shear strains and stresses throw the thickness of the FGM plate while guaranteeing the condition of zero shear stresses on the upper and lower edges. The accuracy and efficacy of this theory has been demonstrated for the buckling behaviour of the FGM plates. All comparative studies have demonstrated that the critical buckling loads obtained using the present refined theory of high-order shear stress and the different theories (FSDT and HSDT) are almost identical. In conclusion, we can confirm that the present refined theory is exact and simple to solve the buckling behavior of FGM plates.

Table 1 – Dimensionless critical buckling load $\bar{N}$ variation of a simply supported rectangular FGM plate ($b = 2a$, $p = 1$).

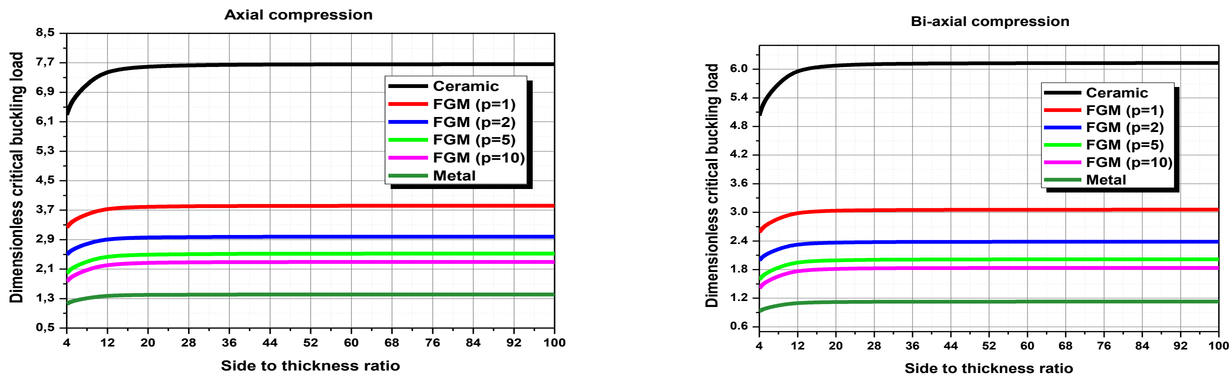
Side to thickness ratio (a/h)	Axial compression (-1,0)			Bi-axial compression (-1, -1)			Compression / tension (-1,1)		
	FSDT	HSDT	Present	FSDT	HSDT	Present	HSDT	TSDT	Present
5	3.7382	3.4163	3.4164	2.9906	2.7330	2.7331	4.9843	4.5551	4.5552
10	3.8000	3.7110	3.7111	3.0400	2.9688	2.9689	5.0667	4.9481	4.9481
20	3.8158	3.7930	3.7930	3.0526	3.0344	3.0344	5.0878	5.0573	5.0574
30	3.8188	3.8086	3.8086	3.0550	3.0468	3.0469	5.0917	5.0781	5.0781
40	3.8198	3.8140	3.8141	3.0558	3.0512	3.0513	5.0931	5.0854	5.0855
50	3.8203	3.8166	3.8166	3.0562	3.0533	3.0533	5.0937	5.0888	5.0888

Table 2 – Dimensionless buckling load $\bar{N}$ Variation of a simply supported FGM plate under axial compression $(\gamma_1 = -1, \gamma_2 = 0)$

Material	Square plate (b = a)					Rectangular plate (b = 2a)				
	Side to thickness ratio (a/h)					Side to thickness ratio (a/h)				
	5	10	20	50	100	5	10	20	50	100
Ceramic (p = 0)	16.0215	18.5786	19.3528	19.5814	19.6145	6.7204	7.4053	7.5993	7.6554	7.6635
FGM (p = 2)	6.3428	7.2629	7.5371	7.6177	7.6293	2.6450	2.8896	2.9581	2.9779	2.9808
FGM (p = 5)	5.0512	6.0346	6.3446	6.4372	6.4507	2.1479	2.4163	2.4944	2.5172	2.5205
FGM (p = 10)	4.4799	5.4525	5.7667	5.8614	5.8751	1.9210	2.1895	2.2690	2.2923	2.2957
Metal (p = ∞)	2.9513	3.4224	3.5650	3.6071	3.6132	1.2380	1.3641	1.3999	1.4102	1.4117

Table 3 – Dimensionless buckling load $\bar{N}$ Variation of a simply supported plate under bi-axial compression $(\gamma_1 = -1, \gamma_2 = -1)$

Material	Square plate (b = a)					Rectangular plate (b = 2a)				
	Side to thickness ratio (a/h)					Side to thickness ratio (a/h)				
	5	10	20	50	100	5	10	20	50	100
Ceramic (p = 0)	8.0108	9.2893	9.6764	9.7907	9.8072	5.3763	5.9243	6.0794	6.1244	6.1308
FGM (p = 2)	3.1714	3.6315	3.7685	3.8088	3.8147	2.1160	2.3117	2.3665	2.3823	2.3846
FGM (p = 5)	2.5256	3.0173	3.1723	3.2186	3.2253	1.7183	1.9330	1.9955	2.0137	2.0164
FGM (p = 10)	2.2399	2.7263	2.8833	2.9307	2.9376	1.5368	1.7516	1.8152	1.8338	1.8365
Metal (p = ∞)	1.4757	1.7112	1.7825	1.8035	1.8066	0.9904	1.0913	1.1199	1.1282	1.1294



a b

Fig. 2 – (a) – Dimensionless buckling load $\bar{N}$ Variation of an FGM rectangular (b = 2a) plate under axial compression $(\gamma_1 = -1, \gamma_2 = 0)$ as a function of the power law index "*p*" and side to thickness ratio (a/h); (b) – Dimensionless buckling load $\bar{N}$ Variation of an FGM rectangular (b = 2a) plate under bi-axial compression $(\gamma_1 = -1, \gamma_2 = -1)$ as a function of the power law index "*p*" and side to thickness ratio (a/h)

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Аналіз поведінки прогинання функціонально градуйованих пластин під механічним навантаженням

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У цій дослідницькій статті представлено уточнену теорію зсуву та деформації для вигину функціонально градуйованих FGM пластин під механічним навантаженням. Зміна напруги зсуву по товщині в параболічній формі враховується в цій теорії та задовольняє умову нульового поперечного напруження зсуву на верхньому та нижньому краях пластини без використання коефіцієнтів поправки на зсув. На відміну від звичайних інших теорій напруги зсуву та деформації, у запропонованій вдосконаленій теорії, яка має багато подібностей до традиційної теорії пластин, бере участь лише чотири невідомих. граничні умови, рівняння рівноваги та вирази напружень. Властивості функціонально градуйованої пластини змінюються відповідно до степеневого закону розподілу об'єму фракції компонентів. Рівняння рівноваги виводяться з принципу віртуальних робіт. Розв'язок функціонально градуйованих пластин з простою опорою виведено, і їх результати порівнюються з результатами теорії першого порядку та теорій вищого порядку. Можна сказати, що запропонована теорія є ефективною та точною у визначенні поведінки вигинання FGM пластин.

Ключові слова: FGM пластина, Уточнена теорія пластин, Механічне видавлювання, Рішення Нав'є, Віртуальний принцип роботи.