JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. 15 No 2, 02011(4pp) (2023)

ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том 15 № 2, 02011(4cc) (2023)

Effect of Fractional Order Time Derivative on the Out-Diffusion Profiles During Degassing a Thin Plate in Vacuum

A. Souigat[1,*], Z. Korichi[1], M.E. Mezabia[2], Y. Benkrima[1], M.T. Meftah[3]

[1] *Ecole Normale Supérieure de Ouargla, 30000 Ouargla, Algeria*
[2] *Laboratoire de Mathématiques Appliquées, Kasdi Merbah University, 30000 Ouargla, Algeria*
[3] *Department of Physics, LRPPS Laboratory, Kasdi Merbah University, 30000 Ouargla, Algeria*

(Received 25 January 2023; revised manuscript received 14 April 2023; published online 27 April 2023)

Fractional calculation methods and their applications in various scientific fields have aroused great interest in recent years. They have been used in the modeling of many physical problems, chemical processes and engineering. Recently, Caputo-Fabrizio proposed a new fractional order derivative without singular kernel, this new operator is suitable for the Laplace transformation and capable of describing physical behavior at different scales. In order to further investigate the possible application of this new operator, we used it to study the effect of fractional order time derivative on the out-diffusion profiles during degassing a thin plate in vacuum. In this work we modulate the impurity transport in the material during degassing by the time fractional diffusion equation, taking into account the Caputo-Fabrizio derivative, and we treated it by using the separation of variables method. An exact solution was obtained, depends on the time fractional-order derivative alpha. We observed that alpha plays an important role in the numerical simulations and it has an effect on the out-diffusion profiles, where the degassing is faster as alpha approaches 1, and the degassing is slow as alpha approaches zero.

Keywords: Fractional derivative, Caputo-Fabrizio, Diffusion equation, Degassing a thin plate.

DOI: 10.21272/jnep.15(2).02011 PACS numbers: 42.90. + p, 45.10.Hj, 66.30. – h

1. INTRODUCTION

In recent years, there has been a great deal of interest in fractional calculus techniques and their applications in a wide range of scientific domains to modeling numerous physical problem, chemical processes, and engineering challenges, see for example [1-4]. The idea of fractional order derivatives is based on various methods of non-integer order derivation, especially, Riemann-Liouville [5, 6], Caputo [7, 8] and others [9, 11]. Caputo and Fabrizio recently proposed a new fractional order derivative without a singular kernel [12, 13]. The modern fractional derivative approach established by Caputo and Fabrizio is suitable for the Laplace transformation, and has several interesting properties stimulated some authors to use it to solve various equations and modeling many phenomena in various branches of science, for instance analysis of logistic equation [14], Korteweg-de Vries-Burgers Equation [15], nonlinear Fisher's reaction diffusion equation [16], the modeling of the electrical flow in a series RLC circuit [17], heat transfer in magnetohydrodynamic [18] and other works see [19-21]. For a function C(t) belongs to the Sobolev space, Caputo and Fabrizio [13] proposed the following fractional derivative.

$$D_t^{\alpha}C(t)=\frac{(2-\alpha)M(\alpha)}{2(1-\alpha)}\int_0^t C'(p)\exp\left[-\alpha\frac{t-p}{1-\alpha}\right]dp\,,\quad(1.1)$$

where $M(\alpha)$ is a normalization constant depending on α $(0<\alpha\leq 1)$

Nieto and Losada [13] suggested a particular method enables to find the normalized function. This method depends on that the fractional integral in equation (1.1) is the average of the function and its anti-derivative for $0<\alpha\leq 1$. Therefore the normalized function by using this way, takes the following form:

$$M(\alpha)=\frac{2}{2-\alpha}\quad(1.2)$$

So what the fractional Caputo-Fabrizio derivative of a function $c(t)$ can redefined as:

$$D_t^{\alpha}C(t)=\frac{1}{1-\alpha}\int_0^t C'(p)\exp\left[-\alpha\frac{t-p}{1-\alpha}\right]dp\,,\quad(1.3)$$

The main aim of this work is to investigate the possibility of applying this new derivative with fractional order to solve the time fractional diffusion equation, and study the effect of fractional order time derivative on the out-diffusion profiles for degassing a thin plate in a vacuum.

2. THEORETICAL IMPLEMENTATION AND CALCULATIONS

Vacuum degassing is a process that enables to obtain a high-strength component by reducing the impurities, like hydrogen, content in the material by being removed in gas form. During this process the impurity captured by traps escapes out of specimen [22, 23]. The impurity transport in the material during degassing can be modeled by the time fractional diffusion equation, this equation is referred to as

$$D_t^{\alpha}C(x,t)=\mu D_x^2C(x,t)\,,\quad(2.1)$$

where the positive constant μ is the diffusion coefficient

* souigataek@gmail.com

of impurity through the material and $0 < \alpha \le 1$.

Consider the case of degassing a thin plate of thickness L in a vacuum, whose surfaces, $x = 0$, $x = L$, are maintained at Zero concentration of impurity. The initial concentration C_0 of impurity is constant at all points of the plate. Therefore, the initial and boundary conditions can be written as:

$$C(x,0) = C_0 ,\ 0 < x < L , \tag{2.2}$$

$$C(0,t) = C(L,t) = 0 , \tag{2.3}$$

As μ is time-independent, we use the separation of variables

$$C(x,t) = C_1(t)C_2(x) , \tag{2.4}$$

then

$$C_2(x)D_t^\alpha C_1(t) = \mu C_1(t)D_x^2 C_2(x) , \tag{2.5}$$

$$\frac{D_t^\alpha C_1(t)}{\mu C_1(t)} = \frac{D_x^2 C_2(x)}{C_2(x)} , \tag{2.6}$$

The left hand side of equation (2.6) is a function of t only, the right hand side is a function of x only. The only way that this can be correct is if both sides equal a constant. In order to simplify the solution, we choose the constant to be equal to $-k^2$

$$\frac{D_t^\alpha C_1(t)}{\mu C_1(t)} = \frac{D_x^2 C_2(x)}{C_2(x)} = -k^2 , \tag{2.7}$$

we have the following two equations to solve

$$D_t^\alpha C_1(t) = -k^2 \mu C_1(t) \tag{2.8}$$

$$D_x^2 C_2(x) = -k^2 C_2(x) , \tag{2.9}$$

Applying the Laplace transform to (2.8), we obtain:

$$\mathcal{L}\left[D_t^\alpha C_1(t)\right] = -k^2\mu\mathcal{L}\left[C_1(t)\right], \tag{2.10}$$

From (1.3) we have

$$\frac{s\mathcal{L}\left[C_1(t)\right] - C_1(0)}{s + \alpha(1-s)} = -k^2\mu\mathcal{L}\left[C_1(t)\right], \tag{2.11}$$

or equivalently,

$$\mathcal{L}\left[C_1(t)\right] = \frac{1}{s}C_1(0) - \frac{\alpha k^2\mu}{s}\mathcal{L}\left[C_1(t)\right] + (\alpha-1)k^2\mu\mathcal{L}\left[C_1(t)\right], \tag{2.12}$$

Using the properties of inverse Laplace transform, we deduce that

$$C_1(t) = (\alpha-1)k^2\mu C_1(t) - \alpha k^2\mu\int_0^t C_1(s)ds + C_1(0) , \tag{2.13}$$

the first derivative of the last equation gives

$$\left(1-(\alpha-1)k^2\mu\right)\frac{dC_1(t)}{dt} = -\alpha k^2\mu C_1(t) , \tag{2.14}$$

$$C_1(t) = C_1(0)\exp\left(\frac{-\alpha k^2\mu}{\left(1-(\alpha-1)k^2\mu\right)}t\right), \tag{2.15}$$

The solution of the equation (2.9) is

$$C_2(x) = a\cos(kx) + b\sin(kx) , \tag{2.16}$$

therefore, the solution can be written as

$$C(x,t) = \exp\left(\frac{-\alpha k^2\mu}{\left(1-(\alpha-1)k^2\mu\right)}t\right)\begin{pmatrix} A\cos(kx) + \\ B\sin(kx) \end{pmatrix}, \tag{2.17}$$

Using the boundary condition we observe that

$$A = 0 , \tag{2.18}$$

and

$$k = \frac{n\pi}{L} , \tag{2.19}$$

we can therefore generalize the solution to the form

$$C(x,t) = \sum_{n=1}^{\infty} B_n \exp\left(\frac{-\alpha\left(\frac{n\pi}{L}\right)^2\mu}{\left(1-(\alpha-1)\left(\frac{n\pi}{L}\right)^2\mu\right)}t\right)\sin(\frac{n\pi}{L}x) , \tag{2.20}$$

Using the initial conditions we have

$$C_0 = \sum_{n=1}^{\infty} B_n \sin(\frac{n\pi}{L}x) , \tag{2.21}$$

We multiply The both sides of equation (2.21) by $\sin\left(\frac{m\pi}{L}x\right)$, where m is another integer, and integrate both sides of the resulting equation from a zero to L we get the following result.

$$\int_0^L C_0 \sin\left(\frac{m\pi}{L}x\right)dx = B_n\sum_{n=1}^{\infty}\int_0^L \sin\left(\frac{m\pi}{L}x\right)\sin\left(\frac{m\pi}{L}x\right)dx , \tag{2.22}$$

We then recognize that the integrals in the summation all vanish except for the one where $n = m$, because the eigenfunctions, $\sin\left(\frac{n\pi}{L}x\right)$ are orthogonal.

Therefore we have

$$\int_0^L C_0 \sin\left(\frac{n\pi}{L}x\right)dx = B_n\int_0^L \sin^2\left(\frac{n\pi}{L}x\right)dx , \tag{2.23}$$

Solving for and evaluating the last integral in equation (2.23) gives the following result

$$B_n = \frac{2}{L}\int_0^L C_0 \sin\left(\frac{n\pi}{L}x\right)dx , \tag{2.24}$$

$$B_n = \frac{2C_0}{n\pi}\left[1-\cos\left(n\pi\right)\right], \tag{2.25}$$

Since $[1 - \cos(n\pi)]$ is zero when n is even and 2 when

n is odd. Then our general solution for $C(x, t)$ becomes

$$C(x,t)=\frac{4C_0}{\pi}\sum_{j=1}^{\infty}\frac{1}{2j+1}\exp(-\lambda t)\sin\left(\frac{(2j+1)\pi}{L}x\right), \quad (2.26)$$

where:

$$\lambda=\frac{\alpha\left(\frac{(2j+1)\pi}{L}\right)^2\mu}{1-(\alpha-1)\left(\frac{(2j+1)\pi}{L}\right)^2\mu}, \quad (2.27)$$

Implication of this solution is that provide estimation of the concentration of impurity in the sample after degassing. Equation (2.26) shows that the $C(x, t)$ depends on the fractional order derivative. In the case of $\alpha = 1$ the concentration is given by the following relationship

$$C(x,t)=\frac{4C_0}{\pi}\sum_{j=1}^{\infty}\frac{1}{2j+1}\exp\left(-\left(\frac{(2j+1)\pi}{L}\right)^2\mu t\right)\sin\left(\frac{(2j+1)\pi}{L}x\right) \quad (2.28)$$

This equation represents the solution of standard diffusion equation for degassing a thin plate in a vacuum, see [24].

3. RESULTS AND DISCUSSIONS

The out-diffusion profiles during degassing with different orders $\alpha = \{0.6, 0.8, 1\}$, $C_0 = 1$ are shown in Fig. 1.

From the Fig. 1, one can see that the out-diffusion profiles depends heavily on the order of the derivative alpha. We observe that degassing is faster as alpha approaches 1, and we have slow degassing as alpha approaches zero. It is clear from the figure.1 that the fractional-order derivative plays an important role in the numerical simulations and it has an effect on the out-diffusion profiles during degassing a thin plate in vacuum.

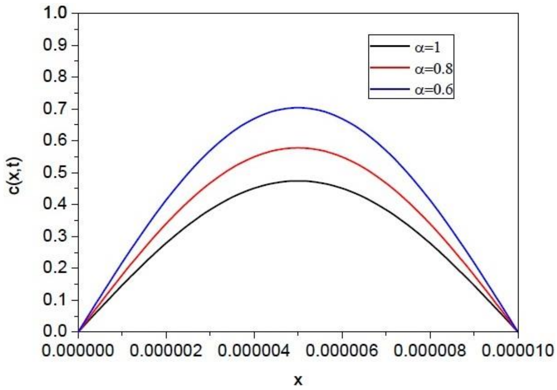

Fig. 1 – The out-diffusion profiles $C(x, t)$ with the parameters $\alpha = \{0.6, 0.8, 1\}$, $C_0 = 1$, $L = 10^{-5}$, $t = 10^5$, and $\mu = 10^{-16}$

4. CONCLUSION

In this work the time fractional diffusion equation was treated for degassing a thin plate in a vacuum by using the new fractional order derivative approach without singular kernel has been established by Caputo and Fabrizio. An exact solution was obtained and we observed that the fractional-order derivative plays an important role in the numerical simulations and it has an effect on the out-diffusion profiles during degassing, where the degassing is faster as alpha approaches 1, and the degassing is slow as alpha approaches zero.

REFERENCES

1. I. Podlubny, *Fractional Calculus and Applied Analysis* **5**, 367 (2002).
2. M. Abdullah, A.R. Butt, N. Raza, *Mechanics of Time-Dependent Materials* **23**, 133 (2019).
3. A.A. Kilbas, H.M. Srivastava, J.J. Trujillo, *Theory and Applications of Fractional Differential Equations*, 204 (Elsevier, Amsterdam: 2006).
4. M. Kavyanpoor, S. Shokrollahi, *Journal of King Saud University – Science* **31**, 14 (2019).
5. I. Podlubny, *Fractional Differential Equations* (Academic Press, New York: 1999).
6. K. Diethelm, *The Analysis of Fractional Differential Equations* (Springer-Verlag, Berlin: 2010).
7. M. Caputo, *Geophysical Journal of the Royal Astronomical Society* **13**, 529 (1967).
8. Y. Ying, Y. Lian, S. Tang, W.K. Liu, *Acta Mech. Sin.* **34**, 515 (2018).
9. D. Zhao, X.J. Yang, H.M. Srivastava, *Thermal Science* **19**, 1867 (2015).
10. X.J. Yang, J.A.T. Machado, H.M. Srivastava, *Applied Mathematics and Computation* **274**, 143 (2016).
11. H. Jafari, H. Tajadodi, J.S. Johnston, *Thermal Science* **19**, 123 (2015).
12. M. Caputo, M. Fabrizio, *Progr. Fract. Differ. Appl.* **1**, 73 (2015).
13. J. Losada, J.J. Nieto, *Progr. Fract. Differ. Appl.* **1**, 87 (2015).
14. D. Kumar, J. Singh, M. Al Qurashi, D. Baleanu, *Adv. Mech. Eng.* **9**, 1 (2017).
15. E.F.D. Goufo, *Mathematical Modeling and Analysis* **21**, 188 (2016).
16. A. Atangana, *Applied Mathematics and Computation* **273**, 948 (2016).
17. A. Atangana, B.S.T. Alkahtani, *Advances in Mechanical Engineering* **7**, 1 (2015).
18. K.A. Abro, M.A. Solangi, *Journal of Mathematics* **49**, 113 (2017).
19. A. Alsaedi, J.J. Nieto, V. Venktesh, *Adv. Mech. Eng.* **7**, 1 (2015).
20. Z. Korichi, A. Souigat, Y. Benkrima, M.T. Meftah, *J. Nano-Electron. Phys.* **14** No 4, 04014 (2022).
21. S. Qureshi, N.A. Rangaigand, D. Baleanu, *Mathematics* **7**, 374 (2019).
22. C.J. Carneiro Filho, M.B. Mansur, P.J. Modenesi, *Materials Science and Engineering: A* **527**, 4947 (2010).
23. E.I. Galindo-Nava, B.I.Y. Basha, P.E.J. Rivera-Diaz-del-Castillo, *Journal of Materials Science & Technology* **33**, 1433 (2017).
24. H. Mehrer, *Diffusion in Solids: Fundamentals, Methods, Materials, Diffusion Diffusion-Controlled Processes* (Springer-Verlag, Berlin Heidelberg: 2007).

Вплив дробової похідної за часом на профілі зовнішньої дифузії під час дегазації тонкої пластини у вакуумі

A. Souigat[1], Z. Korichi[1], M.E. Mezabia[2], Y. Benkrima[1], M.T. Meftah[3]

[1] *Ecole Normale Supérieure de Ouargla, 30000 Ouargla, Algeria*
[2] *Laboratoire de Mathématiques Appliquées, Kasdi Merbah University, 30000 Ouargla, Algeria*
[3] *Department of Physics, LRPPS Laboratory, Kasdi Merbah University, 30000 Ouargla, Algeria*

Методи дробових обчислень та їх застосування в різних наукових галузях викликають великий інтерес в останні роки. Вони використовуються в моделюванні багатьох фізичних задач, хімічних процесів та інженерії. Нещодавно Капуто-Фабріціо запропонував нову похідну дробового порядку без сингулярного ядра. Цей новий оператор підходить для перетворення Лапласа і здатний описувати фізичну поведінку на різних масштабах. З метою подальшого дослідження можливого застосування цього нового оператора ми використали його для вивчення впливу похідної дробового порядку за часом на профілі вихідної дифузії під час дегазації тонкої пластини у вакуумі. У цій роботі змодельований транспорт домішок у матеріалі під час дегазації з допомогою рівняння дифузії дробового порядку за часом, врахувавши похідну Капуто-Фабріціо, і опрацьований за допомогою методу розділення змінних. Отримано точний розв'язок, який залежить від похідної альфа дробового порядку за часом. Змінна альфа відіграє важливу роль у чисельному моделюванні і має вплив на профілі дифузії. Дегазація відбувається швидше, коли альфа наближається до 1, і дегазація сповільнюється, коли альфа наближається до нуля.

Ключові слова: Дробова похідна, Капуто-Фабріціо, Рівняння дифузії, Дегазація тонкої пластини.