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Precision Chaotic Laser Generation

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The task of this work is development of precision chaotic laser generation principles. Its implementation will contribute to evolution of telecommunication systems based on the chaotic generators synchronization effect and other chaotic technology. The key problem for practical use of chaotic regimes is their strong dependence on fluctuations of initial conditions and weak external influences. This is a fundamental property of dynamic chaos. To solve the stated problem, we analyze the semiclassical laser equations for the stable, unstable, and chaotic generation modes. A modified equation for chaotic radiation is obtained. It is supplemented with fluctuations of the pumping parameters, laser components characteristics, and external factors. The equation is the basis for studying of laser dynamics under various initial conditions and for providing of precision chaotic generation.We propose a definition for precision chaotic laser generation. It is the generation of laser radiation, the dynamics of which is classified as chaotic with a given accuracy and is reproducible within the boundaries of the phase portrait. The choice of the phase portrait. as the object of study for precision, is due to the stability of chaotic solutions according to Lagrange. The precision is confirmed by comparing a phase portrait of the system with its reference portrait, obtained with controlled reference parameters of chaotic radiation. As the quantitative estimates of chaotic precision are chosen: the volume of attractor, Lyapunov exponents, and Hurst coefficient with allowable deviations. The precision of chaotic generation and control of chaotic dynamics are ensured by the precision of the pump parameters, by control and stabilization of the components and characteristics of laser, such as the size and dynamics of resonator, quality factor, radiation frequency, temperature, and others.

Keywords: Semiclassical laser equations, Chaotic laser regime, Attractor value, Lyapunov exponents, Hurst coefficient.

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1. INTRODUCTION

A laser is traditionally considered as a source of stable monochromatic radiation. Ideally, this is a deterministic system, in practice it is quasi-deterministic. Indeed, lasers used in information and measuring technology demonstrate high stability of radiation parameters over time. For example, for serial Helium-Neon lasers, frequency stabilization in the time interval of one minute is ± 1 MHz and more, radiation intensity stabilization is ± 0.1 % and more. For lasers used as frequency standards, the degree of monochromaticity and parameter stability are much higher.

Radiation stability is ensured by controlling the characteristics of laser parts and influence of external factors. This is ensured, for example, by thermal stabilization and stabilization of pump parameters. From the point of view of the dynamic systems theories, a laser can be represented as an open nonlinear dynamic system, that is influenced by as external so internal factors, which is the cause of fluctuations in the radiation parameters and affects the nature of processes dynamics.

Thus, lasers can demonstrate both stable regular dynamics and stochastic ones. In this case, the second is considered as undesirable. However, under certain conditions, the dynamics becomes not conditionally determined or random, but chaotic, difficult to manage and predict. The complexity of working with chaotic systems is due to the fundamental property of dynamic chaos that can be described as its strong dependence on fluctuations in the initial conditions and external factors.

The principles of the occurrence of chaotic laser generation were described by H. Haken in the work [1]. Today, a large number of papers have been published about generation and control of chaotic regimes in lasers. For example, the authors of the work [2-5] proposed a chaotic regime control scheme by changing the linear dimensions of the generating system. The task of controlling chaotic generation remains relevant. In the work [6], a method for observing optical chaos in real time is presented. The issues of chaotic laser dynamics parameters measuring are the subject of Yu.P. Machekhin and his colleagues research [7-9].

Traditionally, for information and measuring tasks that require a high degree of stabilization of radiation parameters, both stochastic and chaotic modes have had a negative connotation. However, the unique properties of chaos have found application in secure optical communication systems based on the work of L.M. Pecora and T.L. Carroll, who demonstrated the possibility of spontaneous synchronization of the transmitter and receiver of information operating in a chaotic mode [10-14].

For chaotic telecommunication systems, a controlled chaotic process is a necessary condition. The creation of lasers with given and reproducible parameters of chaotic radiation remains an urgent task. It can be implemented by developing technologies for generation a chaotic laser radiation using methods and tools of Nonlinear measurement theory [7, 15]. In the articles [8, 9], we propose the models for measuring the parameters of chaotic laser radiation and the model for precision synchronization of chaotic dynamical systems, respectively.

The task of this work is to develop the principles of precision generation of chaotic laser radiation. To achieve the goal, the following tasks are solved in the work: analysis of a scenario for chaotic laser generation occurrence using semiclassical laser equations; research and evaluation of factors that influence on chaotic generation; fundamental substantiation for the concept of precision laser generation and development of principles and tools for its provision.

2. LASER CHAOTIC GENERATION

To describe chaotic laser generation, we use the equations of the lasers semiclassical theory for single-mode laser in the form:

$$\frac{db}{dt} = (-i\omega - \kappa)b - ig\sum_{\mu}\alpha_{\mu}, \quad (1)$$

$$\frac{d\alpha_{\mu}}{dt} = (-i\omega - \gamma)\alpha_{\mu} + igbd_{\mu}, \quad (2)$$

$$\frac{dd_{\mu}}{dt} = \frac{1}{T}(d_0 - d_{\mu}) + 2ig(\alpha_{\mu}b^* - \alpha_{\mu}^* b), \quad (3)$$

here: $b-$ dimensionless complex electric field amplitude, $\omega-$ cavity mode circular frequency, $\kappa-$ resonator damping constant, $g-$ coupling constant, $\alpha_{\mu}-$ complex dipole moment of atom μ, $\gamma-$ atomic (natural) line-width, d_0- unsaturated inversion of a single atom, $d_{\mu}-$ inversion of atom μ, $T-$ longitudinal relaxation time [1].

Equation (1) is a field equation whose dynamics can be described by a time function $b(t)$. According to this equation, the reasons for the temporal change in the field amplitude (b) are the oscillations and damping of the field in the resonator ($(-i\omega-\kappa)b$), if there is no interaction between the field and active atoms, and also the action of dipole moments ($ig\sum_{\mu}\alpha_{\mu}$), as a force that forces the field to oscillate. Material equations (2) and (3) describe the dynamics of dipole moments and atomic inversion.

Deterministic equations (1) - (3) under certain conditions give chaotic solutions for the field $b(t)$ parameters. An analysis of the equations demonstrates that the nature of the laser dynamics is mainly influenced by the design and dynamics of the resonator, as well as the magnitude and dynamics of the pumping. Indeed, practice shows that chaotic laser generation can be provided in several ways, namely: time modulation of resonator losses $\kappa(t)$, time modulation of inversion $d_0(t)$, high pump power, injection of modulated coherent electromagnetic radiation $E_p(t)$, change a size $L(t)$ and geometry of resonator in time [1], [2]. Depending on the mechanism of chaotization, chaotic dynamics can be demonstrated by the intensity, phase, frequency, polarization, and periodicity of laser pulses.

Of the above methods for obtaining chaotic radiation, the most interesting for us is the method of modulated pumping, which does not require intervention in the laser design. This is especially important for the implementation of chaotic communication systems using semiconductor lasers [9]. To study this method, we introduce a control parameter into laser equations (1) - (3), which causes chaotic dynamics (modulated pumping κE_p) and changes caused by summation over atoms:

$$b = E = E(t)\exp(-i\omega t), \quad (4)$$

$$\alpha_{\mu} = P = P(t)\exp(-i\omega t), \quad (5)$$

$$\sum_{\mu} d_{\mu} = D, \ \sum_{\mu} d_0 = D_0. \quad (6)$$

Equations (1) - (3) take the form:

$$\frac{dE}{dt} = -\kappa(E - \mathrm{E}_p) - igP, \quad (7)$$

$$\frac{dP}{dt} = -\gamma P + igED, \quad (8)$$

$$\frac{dD}{dt} = \gamma_{||}(D_0 - D) + 2ig(PE^* - P^*E), \quad (9)$$

here: $\gamma_{||}-$ longitudinal relaxation constant.

If we remove the component of the external field from equation (7), we obtain a stationary solution for the field intensity E_S. In the presence of modulated pumping and the fulfillment of the condition $\kappa \ll \gamma_{||} \ll \gamma$, system (7) - (9) gives the following equation for the field strength normalized to a stationary value E_S:

$$\frac{d\hat{E}}{d\tau} = -i\Delta\omega\hat{E} + \left(\frac{g^2 D_0}{\gamma\left(1+\hat{E}^2\right)} - 1\right)\hat{E} + \mathrm{E}_p(\tau), \quad (10)$$

here: $\Delta\omega-$ frequency mismatch, $\Delta\omega = (\omega_p - \omega)/\kappa$; ω_p- external field frequency, $\tau-$ dimensionless time, $\tau = t\kappa$ [1].

Let us analyze equation (10) from the point of view of the laser radiation stability.

First, consider the case of a constant external field E_p. Taking equal to zero the value of the time derivative of the field, we obtain:

$$i\Delta\omega\hat{E} + \left(\frac{g^2 D_0}{\gamma\left(1+\hat{E}^2\right)} - 1\right)\hat{E} + \mathrm{E}_p = 0. \quad (11)$$

In the case when the first term in brackets is more less than one, the equation has a stationary solution $\hat{E} = const.$ If this term is large, then an unstable generation mode takes place. The reason for the instability lies in the large value of the unsaturated inversion D_0 due to the high value of the external field E_p. However, there is no chaotic mode. According to (11), the laser dynamics is different depending on whether the laser operates above or below the generation threshold $D_0 > \kappa\gamma / g^2$.

When the external field is modulated with a frequency ω_m:

$$\mathrm{E}_p(t) = E_p + E_m \cos(\omega_p \tau), E_p > E_m \geq 0, \qquad (12)$$

and E_m has a sufficiently large value, according to the solution of equation (10), a chaotic generation mode arises. This regime has such properties as: a broadband radiation spectrum, a strong dependence on the initial conditions, weak external influences and fluctuations in the system parameters, the phase portrait is a strange attractor, topological mixing and a dense arrangement of periodic trajectories are observed. In addition to the above qualitative signs of chaos, there are also quantitative ones, such as non-zero Lyapunov exponents, the Hurst coefficient tends to 0.5, and the fractal dimension tends to 1.5 [7].

Note that equation (10) is a model one and does not take into account the influence of external factors on the dynamics of lasers, while one of the striking characteristics of chaos is a strong dependence on the initial conditions and the influence of even weak factors on the dynamics [16-18]. Let us consider the factors that should be taken into account to ensure the generation of precision chaotic laser generation.

For the described scenario of chaos, the control parameter is the external field (12). Let us represent the fluctuations of the components E_p, E_m and also the frequency ω_p in the form δE_p, δE_m, $\delta\omega_p$. Now the external field can be represented as:

$$E_p(\Delta, \tau) = E_p(\tau) + \Delta E_p(\tau), \qquad (13)$$

here: $\Delta E_p(\tau)-$ deviation of the external field from the given $E_p(t)$:

$$\Delta E_p(\tau) = \sqrt{\left(\frac{dE_p(\tau)}{dE_p}\delta E_p\right)^2 + \left(\frac{dE_p(\tau)}{dE_m}\delta E_m\right)^2 + \left(\frac{dE_p(\tau)}{d\omega_p}\delta\omega_p\right)^2}$$

For the case of external field modulation, according to (12), we obtain:

$$\Delta E_p(\tau) = \sqrt{\delta E_p^2 + [\cos(\omega_m \tau)\delta E_m]^2 + [E_m \tau \sin(\omega_p \tau)\delta\omega_p]^2}. \qquad (14)$$

According to (10), the laser dynamics also depends on the frequency mismatch $\Delta\omega$, which depends on the frequency of the external field ω_p and cavity mode circular frequency ω. Fluctuations $\delta\omega_p$ have already been taken into account in (14). Consider the fluctuations of natural frequency $\delta\omega$.

There are long-term (time interval more than 1 s) and short-term (time interval less than 1 s) frequency fluctuations. The long-term fluctuations are associated with changes in the length of the resonator and the refractive index of the active medium due to heating or pressure changes in the surrounding atmosphere. The short-term fluctuations are associated with oscillations of the resonator mirrors, which leads to a change in the length of the resonator or to modulation of the refractive index of air and the active medium. In solid-state lasers with modulated optical pumping (12), power fluctuation leads to temperature fluctuations and, as a consequence, to fluctuations in the resonator size and refractive index, which are related to frequency. In lasers with frequency stabilization such fluctuations are minimized by stabilization systems. However, the strong dependence of the chaotic regime on weak fluctuations makes it necessary to take into account small changes in frequency.

Let us consider the case of frequency fluctuation due to a change in the resonator length (ΔL) or laser heating (ΔT), which is a natural scenario. Its value can be given by the expression:

$$|\delta\omega| \equiv \omega\left|\frac{\Delta L}{L}\right| = \alpha\Delta T, \qquad (15)$$

here $\alpha-$ thermal expansion coefficient for the material.

The expression for the frequency mismatch takes the form:

$$\Omega = \frac{\omega_p - \omega - |\delta\omega|}{\kappa}. \qquad (16)$$

In addition to the fluctuations of the parameters described in (14) - (15), other parameters entering into the laser equation (10) fluctuate also. However, the stabilization of the frequency and external field leads to their stabilization as well. The considered parameter fluctuations can be measured and controlled without interfering in laser construction.

Using the results (13) - (16), we obtain the laser equation (10) in an augmented form suitable for providing precision chaotic laser radiation:

$$\frac{d\hat{E}}{d\tau} = -i\Omega\hat{E} + \left(\frac{g^2 D_0}{\gamma\left(1+\hat{E}^2\right)} - 1\right)\hat{E} + \mathrm{E}_p(\Delta, \tau). \qquad (17)$$

The modified laser equation (17) can be used to model and study the laser dynamics for different values of the control parameter and laser characteristics, directly or indirectly present in the equation, and it is the basis for developing the principles of precision chaotic laser generation.

3. PRECISION CHAOTIC GENERATION

Implementation of lasers with chaotic dynamics for information tasks requires the development of the theory and practice of precision chaotic generation, identification and measurement of the dynamic characteristics. The precise chaotic laser generation is generation of laser radiation, the dynamics of which is classified as chaotic, characterized by parameters with a given accuracy, and is reproducible within the boundaries of a phase portrait (in the case of chaotic dynamics, phase portrait is a strange attractor).

The choice of a phase portrait as a tool for ensuring precision is due to the fact that the solutions of the dynamic equation of the form (7) - (9), (17) in the case of chaos are unstable according to Lyapunov due to the exponential divergence of phase trajectories, but stable according to Lagrange, which requires that all solutions do not go beyond the boundaries of a certain area - an attractor. From a physical point of view, an attractor is a state of a dynamic system, to which it tends in the process of its movement (development). In the case of the problem of chaotic generation, chaos with given parameters is a desired state of the system and it is characterized by a strange attractor.

A phase portrait can be characterized with the volume of an attractor (V_A). For conservative systems, this value is constant, but for dissipative systems, it varies depending on many factors. Calculating the value of V_A can be approximated by the problem of calculating the volume of an n-dimensional parallelepiped. It should be noted that strange attractors have a fractal structure, which shows itself in the fractional dimension of the figures. Therefore, in the future, it is necessary to develop principles and methods for calculating the fractal figures volumes.

To ensure precision chaotic laser generation, it is necessary to obtain a reference phase portrait of a system that generates chaotic laser radiation $\hat{E}^e(\tau)$ (where the superscript e means the reference value) with specified parameters. To do this, we fix the values of the controlled pump parameters $E_p^e(\Delta^e,\tau)$ (14), and the laser parameters $\delta\omega^e, \Delta L^e, \Delta T^e$ (15), form the reference phase portrait and calculate its volume V_A^e. During next generations, the attractor volume is also calculated and is compared with the reference value:

$$\Delta V = V_A^e - V_A . \tag{18}$$

If $\Delta V \approx 0$ the reference dynamics maintains. If $\Delta V > 0$ the chaotization decreases. If $\Delta V < 0$ the chaotization grows. The value ΔV is a measure of precision and serves as information for correcting laser parameters that affect generation.

When constructing a chaotic attractor, we have a problem with the number of measurements of the laser parameters that let us information about all states of the system that is necessary for building of full attractor.
The number of points M (the number of joint measurements of variables that form the phase portrait) must be sufficiently large, but finite. According to the formula proposed in [19] we have:

$$M \geq M_{\min} = 10^{2+0.4D_A} , \tag{19}$$

here D_A – dimension of the attractor.

To construct a phase portrait, it is necessary to simultaneously measure the quantities of radiation parameters. In [9], a scheme was presented for the synchronous measurement of laser radiation parameters with a subsequent assessment of the measurement uncertainty. The scheme can be used to measure at the same time intensity, phase, polarization, pulse duration and rate.

As the numerical characteristics of the entire system dynamics we use the group of Lyapunov exponents:

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \ln|u_i| , \tag{20}$$

here: u_i – divergence of two close phase trajectories values.

Their number corresponds to dimension of system. The totality of all Lyapunov exponents forms the Lyapunov spectrum. The presence of at least one positive and limited exponent in the spectrum indicates chaotic dynamics.

These parameters can be used also to forecast of dynamics. The maximum Lyapunov exponent $\lambda_{\max}$ is related to the forecast time T_{for} for the dynamics of system:

$$T_{for}(\lambda) \sim \frac{1}{\lambda_{\max}} \log \frac{1}{|u|} . \tag{21}$$

According to (21) for a deterministic process $\lambda = 0$, the forecast time $T_{for} \to \infty$; for a chaotic process $\lambda \neq 0$, the forecast time has a limited value $T_{for} \sim 1/\lambda_{\max}$; for a random process $\lambda \to \infty$, the forecast time tends to zero $T_{for} \to 0$.

To assess the dynamics of a separate dynamic variable, in our case $\hat{E}(\tau)$, we propose to use the Hurst coefficient H for the time series:

$$H = \frac{\ln(R/S)}{\ln(M/2)} , \tag{22}$$

here R – range of cumulative time series $\hat{E}(\tau)$, S – series standard deviation [20].

If H takes values from the interval $0.5 < H < 1$, the dynamics of studied parameter $\hat{E}(\tau)$ is chaotic, corresponding to chaotic laser generation. We can choose the reference value of the coefficient H^e when the reference chaotic mode of laser operation is. This is a persistent process, with memory and $T_{for} \neq 0$. If $H = 0.5$ the dynamics is completely stochastic and $T_{for} = 0$, the generation is stochastic. If $H = 1$, the process is deterministic with a long prediction time, corresponding to stable laser generation.

After the numerical description of the reference chaotic generation, the permissible values of parameters deviations which are acceptable for precision condition must be introduced in the form: $\delta E_p^e(\Delta^e,\tau)$, δV_A^e, $\delta\lambda^e$, δH^e.

If the parameters of laser radiation deviate from the reference ones, the possibility of correction should be provided. The condition for precision laser generation is the control of the laser parameters directly or indirectly included in the main equation (17). The principles of controlling a chaotic regime can be built on the paradigm that the absorption of energy generates a chaotic regime, and its dissipation decreases chaos. An analysis of equation (17) shows that the radiation regime can be managed both by controlling the pumping system (13) and the laser parameters (15). As about a control mechanism, we can talk about control for the resonator quality factor (Q-factor):

$$Q = 2\pi \frac{W_{full}}{W_{losses}} , \tag{23}$$

here W_{full} – full energy in the resonator, W_{losses} – energy loss in one period.

The quality factor (22) is related to the parameters included in the laser equation (17) by the expressions:

$$Q = \frac{\omega}{\delta\omega} = \omega\tau_c = \omega\frac{2}{c\gamma}, \qquad (24)$$

here τ_c – photon lifetime in a resonator $\tau_c = 2\kappa$, c – speed of light [21].

In addition, we recall the connection between the radiation parameters, the length of the resonator and its dynamics, as well as with temperature (15).

Thus, the precision of chaotic laser generation is ensured by the pumping components precision and the stabilization of its characteristics, such as radiation frequency, temperature, and others. The dynamics and parameters of the chaotic regime can be corrected both by changing the control parameter (pumping system) and by changing the Q-factor and resonator dynamics.

4. CONCLUSSIONS

The paper proposes the foundations for ensuring precision chaotic laser generation.

An analysis of semiclassical laser equations is performed and an equation of chaotic generation is obtained, supplemented by components describing fluctuations of pumping parameter components, laser parameters, and external factors.

A definition of precision chaotic laser generation is proposed as generation of laser radiation, the dynamics of which is classified as chaotic, is determined by parameters with a given accuracy, and is reproducible within the boundaries of the phase portrait. The concept of a reference phase portrait corresponding to a generation with given parameters is introduced.

Quantitative estimates of precision are: attractor volume, Lyapunov exponents, Hurst coefficient.

The precision and control of chaotic laser generation is ensured by the precision and control of pumping system, stabilization and control of laser characteristics (temperature, frequency) and elements (resonator, active medium).

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Прецизійна хаотична лазерна генерація

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Завданням роботи є розробка принципів прецизійної лазерної хаотичної генерації. Її реалізація сприяє розвитку інформаційних систем, що базуються на принципі синхронізації хаотичних генераторів. Ключова проблема практичного використання хаотичних режимів обумовлена фундаментальною властивістю динамічного хаосу – сильною залежністю від флуктуацій початкових умов. У роботі виконано аналіз напівкласичних лазерних рівнянь щодо виникнення нестійких і хаотичних режимів генерації. Отримано рівняння хаотичного випромінювання, доповнене компонентами флуктуацій параметра, що управляє, характеристик лазера і зовнішніх факторів. Рівняння є основою для дослідження лазерної динаміки за різних початкових умов та забезпечення прецизійної хаотичної генерації. Запропоновано визначення прецизійної хаотичної лазерної генерації як генерації лазерного випромінювання, динаміка якого із заданою точністю класифікується як хаотична і є відтвореною в межах фазового портрета. Вибір фазового портрета як об'єкт дослідження на прецизійність обумовлений стійкістю хаотичних рішень щодо Лагранжу. Прецизійність підтверджується порівнянням фазового портрета з еталонним портретом системи, отриманим при контрольованих параметрах хаотичного випромінювання. Кількісними оцінками хаотичної прецизійності обрано: обсяг атрактора, показники Ляпунова, коефіцієнт Херста з допустимими відхиленнями. Прецизійність випромінювання забезпечується прецизійністю накачування і стабілізацією характеристик лазера, таких як частота випромінювання, температура та інші. Динаміка та параметри хаотичного режиму коригуються шляхом зміни керуючого параметра (система накачування), механізмами зміни добротності та динаміки резонатора.

Ключові слова: Напівкласичні лазерні рівняння, Хаотичний лазерний режим, Об'єм атрактора, Показники Ляпунова, Коефіцієнт Херста.