Journal of Nano- and Electronic Physics
Vol. **15** No 1, 01022(5pp) (2023)

Журнал нано- та електронної фізики
Том **15** № 1, 01022(5cc) (2023)

Effect of Idealization Models on Deflection of Functionally Graded Material (FGM) Plate

B. Rebai[1,*], K. Mansouri[2,3,†], M. Chitour[2], A. Berkia[2], T. Messas[1], F. Khadraoui[2], B. Litouche[4]

[1] *University Abbes Laghrour, Civil Engineering Department, 40000 Khenchela, Algeria*
[2] *UniversityAbbes Laghrour, Mechanical Engineering Department, 40000 Khenchela, Algeria*
[3] *Laboratory of Engineering and Sciences of Advanced Materials (ISMA), 40000 Khenchela, Algeria*
[4] *University Center Abdelhafid Boussouf, Mechanic and Electro-Mechanic Department, 43000 Mila, Algeria*

(Received 10 January 2023; revised manuscript received 14 February 2023; published online 24 February 2023)

Functionally graded materials (FGM) are a class of composites, in which the properties of the material gradually change over one or more Cartesian directions, the combination of which results in an assembly with higher performance than components taken separately. This class of composite materials has gained considerable attention in the engineering community, especially in high-temperature applications such as nuclear reactors, aerospace, and power generation industries. The aim of the current work is to study the influence of homogenization (idealization) models and thermal loads on static deflection behavior of sandwich functionally graded plate. Several micromechanical models have been employed to obtain the effective material properties of the two-phase FGM plate. The FGM plate is subjected to linear and no linear thermal loads. The integral theory used contains only four variable functions as against five in the case of other HSDTs. The governing equation are derived and resolved via virtual work principle and Navier's model. The accuracy of the proposed analytical model is confirmed by comparing the results with those given by others model existing in the literature.

Keywords: Homogenization, Sandwich plate, Functionally graded plate, Deflection, Navier solution.

DOI: 10.21272/jnep.15(1).01022 PACS numbers: 77.84. – s, 77.84.Lf, 78.66.Sq

1. INTRODUCTION

Materials are continuously developed from iron, pure metals to composite materials which are in use today. Pure metals have very limited use, since actual application may require contrary property requirement which cannot provide by using single metal. As compared to pure metals, alloys can be stronger and more versatile. Bronze which is alloy of copper and tin was the first alloy that was developed in 4000 BC (Bronze age). Since then, different mixtures of metals and non-metals were tried to combine strengths of multiple materials as per functional requirement[1]. Functionally graded materials (FGM) are a class of composites in which the properties of the material gradually change over one or more cartesian directions [2]. FGMs have a wide range of applications in various sectors due to their flexibility in making a particular composite material according to the requirement. FGMs are used in aerospace structures, military applications, medical applications, photoelectronic devices, automotive parts, and sporting equipment. FGMs are advantageous over conventional structural materials and layered composites due to their continuous change in characteristic properties, thermal stability, good damping properties, and high toughness. Additionally, they are used as coating materials which reduce heat loss from engine exhaust system components and can be used in high and low temperature zones. FGMs are also used in forming tools and cutting tools, making them a useful material for many applications.

There has been a considerable research report on thermal stresses, fracture, thermo mechanical response, buckling, free vibration, etc., of FGM structural elements during the last two decades [3]. Due to the importance and wide technical applications of FGMs structures, they have been addressed by many researchers [4].

The aim of the using the micromechanical models is to accurately predict the effective multiphysics properties. and to quantify the effect of microstructure on the multiphysics behavior of materials by the application of continuum mechanics to a small-scale. Several micromechanical models of FGMs have been reviewed in [5-9]. To assess the effect of the micromechanical models on the structural responses of FG plates.

In this paper at first time the influence of the thermal load with linearity and nonlinearity variations is presented. Then, the effect of the micromechanical models (Voigt, Reuss, LRVE and Tamura) are employed to determine the effects on the Center deflection $\bar{w}$ of the sandwich functionally graded plate.

2. PROBLEM DEFINITION AND GOVERNING EQUATIONS

2.1 Materials Properties

The geometry of the domain problem is assumed to a rectangular plate with thickness "h", length "a", and width "b". The plate has three layers (two faces sheet and a core) (Fig. 1).

The FG-face sheets are made by two materials Titanium (as metal) and Zirconia (as ceramic). The Table 1 resumed the values of The Young modulus, thermal expansion coefficients α and poison ratioν of Ceramic and metal phases.

The volume fraction of the FG-faces sheet are assumed varies as following functions.

* billel.rebai@univ-khenchela.dz
† mansouri.khelifa@univ-khenchela.dz

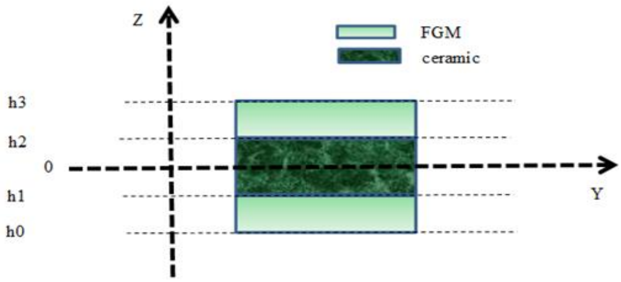

Fig. 1 – Geometry of the FGM sandwich plate

Table 1 – Material properties used in the FG sandwich plate

Properties	Metal: Ti/6A1/4V (Titanium)	Ceramic: ZrO_2 (Zirconia)
$E(z)$(GPa)	66.2	117
ν	1/3	1/3
α (10^{-6}/K)	10.3	7.11

$$V^{(1)} = \left(\frac{z-h_0}{h_1 - h_0}\right)^k, z \in [h_0, h_1]$$
$$V^{(2)} = 1, z \in [h_1, h_2] \quad , \tag{2.1}$$
$$V^{(3)} = \left(\frac{z-h_3}{h_2 - h_3}\right)^k, z \in [h_2, h_3]$$

where k is the material index.

A number of micromechanics models have been proposed for the determination of effective properties of FGMs.

Voigt model: The Voigt model is relatively simple; this model is frequently used in most FGM analyses estimates properties of FGMs.

Reuss model: Reuss assumed the stress uniformity through the material and obtained the effective properties.

***Tamura model*:** The method of Tamura assumes a linear rule of mixture for effective Poisson's ratio of a two-phase composite whereas incorporates an empirical fitting parameter in the effective Young's modulus formulation.

***Volume element (LRVE) model*:** The LRVE is developed based on the assumption that the microstructure of the heterogeneous material is known. The input for the LRVE for the deterministic micromechanical framework is usually volume average or ensemble average of the descriptors of the microstructures.

2.2 Displacement Base Field

Based on the same assumptions of the conventional HSDT (with fives variables or more). The displacement field of the proposed HSDT is only with four unknowns variables and can be written in a simpler form as:

$$\begin{cases} u(x,y,z) = u_0(x,y) - z\dfrac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)\,dx \\ v(x,y,z) = v_0(x,y) - z\dfrac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)\,dy \\ w(x,y,z) = w_0(x,y) \end{cases} , \tag{2.2}$$

where $u_0(x,y)$, $v_0(x,y)$, $w_0(x,y)$ and $\theta_0(x,y)$ are the four-unknown displacement functions of middle surface of the FG-sandwich plate. $f(z)$ is the warping function and (k_1 and k_2) are a constants.

In the current research work the proposed combined (exponential/hyperbolic) warping function ensures the nullity condition of the free surfaces of the FG-sandwich plate (zero transverse shear stresses at top and the Bottom of the FG-sandwich plate). The present exponential/hyperbolic warping function $f(z)$is expressed as:

$$f(z) = \left[\ln\left(\pi \exp(1/20)\right) - (0.1407)^{5/4}\cosh(\pi z)\right] z \tag{2.3}$$

The stresses/strains linear relation of the PFG-sandwich plate can be expressed as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(n)} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}^{(n)} \begin{Bmatrix} \varepsilon_x - \alpha T \\ \varepsilon_y - \alpha T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(n)} \tag{2.4}$$

Where

$$C_{11}{}^{(n)} = C_{22}{}^{(n)} = E^{(n)}(z) / \left(1 - \left(\nu^{(n)}\right)^2\right)$$
$$C_{12}{}^{(n)} = \nu^{(n)} C_{11}{}^{(n)}$$
$$C_{44}{}^{(n)} = C_{55}{}^{(n)} = C_{66}{}^{(n)} = E^{(n)}(z) / 2\left(1 + \nu^{(n)}\right)$$

The variation of the temperature field across the thickness is assumed to be:

$$T(x,y,z) = T_1(x,y) + \frac{z}{h}T_2(x,y) + \frac{\Psi(z)}{h}T_3(x,y) , \tag{2.5}$$

where T_1, T_2 and T_3 are thermal loads, with:

$$\Psi(z) = (h/\pi)\sin(\pi z/h)$$

2.3 Governing Equations

The principle of virtual works of the considered PFG-sandwich plates is expressed as:

$$\delta U + \delta V = 0, \tag{2.6}$$

where δU is the variation of strain energy; δV is the variation of the virtual work done by external load applied to the plate. The governing equations can be obtained as follows:

$$\begin{aligned} &\delta\ u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta\ v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta\ w_0 : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = 0 \\ &\delta\ \theta : -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A'\frac{\partial S_{xz}^s}{\partial x} + \\ &\qquad + k_2 B'\frac{\partial S_{yz}^s}{\partial y} = 0 \end{aligned} \tag{2.7}$$

3. CLOSED-FORM SOLUTION FOR SIMPLY SUPPORTED PLATE

Based on the Navier method, the following expansions of displacements are:

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \begin{Bmatrix} U\cos(\alpha\ x)\sin(\beta\ y) \\ V\sin(\alpha\ x)\cos(\beta\ y) \\ W\sin(\alpha\ x)\sin(\beta\ y) \\ \theta_1\sin(\alpha\ x)\sin(\beta\ y) \end{Bmatrix}, \quad (3.1)$$

where (U, V, W and X) are unknown functions to be determined and $\alpha = \pi/a$ and $\beta = \pi/a$.

In the present work, the transverse temperature loads T_1, T_2, and T_3 in double sinus series form as:

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \end{Bmatrix}\sin(\alpha\ x)\sin(\beta\ y). \quad (3.2)$$

The closed-form solution can be written as following matrix form:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}\begin{Bmatrix} U \\ V \\ W \\ X \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{Bmatrix}, \quad (3.3)$$

where

$$S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right)$$

$$S_{12} = -\alpha\beta\left(A_{12} + A_{66}\right)$$

$$S_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right)$$

$$S_{14} = \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - (k_1 A' + k_2 B')B_{66}^s\beta^2\right)$$

$$S_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right)$$

$$S_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right)$$

$$S_{24} = \beta\left(k_2 B_{22}^s + k_1 B_{12}^s - (k_1 A' + k_2 B')B_{66}^s\alpha^2\right)$$

$$S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right)$$

$$S_{34} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')D_{66}^s\alpha^2\beta^2 - k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right)$$

$$S_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1 A' + k_2 B')^2 H_{66}^s\alpha^2\beta^2 - k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - (k_1 A')^2 A_{55}^s\alpha^2 - (k_2 B')^2 A_{44}^s\beta^2$$

and

$$P_1 = \alpha(A^T T_1 + B^T T_2 + {}^aB^T T_3)$$

$$P_2 = \beta(A^T T_1 + B^T T_2 + {}^aB^T T_3)$$

$$P_3 = -h(\alpha^2 + \beta^2)(B^T T_1 + D^T T_2 + {}^aD^T T_3)$$

$$P_4 = -h(\alpha^2 + \beta^2)({}^sB^T T_1 + {}^sD^T T_2 + {}^sF^T T_3)$$

(A^T, B^T, D^T), (${}^aB^T$, ${}^aD^T$), (${}^sB^T$, ${}^sD^T$, ${}^sF^T$) and (L^T, ${}^aL^T$, R^T) are coefficients calculated by integral summation formulations. In which:

$$\bar{z} = z/h,\ \bar{f}(z) = f(z)/h \text{ and } \bar{\psi}(z) = \psi(z)/h$$

4. NUMERICAL RESULTS AND DISCUSSION

Comparisons are performed to verify the accuracy of the present analytical solution of the thermoelastic bending of the FG-sandwich plate with the others existing models in the literature [10]. The transverse displacement is presented in the dimensionless form as:

$$\bar{w} = \left(h/\left(\alpha_0 \bar{T}_2 a^2\right)\right) w(a/2, b/2) \quad (4.1)$$

With $E_0 = 1$ GPa and $\alpha_0 = 10^6$ K.

In the following, five kinds of PFG-sandwich plates (symmetric and non-symmetric) are examined:

- The (1-0-1) PFG-sandwich plate: the symmetric plate is composed of two thickness layers with ($h_1 = -0$, $h_2 = 0$).
- The (1-1-1) PFG-sandwich plate: the symmetric plate is composed of three equal thickness layers with ($h_1 = -h/6$, $h_2 = h/6$).
- The (1-2-1) PFG-sandwich plate: the symmetric plate with the core thickness equal to the sum of skin thickness with ($h_1 = -h/4$, $h_2 = h/4$).
- The (2-1-2) PFG-sandwich plate: the symmetric plate with core thickness equal half of the skin thickness with ($h_1 = h/10$, $h_2 = h/10$).
- The (2-2-1) PFG-sandwich plate: the non-symmetric plate with the core thickness equal the thickness of the thick layer with ($h_1 = -h/10$, $h_2 = 3h/10$).

4.1 Results Comparaison

Table 2 predicts the dimensionless deflection "$\bar{w}$" of the symmetric and non-symmetric square FG-sandwich plate under linear distribution of the thermal load «$T_3 = 0$» versus power law index "k" and layer thickness ratio. From the table, we can see that the present results obtained via combined exponential/hyperbolic shear deformation theory is in good agreement with different models Voigt, Reuss, LRVE and Tamura for all values of the material index "k" and all configurations of the FG-sandwich plate (1-0-1,1-1-1, 1-2-1, 2-1-2 and 2-2-1).

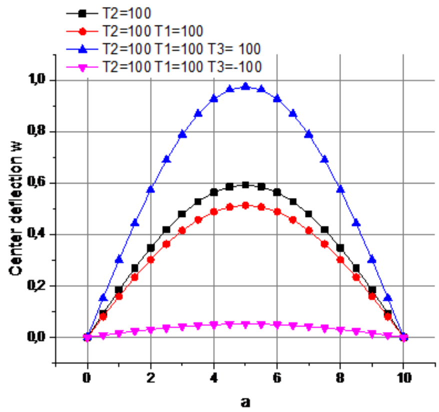

Fig. 2 – Effect of the thermal load T_1, T_2 and T_3 on the Center deflection $\bar{w}$ of the (2-2-1) FG-sandwich plate ($k = 2$) for Voigt model

Table 2 – Center deflection of the FGM sandwich square plates ($T_3 = 0$)

k	Theory		$\bar{w}$				
			1-0-1	1-1-1	1-2-1	2-1-2	2-2-1
0	Zenkour et al [10]	Voigt	0.4802	0.4802	0.4802	0.4802	0.4802
	Present	Reuss	0.4802	0.4802	0.4802	0.4802	0.4802
		LRVE	0.4802	0.4802	0.4802	0.4802	0.4802
		Tamura	0.4802	0.4802	0.4802	0.4802	0.4802
1	Zenkour et al [10]	Voigt	0.6368	0.6062	0.5822	0.6210	0.5925
	Present	Reuss	0.6246	0.5935	0.5704	0.6083	0.5807
		LRVE	0.6316	0.6008	0.5772	0.6156	0.5875
		Tamura	0.6246	0.5935	0.5704	0.6083	0.5807
3	Zenkour et al [10]	Voigt	0.6835	0.6535	0.6223	0.6702	0.6340
	Present	Reuss	0.6803	0.6485	0.6166	0.6658	0.6141
		LRVE	0.6819	0.6512	0.6197	0.6681	0.6318
		Tamura	0.6803	0.6485	0.6166	0.6658	0.6141
5	Zenkour et al [10]	Voigt	0.6914	0.6658	0.6338	0.6813	0.6450
	Present	Reuss	0.6901	0.6630	0.6304	0.6791	0.6421
		LRVE	0.6907	0.6645	0.6322	0.6802	0.6436
		Tamura	0.6901	0.6630	0.6304	0.6791	0.6421

4.2 Effect of the Thermal Loads on the Center Deflection $\bar{w}$

In the present section three types of the temperature distribution across the thickness are considered. The first one, the temperature is linearly distributed through the thickness $T = zT_2$, in the second type the temperatures vary nonlinearly across h; $T = \Psi(z)T_3$ and the third type is reserved for a combination of linear and nonlinear distributions $T = zT_2 + \Psi(z)T_3$.

From Fig. 2, it can be observed that the Center deflection $\bar{w}$ is clearly influenced by the values of the thermal loads T_1, T_2 and T_3. It can be also concluded that the larger values of the Center deflection $\bar{w}$ are obtained for thermal load $T_3 = 100$.

4.3 Effect of the Homogenization Models on Center Deflection $\bar{w}$

This section presents a comparison of the center deflection $\bar{w}$ obtained with the different micromechanical models of Reuss, LRVE and Tamura based on four variable combined exponential/hyperbolic shear deformation theory .The geometry ratio is taken $a/h = 10$, aspect ratio $a/b = 1$, $T_1 = 0$ and $T_2 = 100$.

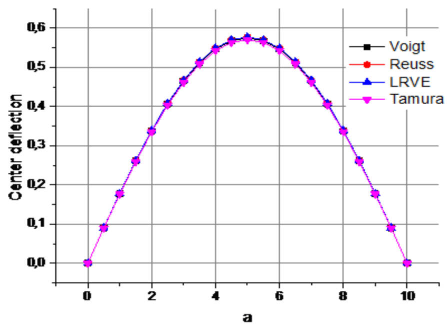

Fig. 3 – Effect of different micromechanical models on the Center deflection $\bar{w}$ of the (1-2-1) FG-sandwich plate ($k = 1$)

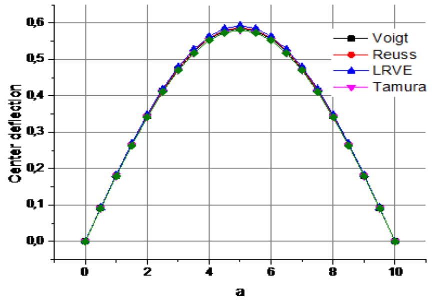

Fig. 4 – Effect of different micromechanical models on the Center deflection $\bar{w}$ of the (2-2-1) FG-sandwich plate ($k = 1$)

Fig. 3 and Fig. 4, show the effect of different micromechanical models on the center deflection of the (1-2-1) and (2-2-1) FG-sandwich plate ($k = 1$).

It can be observed that the Center deflection $\bar{w}$ is not influenced by the variation of different micromechanical models excluded some values in range of $1/10^3$.

5. CONCLUSION

A comparative study has been presented in this paper. The used theory is proposed for the thermo-elastic flexural behavior of the simply supported FG-sandwich plate. is confirmed by the comparisons performed with the other theory used in literature. It can be noted that the present model is simple to predict the thermoelastic deflection behaviors of the simply supported of sandwich functionally graded plate. The plate is strongly influenced by the thermal load however the micromechanical models have approximately the same effect on deflection behaviors of FGM plate.

REFERENCES

1. V. Bhavar, P. Kattire, S. Thakare, S.Patil, R. Singh, *IOP Conf. Ser.: Mater. Sci. Eng.* **229**, 012028 (2017).
2. M. Chitour, A. Bouhadra, M. Benguediab, K. Mansouri, A. Menasria, A. Tounsi, *J. Nano- Electron. Phys.* **14** No 3, 03028 (2022).
3. K. Ravikiran, A. Kashif, N. Ganesan, *Appl. Math. Model.* **32**, 2509 (2008).
4. A. Berkia, M. Benguediab, A. Bouhadra, K. Mansouri, A. Tounsi, M. Chitour, *J. Nano- Electron. Phys.* **14** No 3, 03031 (2022).
5. M.M. Gasik, *Comput. Mater. Sci.* **13**, 42 (1998).
6. J.R. Zuiker, *Comp. Eng.* **5** No 7, 807 (1995).
7. J.H. Kim, G.H. Paulino, *Int. J. Numer. Meth. Eng.* **58**, 1457 (2003).
8. H.S. Shen, Z.X. Wang, *Compos. Struct.* **94**, 2197 (2012).
9. D.K. Jha, T. Kant, R.K. Singh, *Compos. Struct.* **96**, 833 (2013).
10. A.M. Zenkour, N.A. Alghamdi, *J. Mater. Sci.* **43** No 8, 2574 (2008).

Вплив моделей ідеалізації на прогин пластини з функціонально градуйованим матеріалом (ФГМ)

B. Rebai[1], K. Mansouri[2,3], M. Chitour[2], A. Berkia[2], T. Messas[1], F. Khadraoui[2], B. Litouche[4]

[1] *University Abbes Laghrour, Civil Engineering Department, 40000 Khenchela, Algeria*
[2]*UniversityAbbes Laghrour, Mechanical Engineering Department, 40000 Khenchela, Algeria*
[3] *Laboratory of Engineering and Sciences of Advanced Materials (ISMA), 40000 Khenchela, Algeria*
[4] *University Center Abdelhafid Boussouf, Mechanic and Electro-Mechanic Department, 43000 Mila, Algeria*

Функціонально градуйовані матеріали (ФГМ) — це клас композитів, у яких властивості матеріалу поступово змінюються в одному або кількох декартових напрямках, поєднання яких призводить до більш ефективних характеристик, ніж компоненти, взяті окремо. Цей клас композитних матеріалів привернув значну увагу в інженерних галузях, таких як ядерна енергетика, аерокосмічна промисловість та виробництво електроенергії. Мета даної роботи полягала у дослідженні впливу моделей гомогенізації (ідеалізації) та термічних навантажень на статичну поведінку прогину сендвічової функціонально градуйованої плити. Кілька мікромеханічних моделей було використано для отримання ефективних властивостей матеріалу двофазної плівки ФГМ. Плівка ФГМ піддається лінійним і нелінійним тепловим навантаженням. Використана інтегральна теорія містить лише чотири функції змінних на відміну від п'яти у випадку інших моделей. Базове рівняння отримано та розв'язано за допомогою принципу віртуальної роботи та моделі Нав'є. Точність запропонованої аналітичної моделі підтверджується порівнянням результатів з літературними даними.

Ключові слова: Гомогенізація, Сендвіч-плівка, Функціонально градуйований матеріал, Прогин, Рівняння Нав'є.