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Precision Synchronization of Chaotic Optical Systems

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The ideas of this article develop the technologies for synchronization of chaotic information systems components and parameters. Such systems hide information by embedding data into a chaotic carrier signal. Precision chaotic synchronization requires the correct measurement and analysis of chaotic dynamic variables. A model for estimating the degree of synchronization of chaotic laser modes is proposed. The model is based on the principles and methods of nonlinear chaotic values measurement. It provides the measurement and analysis of chaotic variables, the formation of measurement portrait that represents the states and dynamics of the system, and completes the methods for estimating the chaos degree, radiation parameters stability, and degree of synchronization of dynamic variables. A scheme for studying and controlling the chaotic dynamics of pulsed lasers, which includes a laser, a pulsed energy meter, a spectrum analyzer, a pulse frequency measurement unit, and a system for control, synchronization and recording measurement results is proposed. The divergence criteria for the dynamic variables' values, fractal dimensions, measurement phase? portraits are offered for evaluation of synchronization. The equation that connects the fractal dimension and system parameters stability is obtained. It can be used for control of chaotic information systems components and parameters dynamics.

Keywords: Chaotic system, Non-linear dynamic, Synchronization.

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1. INTRODUCTION

The chaotic information methods and systems have an important position among the future directions of information and telecommunication technologies. Their features are the combination of data with a chaotic carrier signal, whose parameters are known only to the sender and recipient. Furthermore, chaotic modulation of digital data, in addition to secrecy, provides low sensitivity to electronic nonlinearity in transmission and reception. Both chaotic radio and microwave systems and optical chaotic communication systems are developing today. The creation of a two-way optical chaotic communication system is an urgent task [1-5].

A condition for the stable functioning of chaotic systems is stable, precision synchronization of the transmitter and receiver (or the group of receivers) of information [6], that in chaotic, unstable dynamics is a difficult task. The occurrence, course and characteristics of nonlinear, chaotic processes depend on the number of internal and external factors. Such systems demonstrate strong fluctuations because of weak effects and exponential divergence of phase trajectories.

Chaotic information technologies are based on the physical and mathematical foundations of the information theory, dynamic chaos, and synergies theories. The implementation of precision synchronization in chaotic systems can be achieved by methods and tools of nonlinear measurement theory, the task of which is to measure dynamic variables (DVs) of open, dissipative nonlinear dynamic systems (NDSs). Its main features and models take into account that the measured values are interval DVs with nonlinear stochastic or chaotic dynamics, leading to stochastic and chaotic dynamics of measurement results. The time series of measurement results are a non-Markov process, have the Hausdorff dimension, small changes in the initial conditions and fluctuations lead to significant changes of measurement results. It is proposed to use the system phase portrait, fractal and entropy analysis, Lyapunov indicators, prediction time and other tools to study and classify the dynamics of chaotic DVs.

The aim of the article is to develop technologies for precision synchronization of chaotic optical systems by methods and tools of nonlinear measurement theory. To achieve the goal, the following tasks were set and solved: to analyze properties of dynamic chaos, which became a basis for chaotic information technologies, to analyze advantages of a chaotic signal as a carrier of information, to study the method of chaotic synchronization, to develop the scheme for studying chaotic lasers and tools for evaluation of their parameters synchronization.

2. SYNCHRONIZATION OF CHAOTIC SYSTEMS

Dynamic (or deterministic) chaos is a phenomenon that occurs in NDSs of different origins. In biological, physical and technical NDSs, however, chaos arises and insists in the same way. H. Haken defined chaos as irregular motion described by deterministic equations [7, 8]. This is a phenomenon when NDS behavior appears to be random even though it is determined by deterministic laws. In electronic and optoelectronic devices, chaos can be both spontaneous and controlled process. Chaos demonstrates the following main characteristics: a continuous power spectrum, an exponential decrease in the correlation function, a short prediction time for the system dynamics, strong sensitivity to initial conditions and noise, that results in the exponential blurring of the phase trajectory, topological mixing, a dense arrangement of periodic orbits in phase space, the presence of an attractor.

Research into the causes and properties of chaos led to the idea of developing new information technologies based on it. The perspective of this idea is based on the following properties of chaos in technical systems: possibility of implementing a large number of chaotic modes in one device, possibility of controlling a chaotic mode with a small change in the control parameter of the system, high information capacity of a broadband chaotic by nature signal, an increase in the modulation rate compared to regular signals and increase in the information security. The studies by Pecora L.M. and Carroll T.L. demonstrate the possibility of spontaneous synchronization of the transmitter and receiver operating in a chaotic mode.

Chaotic communication systems differ in the ways in which an information signal enters a chaotic carrier signal. The literature describes the method of nonlinear mixing of the signal, the method of trajectory correction by small perturbations, the method of using the thin attractor structure [9]. According to the properties of chaos, even small changes in such a signal initiate a change in its structure and chaotic mode. These changes are difficult to detect by an outside observer but can be reliably detected by an information receiver, which has a synchronized receiver and a special equipment. The input of information is accompanied by a change in parameters, and its extraction is ensured by selecting parameters that synchronize the receiver and the transmitter. The device synchronized with the transmitter is a nonlinear filter that allows to emit the desired chaotic signal among others. The key task of the described technology is evaluation and precision reproduction of the synchronization parameters. The theory of chaotic synchronization is under development today [10].

Let us consider two NDSs that can generate chaotic fluctuations. The state of each system is unambiguously defined as the sum of DV values $\left[X_1(t),...,X_n(t)\right]$ at a certain moment of time t. The evolution law $F(X, t)$ describes the evolution of the initial state $\left[X_1(t_0),...,X_n(t_0)\right]$:

$$F\left[X(t_0)\right]\rightarrow X(t)\,, \tag{1}$$

where X is the state vector of an n-dimensional NDS, n is the number of DV.

NDS can be described by the following differential equation:

$$\frac{dX(t)}{dt}=F\left[X(t)\right]. \tag{2}$$

Chaotic synchronization of two or more identical NDSs is called such time-identical change of the same DVs characterized by chaotic oscillations, when the systems maintain their stable conditions despite external influences. It can be described by the formula:

$$\Delta X(t)=\lim_{t\to\infty}\left|X_i^{'}(t)-X_i^{''}(t)\right|=0\,, \tag{3}$$

where $X_i^{'}(t)$ and $X_i^{''}(t)$ are the same DVs of two systems.

That is, the chaotic dynamics of both systems must follow the same evolutionary law:

$$F(X^{'},t)=F^{'}(X^{''},t)\,. \tag{4}$$

During synchronization of different systems, condition (4) cannot be completed and synchronization is considered in a topological sense when the phase trajectories $X^{'}(t)$ and $X^{''}(t)$ are not synchronous but repeat each other and the function $\Delta X(t)$ is random with zero mean value.

Let us consider a hierarchical NDS characterized by a vector X (2) consisting of two subsystems with state vectors $X^{'}$ (a leading system) and $X^{''}$ (a following system) (Fig. 1). Then equation (2) transforms into the system:

$$\frac{dX^{'}}{dt}=F^{'}\left[X^{'},X^{''}\right],\frac{dX^{''}}{dt}=F^{''}\left[X^{''},X^{'}\right]. \tag{5}$$

If signal $X_1^{'}(t)$ from the leading subsystem comes into the following subsystem, then under certain conditions the difference between the input and output $X_2^{'}(t)$ signals is almost zero (3). Thus, there is a situation when the following system reproduces the dynamics and values of the leading system DVs. An example of the described procedure is an injection of modulated radiation from a leading laser as pumping energy into a following laser, when they form a chaotic optical information system.

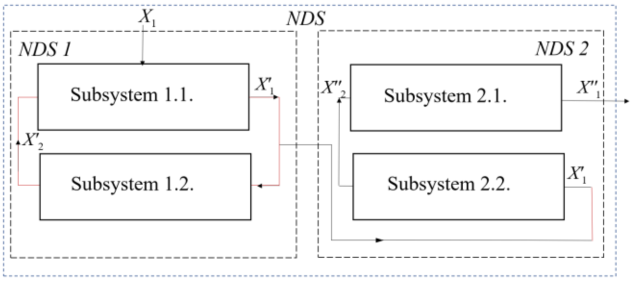

Fig.1 – Synchronization schema: NDS is a communication system consisting of the transmitter NDS 1 (leading system) and the receiver NDS 2 (following system), X_1 is the input signal, $X_1^{'}$ is the control signal, $X_1^{''}$ is the output signal

Note that the condition (3) requiring the fulfilment of the condition $t\to\infty$ cannot be achieved in practice. This ideal synchronous response can only be obtained if there is no noise at all. Real NDSs, such as lasers, are dissipative open systems subject to external influences, leading to fluctuations in their DVs. In the case of chaotic modes, such systems are highly dependent on weak control changes. For this reason, it is advisable to talk about a small interval of DV $\Delta X(t)$ (3). Stable and precision synchronization of two and more NDSs can be provided by chaotic quantities measurement methods developed within a nonlinear measurement theory.

3. IMPLEMENTATION OF PRECISION SYNCHRONIZATION

According to [11], measurement precision is a closeness of agreement between indications or measured quantity values obtained by replicate measurements on the same or similar objects under specified conditions. Measurement precision is usually expressed numerically by measures of imprecision, such as standard deviation, variance, or coefficient of variation under the specified conditions of measurement. The ‘specified conditions’ can be, for example, repeatability conditions of measurement, intermediate precision conditions of measurement, or reproducibility conditions of measurement. A chaotic signal generator is precise if it steadily keeps a set of chaotic modes, has low sensitivity to changes in the external conditions, in a pair ‘leading system – following system’ demonstrates a stable synchronous response, etc. [1].

Analysis of these definitions made it possible to give a definition for ‘precision synchronization of chaotic systems. Precision synchronization of chaotic NDSs is the state of two or more systems when the divergence of DVs values does not exceed a given value ΔX within a time interval T. At the same time, in a narrow sense, it is the synchronization of output signals $X_1^{'}$ and $X_1^{''}$ (Fig. 1). In a broad sense, it is possible to talk about synchronization of a number of DVs characterizing the systems states, proximity of NDSs phase portraits generated by vectors $X^{'}$ (leading system) and $X^{''}$ (following system) and proximity of their topological characteristics (Kolmogorov-Sinai entropy, Lyapunov exponents, fractal dimension, etc.). The implementation of precision synchronization requires correct and precise measurements of DVs, their fluctuations and external influences that can start the modification of oscillating modes.

The authors have developed a model for estimating the synchronization of laser chaotic regimes. The schema for studying and controlling chaotic dynamics of pulse lasers is presented in Fig. 2. It can be used to measure and evaluate the dynamics and stability of DVs such as: pulse energy, duration and frequency, radiation spectral characteristics. Blocks 1, 2, 10 are a system consisting of two closed feedback loop subsystems (NDS 1, Fig. 1) [12].

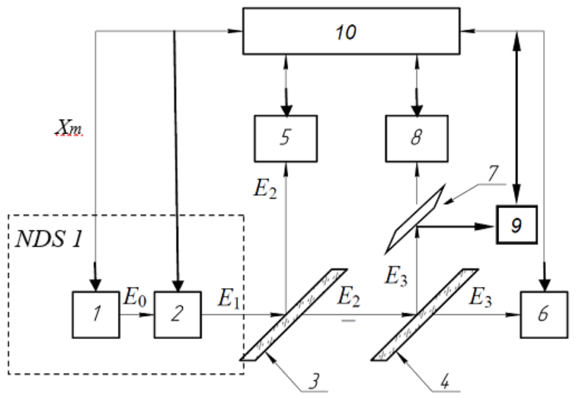

Fig. 2 – Schema for studying and controlling chaotic dynamics of pulse lasers where X_m is the control signal, E_i are the DVs (i = 0, …, n): 1 – pumping system, 2 – laser, 3, 4, 7 – dividing plates, 5 – pulse energy meter, 6 – spectrum analyzer, 8 – pulse duration measurement unit, 9 – pulse frequency measurement unit, 10 – system of control, synchronization and recording of measurement results

During the experiment, the values of DVs are measured and their dynamics is studied by a controlled change in the signal X_m. The experiment generates the time series of DVs $\{x_i\}$, series of measurement results $\{y_i\}$ and series of measurement estimates corrected for all known systematic sources of uncertainty and standard uncertainty $\{u_i\}$:

$$(y_1 - u_1, y_1 + u_1), \ldots, (y_n - u_n, y_n + u_n) . \quad (6)$$

The dataset (6) is the vector Y which is the result of the measurement evaluation of the vector X.

The DVs possible values intervals should be calculated next. When chaotic dynamics demonstrates the Lyapunov stability of phase trajectories in the space X, the values of DVs form an attractor bounded by values $\Delta X_i = X_i^{\max} - X_i^{\min}$ or, taking into account (6):

$$\Delta y_i = (y_{i\min} - u_{i\min}, y_{i\max} + u_{i\max}) , \quad (7)$$

where $y_{i\min}$, $y_{i\max}$ are estimates of the maximum and minimum measurement X_i results, $u_{i\min}, u_{i\max}$ are their uncertainties.

According to the obtained data of Y, a measurement portrait is completed. A measurement portrait is a graphical and numerical representation of the DVs measurement results, which is an extended phase portrait of NDSs, where each value X_i is displayed by its evaluations $Y_i(x_i, y_i)$. The presence of measurement uncertainties $\{u_i\}$ in the measurement portrait, Y is a necessary condition for assessing processes precision. Furthermore, the synchronization condition for the dynamics of two chaotic NDSs (3) is described by the formula:

$$\Delta Y = \lim_{t \to T} \left| Y^{'} - Y^{''} \right| \to 0 , \quad (8)$$

where T is a limited time, 0 is the zero vector.

The study of the measurement portrait (8) gives the possibility to define topological characteristics of NDSs.

In the case of non-identical systems, when condition (4) is not met, it is advisable to use the topological synchronization criterion based on the Lyapunov exponents λ_i [12]:

$$\Delta Y = \lim_{t \to T} \left| \lambda_i^{'} - \lambda_i^{''} \right| \to 0 . \quad (9)$$

Application of the criterion (9) is effective in the case when the trajectories $X_i^{'}(t)$ and $X_i^{''}(t)$ in the phase space are not synchronous but repeat each other.

A numerical assessment of the degree of chaotic dynamics (2) is completed using a fractal analysis of the measurement results (6) and expressed by the fractal dimension D:

$$D = 2 - H , \quad (10)$$

where H is the Hurst parameter [12]:

$$H = \ln(R/s)/\ln(n/2),$$
$$R(i) = \max_{1 \le i \le n} x(i,n) - \min_{1 \le i \le n} x(i,n),$$
$$x(i,n) = \sum_{i=1}^{n} (x_i - \bar{x}_i)$$

where s is the standard deviation, $\bar{x}_i$ is the arithmetic mean of the values $\{x_i\}$.

The dynamics can be classified using a fractal scale with D values from 1 to 2. If $D = 1$, the process (2) is deterministic; if $1 < D < 1.5$, the process is chaotic; when $D = 1.5$, the process is random; when $1.5 < D < 2$, the process has a noise range; if $D = 2$, the range of measured values is very big for analysis. Thus, the fractal dimension D is a numerical characteristic of the order or chaos of NDSs. Furthermore, the condition for synchronizing the chaotic dynamics of two DVs for different NDSs is written as:

$$\Delta D = \lim_{t\to T}\left|D_i^{'}(t) - D_i^{''}(t)\right| \to 0 . \qquad (11)$$

To achieve precision, it is necessary to know the DV stability value that is defined as:

$$\Delta x = \pm\frac{2s}{\bar{x}} . \qquad (12)$$

The analysis of expressions (11) and (12) makes it possible to complete a new equation that demonstrates the relationship between the DV stability and fractal dimension of its dynamics in the form:

$$D = 2 - \frac{\ln(2R)}{\ln(\frac{\Delta x\bar{x}n}{2})} . \qquad (13)$$

According to (13), changes in the values of R and Δx because of the control of X_m and device parameters changes lead to changes in the NDS dynamics. Formula (13) can be used for precision synchronization of chaotic optical systems (11) with changing stability values Δx (12) and control of the fractal dimension D (13). Unstable, chaotic dynamics is possible in both multimode and single-mode lasers [13-15]. In a single-mode laser, chaos is possible in the case of low resonator quality and high pump power. It is possible to achieve chaotic modes with low pump power and modulation effect, such as temporary modulation of resonator loss or inversion, and injection of a modulated coherent electromagnetic field. Doubling the generation period may also lead to chaos.

Thus, application of the model for estimating the synchronization of laser chaotic regimes (Fig. 2) allows to perform the following operations: to measure the DVs of NDSs and evaluate the results (6), (7); to form the NDS state and dynamics measurement portrait $Y(x,u)$; to calculate the NDS chaoticity degree (10) and DVs stability value (12); to estimate the degree of synchronization of both individual DVs (11), (13) and full systems (8). For non-identical systems, the topological synchronization criterion based on Lyapunov exponents (9) is proposed.

4. CONCLUSION

One of the most important scientific tasks of nowadays is practical application of the chaotic NDSs characteristics research results. We try to use such properties as continuous power spectrum, exponential decline of correlation function, short system dynamics forecast time, strong sensitivity to initial conditions and noise etc. So, the ideas of our article develop the technologies for synchronization of optical chaotic information systems and their components. The main advantage of these systems is an ability to hide useful information into a chaotic carrier signal that looks like a nose.

As chaotic system we have studied optical information systems with chaotic lasers. Correct operation of such systems can be ensured by precision synchronization of their main elements (lasers) which requires the correct measurement and analysis of chaotic dynamic variables. For this aim we have developed the model for estimating the synchronization of laser chaotic regimes. Our model provides the measurement and analysis of chaotic variables, formation of measurement portrait that represents the states and dynamic of NDSs. The model has the methods for estimation of chaos degree and radiation parameters stability with fractal tools and degree of dynamic variables synchronization. For evaluation of synchronization, it is proposed to use the divergence criteria for the dynamic variables' values, fractal dimensions, measurement portraits divergence. We have also obtained the equation that connects the fractal dimension and stability of the NDSs parameters.

Thus, application of the work's results supports the precision synchronization of chaotic NDSs as a condition for the stable functioning of chaotic communication systems and helps to develop a bilateral optical, chaotic communication systems.

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Прецизійна синхронізація хаотичних оптичних систем

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Стаття присвячена питанню розвитку технології синхронізації компонент і параметрів хаотичних, оптичних інформаційних систем, особливість яких полягає у вбудові даних в хаотичний сигнал. Прецизійна синхронізація вимагає коректного вимірювання і аналізу хаотичних динамічних змінних. На основі принципів і методів вимірювань нелінійних хаотичних величин розроблена модель оцінки ступеня синхронізації хаотичних режимів лазерів. Модель забезпечує вимірювання та аналіз динамічних змінних, формування портрета вимірювання станів і динаміки системи, дозволяє оцінювати ступінь хаотичності, значення стабільності параметрів випромінювання, а також ступінь синхронізації динамічних змінних. Запропонована схема вивчення та управління хаотичною динамікою імпульсних лазерів, до складу якої входять лазер, лічильник імпульсної енергії, аналізатор спектра, блок вимірювання частоти імпульсів та система управління, синхронізації і запису результатів вимірювань. Для оцінки синхронізації запропоновано критерії розбіжності значень динамічних змінних, значень фрактальної розмірності, фазових портретів вимірювань. Для контролю динаміки системи в роботі отримано співвідношення зв'язку фрактальної розмірності, як характеристики динаміки змінної, та її стабільності.

Ключові слова: Хаотична система, Нелінійна динаміка, Синхронізація.