JOURNAL OF NANO- AND ELECTRONIC PHYSICS *ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ*
Vol. **12** No 4, 04034(7pp) (2020) Том **12** № 4, 04034(7cc) (2020)

Dynamics of Receiving Electroelastic Spherical Shell with a Filler

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(Received 29 February 2020; revised manuscript received 20 August 2020; published online 25 August 2020)

Numerical calculations of the amplitude-frequency characteristics of the difference of electric potentials at the output of an elastic spherical converter with internal filling were obtained. Vacuum, helium and water were used as the internal filler. Mathematically the operation of the specified oscillatory system is described using the state equations for piezoceramics, which linearly relate components of mechanical stresses, deformations, electrical tensions and induction; the equations of motion of a thin shell involving equations of Cauchy ratios which are connecting components of the strain tensor and the displacement vector; equations of forced electrostatics. The output electrical signal of the investigated spherical receiving transducer with a fully electrode surface is determined by the centrally symmetric component of the stress-strain state of the piezoceramic shell. It is established that the oscillatory system is characterized by the presence of a basic resonance of zero mode and an additional position whose position depends on the electrical load and the characteristics of the aggregate. It is shown that the presence of a filler makes it difficult to match the resistance of the converter with the input resistance of the receiving path and leads to a decrease in the width of its working strip. The resonance is no longer accompanied by antiresonance, as in the case of air or helium filling. Since helium is very similar to air in its characteristics, the frequency response is similar to the frequency response of air. The resonance region accompanied by antiresonance almost coincides in frequency, and the local extremum of the frequency response in the low-frequency region is as weak as in the case of filling the converter with air.

Keywords: Piezoceramic spherical transducer, Aggregate, Amplitude-frequency characteristics, Electric field, Electroelastic properties, Sensitivity, Sound reception, Potential difference.

DOI: 10.21272/jnep.12(4).04034 PACS number: 43.38.n

1. INTRODUCTION

Design of underwater electroacoustic spherical transducers often involves the use of compensated power structures which are resistant to the effects of the hydrostatic pressure. One of the possible ways of designing, implementing such a property, is to fill the inner cavity of the sphere with acoustic environments, such as liquids or gases [1-3].

A positive quality of this approach is an increase in the range of the working depths and the ability to adjust the frequency dependences of the main characteristics of acoustic, mechanical, and electric fields that take part in the process of converting an acoustic field into a mechanical and then – into an electrical one. However, this may reduce the efficiency of the converter in terms of possible reduction of the sensitivity, increasing the size and cost of the converter, as well as complicating the assembly process and the structure itself [4, 5]. The dynamic properties of such an oscillatory system manifest themselves in a changing sensitivity, which is conveniently and prospectively described, based on the methodology of cross-cutting tasks, which actually involve the presentation of the reception process from pressure in the acoustic field to the voltage on the load of the transducer electrodes by a joint solution of equations for acoustic, mechanical and electric fields [1-3, 6, 7].

The present work is devoted to the study of the electroelastic properties of electroacoustic transducers made of piezoceramics and belongs to the class of problems of stationary hydroelectroelasticity.

The paper considers the results of numerical studies of the cross-cutting issues of the problem of receiving sound waves by a spherical piezoelectric transducer with fully electrodated surfaces, and the article is a continuation of [8]. The presented materials contain brief information about the main relationships and the solution of the cross-cutting reception problem [8], calculation results and analysis of the frequency characteristics of the electric voltage at the load of the converter, the construction of which implies either vacuuming the internal region or filling it with ideal environments.

In our opinion, the proposed material regarding the results of numerical studies of the amplitude-frequency characteristics of the electrical voltage (hereinafter the frequency response) of these converters seems relevant and modern. And the aim of the work is to develop recommendations on the use of gaseous and liquid fillers of the main cavities of spherical piezoelectric transducers of compensated design, with the purpose of regulating the bias of the resonance regions of the amplitude-frequency characteristics.

2. FORMULATION OF THE PROBLEM

It is assumed that a receiving piezoceramic (piezoelectric material: density ρ_m, speed of sound c_m) transducer in the form of a spherical shell (radius R_0, wall thickness h_0) is placed in an ideal fluid with density ρ and speed of sound c (Fig. 1). The shell is radially polarized.

The task used:

– general rectangular coordinate system O, X_1, X_2, X_3 which is located so that the axes OX_1, OX_2 are lying in a plane crossing the latitude of the converter and the axis OX_3 – in the plane of its meridian section;

– spherical coordinate system O, r, φ, ϑ whose center coincides with the center of the rectangular coordinate system.

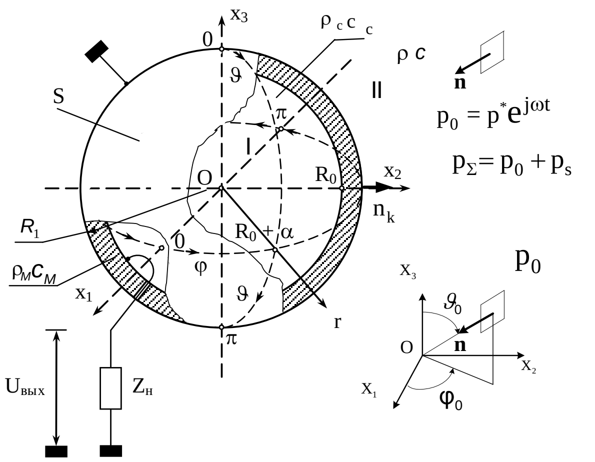


Fig. 1 – Spherical piezoceramic converter

Continuous electrodes are applied to the outer and inner surfaces of the converter, which completely cover them and are connected to an arbitrary electrical load Z_n. We consider the electrode thickness to be small, does not affect the mechanical characteristics of the transducer and does not require additional conditions for the mechanical and electric fields. The electrical voltage U_n ($U_{вых}$ that arises on the load Z_n is required).

Flat sound wave p_0 with amplitude p_0^* falls on the spherical converter from infinity in the direction of the vector n (which in turn is positioned by angles ϑ_0, φ_0) $p_0 = p_0^* e^{j\omega t}$ resulting in a diffused $p_s(r, \varphi, \theta)$ and penetrated external and internal fields $p_I(r, \varphi, \theta)$.

Mathematically the operation of the specified oscillatory system is described using [1, 8]:

– state equations for piezoceramics, which linearly relate components of mechanical stresses, deformations, electrical tensions and induction;

– equations of motion of a thin shell involving equations of Cauchy ratios which are connecting components of the strain tensor and the displacement vector;

– equations of forced electrostatics [9].

The compatible solution of these equations allows to determine the characteristics of the converter, taking into account the relationship of three fields: electrical, mechanical and acoustic. The solution of the problem are performed by the method of partial regions [9] with the use of the Fourier method, and the properties of orthogonality of the associated Legendre functions and trigonometric functions at intervals $\vartheta \in [0;\pi]$, $\varphi \in [0;2\pi]$.

Recall that the method of partial domains is based on the division of the total space of existence of a sound field into canonical domains so that in each such formed region the field satisfies the Helmholtz equation.

It is proposed to split the workspace into two areas I and II (Fig. 1) so that the area I ($0 \le r \le R_1 - h_{0s}$, $R_0 = R_1 - h_{0s}$, $\varphi \in [0;2\pi]$, $\vartheta \in [0;\pm\pi]$) corresponds to the closed space of the internal volume of the converter and the area II ($R_1 \le r \le \infty, \varphi \in [0;2\pi], \vartheta \in [0;\pm\pi]$) corresponds to the outer space.

The field in region I is formed by a penetrating wave $p_I(r, \varphi, \theta)$, and the field in region II – by the superposition of flat $p^0(r, \varphi, \theta)$ and scattered $p_s(r, \varphi, \theta)$ waves

$$p_{II}(r,\varphi,\theta) = p^0(r,\varphi,\theta) + p_s(r,\varphi,\theta)$$

Thus, the solution of the problem is to find unknown coefficients of decomposition for the acoustic, mechanical and electrical fields of the "external working environment – spherical shell – internal volume filled with acoustic medium" system, which are the consequences of the influence of external (acoustic load $p_{II}(r, \varphi, \theta)$).

3. BOUNDARY CONDITIONS

Acoustic-mechanical Conditions

According to the statements [1-3, 11-13], acoustic and mechanical conditions are defined for the boundaries of regions I and II in the form of conditions of conjugation of force and kinematic type as:

$$\begin{cases} p_I - p_{II} = \sigma_{rr}; \\ \qquad r = R_1, r = R_0; \varphi \in [0;2\pi], \vartheta \in [0;\pi] \\ \upsilon_{rI} = \upsilon_{r\,II}; \end{cases} \quad (3.1)$$

$$\upsilon_{rI}\big|_{r=R_0} = \upsilon_{rII}\big|_{r=R_1} \Rightarrow \frac{1}{j\rho_c c_c}\frac{\partial p_I}{\partial (k_c r)}\bigg|_{r=R_0} = \frac{1}{j\rho c}\frac{\partial p_{II}}{\partial (kr)}\bigg|_{r=R_1} \quad (3.2)$$

υ_{rI}, υ_{rII} are the radial components of the oscillatory velocities of the points of the inner and outer surfaces of the shell considered equal; $\sigma_{rr} = \sigma_{kj}$ are the radial component of the tensor of the resultant mechanical stresses that appear in the piezo-material of the shell during its deformation;

$$\upsilon_{rII}\big|_{r=R_1} = \upsilon_{r0}\big|_{r=R_1} \Rightarrow \frac{1}{j\rho c}\frac{\partial p_{II}}{\partial (kr)}\bigg|_{r=R_1} = \upsilon_{r0}\big|_{r=R_1}, \quad (3.3)$$

υ_{r0} are the radial components of the oscillatory velocities of the particles of the medium $\upsilon_{rII}\big|_{r=R_1} = \upsilon_{r0}\big|_{r=R_1}$.

In addition, for the development of the condition determined by the first equation of the system of functional equations (3.1), when contacting a piezoceramic shell with a fluid of low dynamic viscosity, it is appropriate to use Newton's third law as the following equality:

$$n_k(\sigma_{kj} - \delta_{kj}\Delta p) = 0, \ \forall \ x_k \in S, \quad (3.4)$$

n_k is the k-th component of the vector of external normal to the surface of the transducer s; δ_{kj} is the Kronecker symbol for indexes k, j; $\Delta p = p_I(r,\varphi,\vartheta) - p_{II}(r,\varphi,\vartheta)$ is the excess pressure that is brought to the surface of the shell $r = R_1$ or $r = R_0$; $x_k \in S$ is the condition of belonging to the spatial coordinate of the shell environment.

By involving the conjugation conditions, the displacement $u_r, u_\varphi, u_\vartheta$ can be written as

$$\upsilon_{rII}\big|_{r=R_1} = \upsilon_r\big|_{r=R_1} = j\omega u_r, \ r \in [R_0, R_1],$$

υ_r is the radial component of the oscillatory speed of the material particles of the surface of the shell, u_r is the radial component of the displacement of the material particles of the surface of the shell.

Electrical Conditions

Using the position of works [1, 11], we consider that the shell thickness is much smaller than the radial dimensions: $h_0 \ll R_0, R_1$.

Therefore, the electric polarization of the deformed piezoceramic shell is determined by the radial component of the electric induction vector $D_m \to D_r$.

In this case, the electric charge Q formed on the electroded surfaces by the free carriers of electric current as a result of the polarization charges of the deformed piezoceramic element on them, taking into account [1, 9, 12], is represented as:

$$Q = -\int_S D_r dS = -R_0^2 \int_0^{2\pi}\int_0^{\pi} D_r(\varphi,\vartheta)\sin\vartheta d\vartheta d\varphi .$$

$D_r(\varphi, \vartheta)$ corresponds to the equation

$$D_r = e_{11}\varepsilon_{rr} + e_{12}\left(\varepsilon_{\varphi\varphi} + \varepsilon_{\vartheta\vartheta}\right) + \chi_{11}^{\varepsilon} E_r ,$$

where S is the area of the electrode, $dS = R_0^2 \sin\vartheta d\vartheta d\varphi$.

The current I_n of the load circuit (that is, the current due to the load of the electrode Z_n) is represented as the rate of change of Q, which will be determined by the time derivative:

$$I_n = \frac{\partial Q}{\partial t} = j\omega Q = -j\omega \int_S D_r dS =$$
$$= -j\omega R_0^2 \int_0^{2\pi}\int_0^{\pi} D_r(\varphi,\vartheta)\sin\vartheta d\vartheta d\varphi = -j\omega R_0^2 \cdot I ,$$
$$I = \int_0^{2\pi}\int_0^{\pi} D_r(\varphi,\vartheta)\sin\vartheta d\vartheta d\varphi =$$
$$= \int_0^{2\pi}\cos(m\varphi)d\varphi \int_0^{\pi} D_r^{\nu m}(\varphi,\vartheta) P_\nu^m(\cos\vartheta)\sin\vartheta d\vartheta$$

Applying [13], we pass to the form:

$$I = 4\pi D_r^0 .$$

Therefore, the current will be determined by the expression $I_n = -j\omega 4\pi R_0^2 D_r^0$, D_r^0 corresponds to the centrally symmetric component of the expression

$$D_r^0 = e_{11}\varepsilon_{rr}^0 + e_{12}\left(\varepsilon_{\varphi\varphi}^0 + \varepsilon_{\vartheta\vartheta}^0\right) + \chi_{11}^{\varepsilon} E_r^0 .$$

Using the boundary conditions of the types $divD = divD_r = divD_r^0 = 0$ and $E_r = E_r^0 = -\frac{\partial\Psi}{\partial x_k}$, we have the opportunity to get Ψ is the scalar electrical potential and, accordingly, the electrical voltage U_n of the shell electrodes due to the corresponding potential difference.

Note that the definition E_r^0 corresponds to the condition that there is no charge in the piezoceramics. Thus, for a vector D_r^0, using the form of an operator div in spherical coordinates, we have:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D_r^0\right) = 0 \text{ and } D_r^0 = \frac{C_1}{r^2},$$

C_1 is the constant to be determined.

4. SHORT DESCRIPTION OF THE SOLUTION THROUGH THE RECEPTION

According to works [1-4, 8], the part of the acoustic field representation in partial regions I, II leads to the scattered and penetrated wave as a result of a plane wave falling on the surface of the sphere. The full field must satisfy the boundary conditions on the surface of the sphere: the equality of pressures and normal constituents of the velocity of the particles on the surfaces of the transducer represented by equations (3.1)-(3.3). Note that the decomposition for pressures can be represented as an algebraic sum of centrally symmetric spherical harmonics, a set of zero-order tesseral harmonics (or an axisymmetric solution) and higher-order tesseral harmonics (or a non-asymmetric solution):

$$\begin{aligned} p_{II}(r,\varphi,\vartheta) &= \left[p_0^* \sqrt{\frac{\pi}{2kr}} J_{1/2}(kr) + A_0 \sqrt{\frac{\pi}{2kr}} H_{1/2}^{(2)}(kr) \right] + \\ &+ \left[p_0^* \sum_{n=1}^{\infty} (2n+1) j^n \sqrt{\frac{\pi}{2kr}} J_{n+1/2}(kr) P_n(\cos\vartheta_0) P_n(\cos\vartheta) + \right. \\ &\left. + \sqrt{\frac{\pi}{2kr}} \sum_{n=1}^{\infty} A_n H_{n+1/2}^{(2)}(kr) P_n(\cos\vartheta) \right] + \\ &+ \left[p_0^* \sum_{n=1}^{\infty}\sum_{m=1}^{n} (2n+1) j^n \frac{(n-m)!}{(n+m)!} \sqrt{\frac{\pi}{2kr}} J_{n+1/2}(kr) \times \right. \\ &\times P_n^m(\cos\vartheta_0) P_n^m(\cos\vartheta)\cos(m(\varphi-\varphi_0)) + \\ &\left. + \sqrt{\frac{\pi}{2kr}} \sum_{n=1}^{\infty}\sum_{m=1}^{n} A_{nm} H_{n+1/2}^{(2)}(kr) P_n^m(\cos\vartheta)\cos(m\varphi) \right], \end{aligned} \quad (4.1)$$

$$\begin{aligned} p_I(r,\varphi,\vartheta) &= \left[B_0 \sqrt{\frac{\pi}{2kr}} J_{1/2}(k_c r) \right] + \\ &+ \left[\sqrt{\frac{\pi}{2kr}} \sum_{n=1}^{\infty} B_n J_{n+1/2}(k_c r) P_n(\cos\vartheta) \right] + \\ &+ \left[\sqrt{\frac{\pi}{2kr}} \sum_{n=1}^{\infty}\sum_{m=1}^{n} B_{nm} J_{n+1/2}(k_c r) P_n^m(\cos\vartheta)\cos(m\varphi) \right], \end{aligned} \quad (4.2)$$

$A_0, B_0, A_{nm}, B_{nm}, \tilde{C}_{nm}$ are the unknown coefficients of expansions (4.1), (4.2); $J_{1/2}(kr)$, $H_{1/2}^{(2)}(kr)$ are the Bessel and Hankel functions of a non-integral character; $I_n(kr)$ is the spherical Bessel function of integer *-th* order, $n = 0, 1, 2, 3, \ldots$; $H_n^{(2)}(kr)$ is the second n-order Hankel spherical function; $P_n^m(\ldots)$ is the Legendre function of the first kind of integer degree and m, $n = 0, 1, 2, 3, \ldots$

Under the influence of the excess pressure, which is represented by condition (3.4), the shell is deformed, which corresponds to certain displacements of the material points of the shell and the appearance of the corresponding mechanical strains by the Hooke law:

$$\varepsilon_{\lambda\beta}(\varphi,\vartheta) = \varepsilon_{\lambda\beta}^0 + \sum_{n=1}^{\infty} \varepsilon_{\lambda\beta}^n P_n(\cos\vartheta) +$$
$$+ \sum_{n=1}^{\infty}\sum_{m=1}^{n} \varepsilon_{\lambda\beta}^{nm} P_n^m(\cos\vartheta)\cos(m\varphi),$$

which, by analogy with (4.1) and (4.2), is an algebraic sum of centrally symmetric, axisymmetric, and non-symmetric solutions of a problem.

Understanding this, we determine the equation of oscillations of the shell based on the general conditions of motion, which are given in [4, 8, 11, 13].

We use the expressions to calculate the tensor components of the resultant mechanical strains in the piezoceramics:

$$\begin{aligned}
\sigma_{rr} &= c_{11}^{E}\varepsilon_{rr} + c_{12}^{E}\varepsilon_{\varphi\varphi} + c_{12}^{E}\varepsilon_{\vartheta\vartheta} - e_{11}E_r \\
\sigma_{\varphi\varphi} &= c_{12}^{E}\varepsilon_{rr} + c_{22}^{E}\varepsilon_{\varphi\varphi} + c_{12}^{E}\varepsilon_{\vartheta\vartheta} - e_{12}E_r \\
\sigma_{r\vartheta} &= 2c_{55}^{\mathrm{E}}\varepsilon_{r\vartheta} - e_{26}E_{\vartheta}\,, \\
\sigma_{r\varphi} &= 2c_{55}^{\mathrm{E}}\varepsilon_{r\varphi} - e_{26}E_{\varphi}\,. \\
\sigma_{\varphi\vartheta} &= 2c_{44}^{E}\varepsilon_{\varphi\vartheta},
\end{aligned} \tag{4.3}$$

$c_{11}^{E};c_{12,}^{E}c_{22}^{E};c_{44}^{E};c_{55}^{E}$ are the components of the elastic modulus tensor (the radial axis of the sphere coincides with the OX_1 axis); $\sigma_{\varphi\varphi};\sigma_{\vartheta\vartheta};\sigma_{\varphi\vartheta};\sigma_{r\varphi};\sigma_{r\vartheta}$ are the components of the tensor of the resulting mechanical strains; $e_{11};e_{12};e_{26}$ are the piezomodules; $\varepsilon_{rr};\varepsilon_{\varphi\varphi};\varepsilon_{\vartheta\vartheta};\varepsilon_{r\varphi};\varepsilon_{r\vartheta};\varepsilon_{\varphi\vartheta}$ are the deformation tensor components; $E_r,E_{\varphi};E_{\vartheta}$ are the components of the electric field intensity vector in piezoceramics. We determine the equation of state for the components of the electric induction vector in the form:

$$D_m = e_{mij}\varepsilon_{ij} + \chi_{mn}^{\varepsilon}E_n\,, \tag{4.4}$$

D_m is the component vector of electric induction determined by the algebraic sum of "electric polarization" $D_m^d = e_{mij}\varepsilon_{ij}$ (caused by elastic mechanical deformations ε_{ij} that is, it is a dynamic component) and a "Coulomb" component $D_m^s = \chi_{mn}^{\varepsilon}E_n$, which is due to the pre-polarization of the ceramics (this is a static component); χ_{mn}^{ε} is the dielectric constant tensor components.

Further, based on [1, 8] and realizing the condition of satisfying the oscillations of the second Newton law in differential form, the equations of motion of the sphere with respect to mechanical strains σ_{kj} and displacements of the material particles of the shell $u_r,u_{\varphi},u_{\vartheta}$ will be:

$$\frac{1}{R_0}\left(2\sigma_{rr} - \sigma_{\varphi\varphi} - \sigma_{\vartheta\vartheta}\right) + \rho_M\omega^2 u_r = 0\,,$$

$$\frac{1}{R_0\sin\vartheta}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi} + \frac{1}{R_0}\frac{\partial\sigma_{\varphi\vartheta}}{\partial\vartheta} + \frac{2\sigma_{\varphi\vartheta}}{R_0}ctg\vartheta + \rho_M\omega^2 u_{\varphi} = 0\,,$$

$$\frac{1}{R_0\sin\vartheta}\frac{\partial\sigma_{\varphi\vartheta}}{\partial\varphi} + \frac{1}{R_0}\frac{\partial\sigma_{\vartheta\vartheta}}{\partial\vartheta} + \frac{1}{R_0}\left(\sigma_{\vartheta\vartheta} - \sigma_{\varphi\varphi}\right)ctg\vartheta + \rho_M\omega^2 u_{\vartheta} = 0\,,$$

$u_{\varphi},u_{\vartheta}$ are the angular components of the displacement vector of material particles of deformed spherical shell.

Based on the piezoelectric equations (4.3) and (4.4), which are supplemented by the components of vector $\mathbf{D}_m(D_r,D_{\varphi},D_{\vartheta})$ in the form:

$$\begin{aligned}
D_r &= e_{11}\varepsilon_{rr} + e_{12}\left(\varepsilon_{\varphi\varphi} + \varepsilon_{\vartheta\vartheta}\right) + \chi_{11}^{\varepsilon}E_r \\
D_{\varphi} &= 2e_{26}\varepsilon_{r\varphi} + \chi_{22}^{\varepsilon}E_{\varphi} \\
D_{\vartheta} &= 2e_{26}\varepsilon_{r\vartheta} + \chi_{22}^{\varepsilon}E_{\vartheta}
\end{aligned} \tag{4.5}$$

and considering the continuous electroding of the transducer surfaces, $D_{\varphi} = D_{\vartheta} = 0$ and the Cauchy ratio for displacements and deformations [1, 8, 13]:

$$\begin{aligned}
u_{\varphi} &= \frac{1}{\sin\vartheta}\frac{\partial u_r}{\partial\varphi}, \qquad u_{\vartheta} = \frac{\partial u_r}{\partial\vartheta} \\
\varepsilon_{\varphi\varphi} &= \frac{1}{R_0}\frac{1}{\sin\vartheta}\frac{\partial u_{\varphi}}{\partial\varphi} + \frac{u_{\vartheta}}{R_0}ctg\vartheta + \frac{u_r}{R_0} \\
\varepsilon_{\vartheta\vartheta} &= \frac{1}{R_0}\frac{\partial u_{\vartheta}}{\partial\vartheta} + \frac{u_r}{R_0}
\end{aligned} \tag{4.6}$$

After the series of mutual transformations (4.5)-(4.6) ([8, 9]) for the displacement component u_r we obtain in the simplified form:

$$\alpha_{\nu}u_r + \frac{c_{11}^{**}}{R_0}\Delta p - \frac{e_{11}^{**}}{R_0}E_r = 0\,,$$

where

$$\alpha_{\nu} = \frac{c_{12}^{**}}{R_0^2}\left(2 - \nu(\nu+1)\right) + \rho_M\omega^2\,. \tag{4.7}$$

Note that the component u_r can be represented by:

$$u_r = u_r(r,\varphi,\vartheta) = \sum_{\nu=0}^{\infty}\sum_{m=0}^{\nu}u_r^{nm}(r)P_{\nu}^{m}(\cos\vartheta)\cos(m\varphi) = $$
$$= \sum_{n=0}^{\infty}\sum_{m=0}^{n}u_r^{nm}f_n^m\left(\varphi,\vartheta\right),\; r = R_1$$

Let us determine the electric field. To find electrical characteristics: current in the external load circuit I, electrical induction D, and tension E_r, it is necessary to determine the centrally symmetric components of the physical fields under consideration, namely:

$$\sigma_{\vartheta\vartheta}^0, \sigma_{\varphi\varphi}^0, \varepsilon_{rr}^0, \varepsilon_{\varphi\varphi}^0, \varepsilon_{\vartheta\vartheta}^0, E_r^0, D_r^0, u_r^0, \Delta p^0\,.$$

So, after simplification of the relations for deformations and mechanical strains in accordance with the selected type of electroding of the surfaces of the transducer, the centrally symmetric components of induction and tension will be presented as:

$$\begin{aligned}
D_r^0 &= e_{11}\varepsilon_{rr}^0 + e_{12}\left(\varepsilon_{\varphi\varphi}^0 + \varepsilon_{\vartheta\vartheta}^0\right) + \chi_{11}^{\varepsilon}E_r^0 \\
E_r^0 &= \frac{C_1}{\chi_{11}^{*}r^2} + \frac{2e_{12}^{*}}{\chi_{11}^{*}R_0}u_r^0 - \frac{e_{11}}{\chi_{11}^{*}c_{11}^{E}}\Delta p^0
\end{aligned}$$

Application of boundary conditions of the electric field (item 2) for centrally symmetric solution of the form:

$$divD = divD_r = divD_r^0 = 0 \text{ and } E_r^0 = -\frac{\partial\Psi^0}{\partial r}\,, \tag{4.8}$$

Ψ^0 is the potential of a centrally symmetric electric field.

After integrating the both sides of equation (4.8) along the radial coordinate for the potential, we obtain:

$$\Psi^0 = \frac{C_1}{\chi_{11}^* r} + C_2 + \left(\frac{e_{11}}{\chi_{11}^* c_{11}^E} \Delta p^0 - \frac{2e_{12}^*}{\chi_{11}^* R_0} u_r^0 \right)(r - R_0) , \quad (4.9)$$

C_2 is the constant to be determined.

Considering the potential of the external electrode is zero, the potential difference "external electrode – internal electrode" will be:

$$\Psi^0\Big|_{r=R_0+h_0} = 0;$$

$$\Psi^0\Big|_{r=R_0} - \Psi^0\Big|_{r=R_1} = U_n = j\omega Z_n 4\pi r^2 D_r^0\Big|_{r=R_0} =$$

$$= j\omega Z_n 4\pi R_0^2 D_r^0 = j\omega Z_n 4\pi C_1 ;$$

where $U_n = I_n Z_n$ and $Z_n = R_n + jX_n$.

Thus, as a result of the general solution of the problem for the given boundary conditions and type of electroding, we have five unknown coefficients $u_0^R, A_0, B_0, C_1, C_2$, the finding of which requires five algebraic equations. Namely:

– two equations arising from the force and kinematic conditions of conjugation (3.1), (3.2) involving the expansions (4.1) and (4.2);

– two equations (4.3), which use (4.9) to find the coefficients C_1, C_2 for the known one Z_n.

The last fifth equation for finding the coefficient u_r^0 follows from (4.7) written for a centrally symmetric solution $\nu = 0$, which gives:

$$\alpha_0 u_r^0 + \frac{c_{11}^{**}}{R_0} \Delta p^0 - \frac{e_{11}^{**}}{R_0} E_r^0 = 0 ,$$

where $\alpha_0 = \dfrac{2c_{12}^{**}}{R_0^2} + \rho_M \omega^2$.

Thus, a system of five algebraic equations is obtained, the number of which corresponds to the number of unknown coefficients of expansions and equations for the electric field, which makes it possible to find them uniquely.

5. THE RESULTS OF THE CALCULATIONS

The calculations were carried out for a piezoceramic spherical transducer made of piezomaterial (speed of sound $c_m = 3400$ m/c, density $\rho_m = 7210$ kg/m^3, components of the elastic modulus tensor $c_{11}^E = c_{22}^E = 15.1 \cdot 10^{10}$ N/m^2, $c_{22}^E = 7.9 \cdot 10^{10}$ N/m^2, piezo modules $e_{11} = 17.7$ kg/m^2 and $e_{12} = -7.9$ kg/m^2) with a diameter of 12.5 mm and a wall thickness of 1 mm.

Air, helium and water were chosen as the fillers for the internal volume of the sphere.

5.1 Frequency Load Voltage Dependence for Vacuum Converter

Mathematically, the potential difference at the output of a spherical piezoceramic transducer of acoustic waves is determined by the relation:

$$U_{out} = f_e(\omega) \frac{2\alpha}{\chi_{33}^{\sigma}} \left\{ e_{31}^* \left[\varepsilon_{\vartheta\vartheta}^{(0,0)} + \varepsilon_{\phi\phi}^{(0,0)} \right] - \frac{e_{33}}{c_{33}^E} \Delta P^{(0,0)} \right\} ,$$

where $f_e(\omega) = i\omega C_0^{\sigma} Z_n \Big/ \left(1 + i\omega C_0^{\sigma} Z_n\right)$ is the function of turning of the piezoceramic acoustic wave receiver. Note that, electrically, the converter shows only zero mode due to the selected type of electrode.

Fig. 2 shows the results of the calculations of the amplitude-frequency characteristics (frequency response) of the difference of electric potentials at the output of a converter with classical construction (vacuum filling) for different electrical loads in the range from $Z_n = 100\ \Omega$ to $Z_n = 1$ MΩ.

In order to confirm the theoretical results in the laboratory conditions, the frequency response at the converter operation at a load of 100 kΩ was measured.

It can be seen that as the electrical load increases, local frequency extremal extremum in the low frequency region appears. As the impedance increases, the extremum becomes more gentle and sharp and its peak approaches the y-axis. The sensitivity in the field, however, in the low-frequency region, should obviously increase. A similar result is confirmed by [11-13].

In short-circuit mode, when $Z_n \to 0$, the function is on $f_e(\omega) = 0$ and off $U_{вых} = 0$. In idle mode, when $Z_n \to \infty$ at any low frequency, the function is on $f_e(\omega) = 0$. In this mode, the function is a Heaviside function. It follows that the piezoceramic acoustic wave receiver is not capable of detecting static pressure.

The output electrical signal of the investigated spherical receiving transducer with a fully electrode surface is determined by the centrally symmetric component of the stress-strain state of the piezoceramic shell.

The dashed curve is experimental. As can be seen from comparing the calculation materials with the experimental ones, there is a good coincidence of the results.

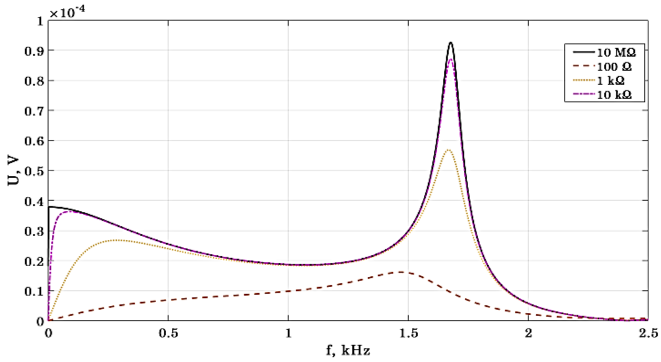
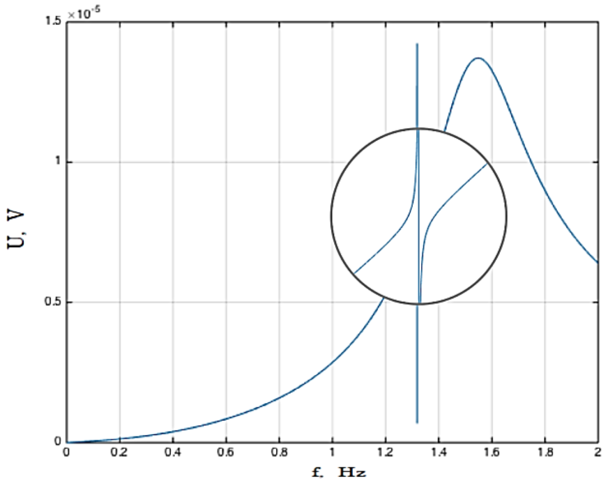


Fig 2 – Frequency response of electrical potentials at the receiver (range of electrical load from $Z_n = 100\ \Omega$ to $Z_n = 1$ MΩ

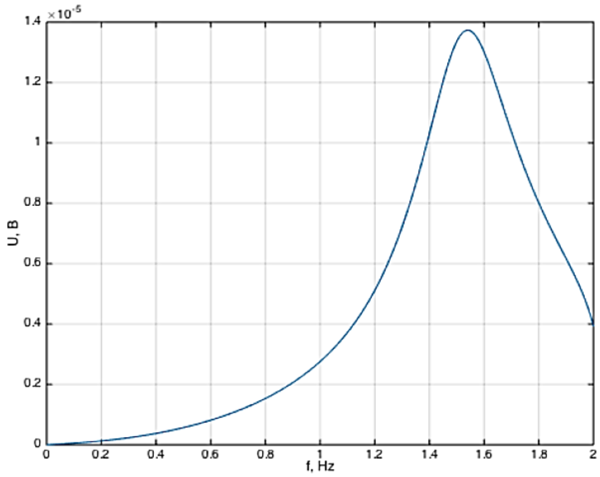
5.2 Frequency Response of the Converter with Helium Filling

The calculated frequency response of the electrical voltage (potential difference) at the output of the converter with internal helium filling for different loads in the range from $Z_n = 100\ \Omega$ to $Z_n = 1$ MΩ is shown in Fig. 3.

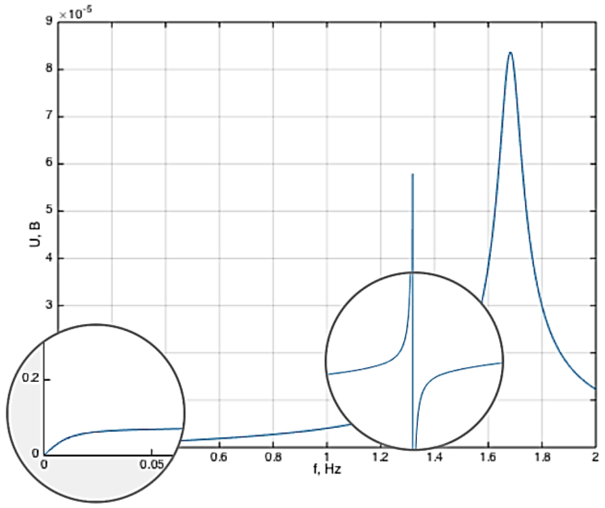
Calculation materials of Fig. 3 show that since helium is quite similar in characteristics to air, the frequency response is similar (blue curve in Fig. 2). The resonance region accompanied by antiresonance almost coincides in frequency, and the local extremum of the frequency response in the low-frequency region is as weak as in the case of filling the converter with air.



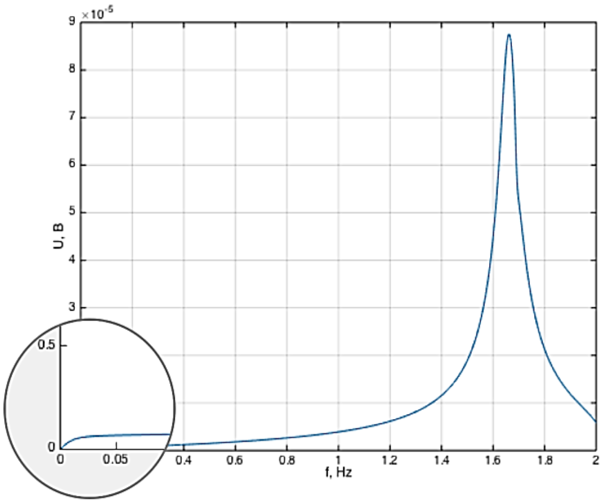
a



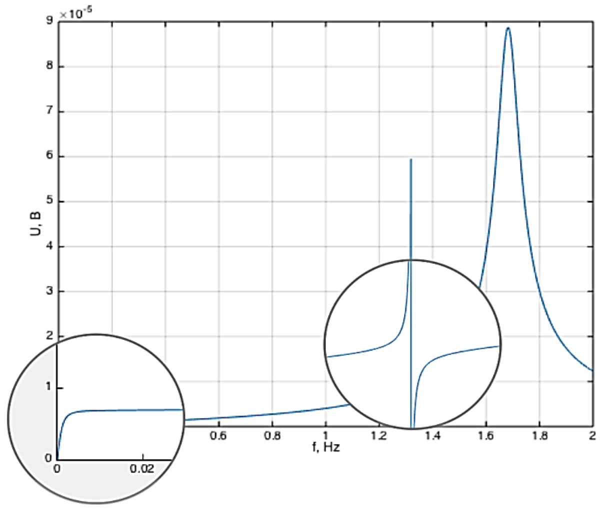
a



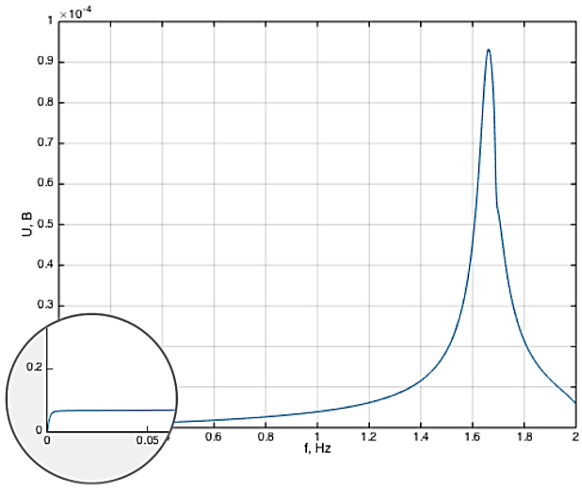
b



b



c



c

Fig 3 – Frequency response of electrical potentials at the receiver (range of electrical loads: a) Z_n = 100 Ω, b) Z_n = 10 kΩ, c) Z_n = 1 MΩ)

Fig. 4 – Frequency response of electrical potentials at the receiver (range of electrical loads: a) Z_n = 100 Ω, b) Z_n = 10 kΩ, c) Z_n = 1 MΩ)

5.3 Filling the Converter with Liquid

Fig. 4 presents the frequency response of the potential difference at the output of the converter with internal water filling for different electrical loads in the range from $Z_n = 100\ \Omega$ to $Z_n = 1\ \text{M}\Omega$. The results demonstrate the peculiarity of counteracting the filling of the environment. That is the water inside counteracts the deformation of the piezoelectric compensating the external (static) pressure. Here we see that the resonance is no longer accompanied by antiresonance, as in the case of air or helium filling.

Local extremum in the low-frequency range [0 Hz; 0.05 Hz] is less pronounced, and the absolute value of the sensitivity drops sharply.

6. CONCLUSIONS

As a result of the solution of the "through" problem of receiving sound by an elastic spherical converter with internal filling it is established that:

– the oscillatory system is characterized by the presence of a fundamental resonance of zero mode and an additional position whose position depends on the electrical load and the characteristics of the aggregate;

– additional resonance arises due to the resilience of the acoustic filler. Reducing elasticity (increasing inertia) leads to a significant decrease in the amplitude of the additional resonance, shifting it along the frequency domain up to the complete exclusion of this resonance;

– use of a filler fluid eliminates the elasticity of the filler as such in the specified operating conditions. The frequency response of the converter does not contain low-frequency resonances related to the inertial-elastic properties of the converter-filler system;

– the frequency response of the inverter output voltage indicates that, if any aggregate is used, the piezoceramic acoustic wave receiver is not capable of recording static pressure.

It is shown that the presence of a filler makes it difficult to coordinate the resistance of the converter with the input resistance of the receiving path and leads to a decrease in the width of its working strip.

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Динаміка електропружної сферичної оболонки з заповнювачем при прийомі звуку

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Отримані чисельні розрахунки амплітудно-частотних характеристик різниці електричних потенціалів на виході пружного сферичного перетворювача з внутрішнім наповненням. В якості внутрішнього завпонювача використано вакуум, гелій та воду. Математично робота зазначеної коливальної системи описується з використанням рівнянь стану п'єзокераміки, які лінійно співвідносять компоненти механічних напруг, деформацій, електричних напружень та індукції; рівнянь руху тонкої оболонки з рівняннями коефіцієнта Коші, що з'єднують компоненти тензора деформації та вектора зміщення; рівнянь вимушеної електростатики. Вихідний електричний сигнал досліджуваного сферичного приймального перетворювача з повністю електродною поверхнею визначається центрально симетричним компонентом напружено-деформованого стану п'езокерамічної оболонки. Встановлено, що коливальна система характеризується наявністю основного резонансу нульового режиму та додаткового положення, положення якого залежить від електричного навантаження та характеристик наповнювача. Показано, що наявність наповнювача ускладнює співставлення опору перетворювача з вхідним опором приймального тракту і призводить до зменшення ширини його робочої смуги. При використанні наповнювача у вигляді рідини результати досліджень демонструють особливість протидії наповненню середовища. Резонансна область, що супроводжується антирезонансною частотою, майже співпадає по частоті, а локальний екстремум частотної характеристики в низькочастотній області такий же слабкий, як у випадку заповнення перетворювача повітрям.

Ключові слова: П'єзокерамічний сферичний перетворювач, Заповнювач, Амплітудно-частотні характеристики, Електричне поле, Електропружні властивості, Чутливість, Прийом звуку, Різниця потенціалів.