Journal of Nano- and Electronic Physics
Vol. **12** No 4, 04002(6pp) (2020)
Журнал нано- та електронної фізики
Том **12** № 4, 04002(6cc) (2020)

Nonlinear Model of Ice Surface Softening during Friction Taking into Account Spatial Heterogeneity of Temperature

A.V. Khomenko, D.T. Logvinenko, Ya.V. Khyzhnya

Sumy State University, 2, Rymsky-Korsakov St., 40007 Sumy, Ukraine

(Received 18 June 2020; revised manuscript received 15 August 2020; published online 25 August 2020)

A nonlinear model of a viscoelastic medium is proposed, which describes softening of a thin layer of the ice surface during friction. The description of this transformation is based on the three following basic equations: the Kelvin-Voigt equation for a viscoelastic medium, the relaxation equations of Landau-Khalatnikov-type and for heat conductivity. It is revealed that mentioned equations coincide formally with the synergetic Lorenz system, where the order parameter is reduced to the shear strain, the stress acts as the conjugate field, and the temperature plays the role of the control parameter. The work further develops a nonlinear model of ice surface softening during friction, taking into account the spatial inhomogeneity of temperature in the equation of heat conductivity. In the framework of one-mode and adiabatic approximations an analytical soliton solution of a one-dimensional parabolic equation for the spatial normal distribution of shear strain to the ice surface is found. Due to the numerical solution of the one-dimensional Ginzburg-Landau differential equation, the distribution of friction force over the softened surface layer of ice is obtained and described. Two physical situations are considered: 1) the upper and lower surfaces move with equal velocities in opposite directions; 2) the upper surface is sheared along the fixed lower one. The coordinate dependencies of the friction force at different times are constructed and the evolution of the system to a stationary state is described. It is shown that the growth of time and background ice temperature leads to a sharper change of the friction force along the thickness of the premelted surface layer of ice, i.e. the relative shear velocity of the rubbing surfaces increases.

Keywords: Ice friction, Phase transition, Rheology, Plasticity, Friction force, Shear strain and stress.

DOI: 10.21272/jnep.12(4).04002 PACS numbers: 05.65. + b, 05.70.Ln, 46.55. + d, 62.20.F –, 64.60. – i, 81.40.Pq

1. INTRODUCTION

Ice friction is an important problem both from a practical point of view (for example, driving on ice) and in nature (in particular, the movement of glaciers), which includes the following processes: creep, destruction and melting. In the applied aspect, modeling ice friction is useful in the study of processes in the design of artificial ice environments, materials moving on ice, in particular, to create appropriate elements in mechanical engineering and winter sports [1-12].

The topic of ice medium formation from heterogeneous matter has recently gained great interest among scientists, for example, the model for creating lakes on ice shelves [1]. It describes the full cycle of creating a lake so that it is possible to study the effect of thawed lakes depending on density and temperature profiles on the ice shelf. Also, a model developed on real data taken from observations in the Barents Sea [2] describes the consolidation of ice rubble due to the penetration of low salinity water at freezing point inside the rubble when the water salinity decreases with time.

As part of the synergetic representation of boundary friction using a system of three differential equations for stresses, strains and temperature of the softened near-surface layer of ice, the nontrivial behavior of the ice layer is proposed [10-12]. This work is devoted to the further development of the model, and it studies the time and spatial evolution of the system taking into account the inhomogeneous temperature distribution along the thickness of the near-surface ice layer.

2. BASIC EQUATIONS AND MODEL

Consider a system of basic equations in the dimensionless form [10] taking into account the inhomogeneity of temperature:

$$\tau_\varepsilon \dot{\varepsilon} = -\varepsilon + \sigma\,, \tag{1}$$

$$\tau_\sigma \dot{\sigma} = -\sigma + g\left(T-1\right)\varepsilon\,, \tag{2}$$

$$\tau_T \dot{T} = \nabla^2 T + \left(\tau_T Q - T\right) - \sigma\varepsilon\,, \tag{3}$$

where ε is the shear component of the relative strains, σ is the shear component of the stresses, T is the ice surface temperature, $\tau_{\varepsilon\sigma T}$ are the relaxation times of the strain, stress and temperature, Q is the heat flow from the sliding block on the ice surface ($T_e = \tau_T Q$ is the background ice and block temperature). A constant $g < 1$ is introduced, which is equal to the ratio of the characteristic shear modulus of ice to its relaxed value.

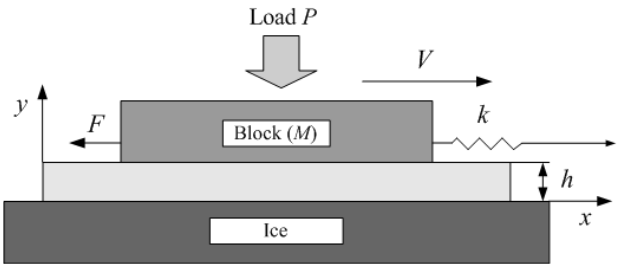

Fig. 1 – Mechanical analogue of the system

In our case, the system of equations (1)-(3) represents a nonlinear model of a viscoelastic medium, which describes softening of a thin layer of ice surface during friction [10-12]. The basic idea is that the softened surface layer of ice is a solid body with a high density of defects. The system of differential equations with partial derivatives (1)-(3) specifies the features of the behavior of an inhomogeneous system operating in the boundary friction mode, which is shown in Fig. 1.

Fig. 1 shows a system consisting of a block with mass M, located under a layer of softened ice with thickness h, and a spring with stiffness k. An additional load P is applied and the block moves with speed V. Friction force F acts on the block.

3. RESULTS

3.1 Analytical Solution

The selected equations (1)-(3) have no solution in general form, so to solve this problem we use the following adiabatic hierarchical approximation [10-13]:

$$\tau_\varepsilon >> \tau_T, \tau_\sigma . \tag{4}$$

This approach to solving the problem is due to the fact that in the process of evolution the stress $\sigma(t)$ and the ice surface temperature $T(t)$ follow the changes in the strain $\varepsilon(t)$. According to [10-12], the minimum deformation time can be estimated by $\tau_\varepsilon \sim 10^{-5}$ s that is determined by the reorientation time of water molecules at the freezing point of fresh water. The microscopic Debye time is equal to $\tau_\sigma \approx a/c \sim 10^{-12}$ s, where $a \sim 1$ nm is the lattice constant or intermolecular distance, and $c \sim 10^3$ m/s is the speed of sound.

In addition, within the one-mode approximation, for the operator ∇^2 the replacement is used, $\nabla^2 \to (l/L)^2$, where l is the length of thermal conductivity or the thickness of the ice film, L is the maximum value of the length of thermal conductivity. Then, equating the left parts of equations (2) and (3) to zero, we can express the stress σ and temperature T as a function of the strain ε:

$$\sigma = g\varepsilon\left(1 - \frac{2\left[(l/L)^2 - 1\right] + T_e}{(l/L)^2 - 1 - g\varepsilon^2}\right), \tag{5}$$

$$T = \frac{T_e - g\varepsilon^2}{(l/L)^2 - 1 - g\varepsilon^2} . \tag{6}$$

According to equations (5) and (6) at $(l/L)^2 = 0$, the stationary values of the stress σ and temperature T are as follows:

$$\sigma_0 = g\varepsilon_0\left(\frac{3 + g\varepsilon_0^2 - T_e}{1 + g\varepsilon_0^2}\right), \tag{7}$$

$$T_0 = \frac{g\varepsilon_0^2 - T_e}{1 + g\varepsilon_0^2} . \tag{8}$$

Substituting equation (5) into (1) provided that $\varepsilon^2 << 1 + (l/L)^2$, we obtain:

$$\tau_\varepsilon\dot{\varepsilon} = -\varepsilon\left[1 + g(3 - T_e)\right] + g^2\varepsilon^3(2 - T_e) + g(l/L)^2 \times\left(\varepsilon T_e - 2\varepsilon^3\right) + 2g\varepsilon(l/L)^4 . \tag{9}$$

In the following equation (10), the inverse transition is performed from $(l/L)^2$ to the operator ∇^2:

$$\tau_\varepsilon\dot{\varepsilon} = -\varepsilon\left[1 + g(3 - T_e)\right] + g^2\varepsilon^3(2 - T_e) + g \times\left(\varepsilon T_e - 2\varepsilon^3\right)\nabla^2 + 2g\varepsilon\nabla^4 . \tag{10}$$

Using approximations that $gT_e \approx 1$, as well as each derivative ∇ and ε add an order of smallness, so the powers of orders more than a third are neglected, then the following equation is obtained:

$$\tau_\varepsilon\dot{\varepsilon} - \nabla^2\varepsilon = -\varepsilon\left[1 + g(3 - T_e)\right] + g(2g - 1)\varepsilon^3 . \tag{11}$$

Thus, the system of equations (1)-(3) is reduced to the time-dependent Ginzburg-Landau equation:

$$\tau_\varepsilon\dot{\varepsilon} = -\frac{\delta V_1}{\delta\varepsilon(y)} , \tag{12}$$

$$V_1 = \int\left\{\left[1 + g(3 - T_e)\right]\frac{\varepsilon^2}{2} + g(1 - 2g)\frac{\varepsilon^4}{4} + \frac{1}{2}(\nabla\varepsilon)^2\right\}dy$$

that can be rewritten in the following form:

$$\tau_\varepsilon\dot{\varepsilon} = \nabla^2\varepsilon - \frac{\partial V}{\partial\varepsilon} . \tag{13}$$

Here the synergetic potential

$$V = \left[1 + g(3 - T_e)\right]\frac{\varepsilon^2}{2} + g(1 - 2g)\frac{\varepsilon^4}{4} \tag{14}$$

in which the condition $g < 0.5$ should be fulfilled.

If the temperature far away from the contact surfaces $T_e = \tau_T Q$ is less than the critical value

$$T_{c0} = 3 + g^{-1} , \tag{15}$$

then the potential (14) takes a minimum, which corresponds to the shear strain $\varepsilon_0 = 0$, so the softening cannot occur and ice stays in a crystalline state. In the opposite case $T_e > T_{c0}$ the ice is softened and the steady-state shear strain has a non-zero value, which we obtain by equating to zero the derivative with respect to ε from (14)

$$\varepsilon_0 = \left(\frac{gT_e - (3g + 1)}{(1 - 2g)g}\right)^{1/2} . \tag{17}$$

In the steady state $\dot{\varepsilon} = 0$ equation (13) has the first integral

$$\frac{1}{2}\left(\frac{d\varepsilon}{dy}\right)^2 = V + |V_0| . \tag{18}$$

Here it is taken into account that in the "ordered" phase (the softened ice state) $y = -\infty$ and under the conditions $\varepsilon = \varepsilon_0, \nabla\varepsilon = 0$ the equality of the integration constant to the absolute value of the synergetic potential V_0 at ordering is required

$$V_0 \equiv V(\varepsilon_0) \cong -\frac{\left(gT_e - (3g+1)\right)^2}{4g(1-2g)}. \tag{19}$$

The solution of equation (13) taking into account stationary conditions shows that the distribution of shear strain is represented by the kink

$$\varepsilon = \varepsilon_0 \tanh\left(\frac{y_0 - y}{\xi}\right), \tag{20}$$

$$\xi^2 \equiv \frac{2}{gT_e - (3g+1)}, \tag{21}$$

where ξ is the correlation length diverging at the critical value of the friction surfaces temperature (15). The integration constant $y_0 >> \xi$ determines the width of the boundary region in which the shear strain descents from the stationary value (17) to zero. Thus, within the framework of adiabatic and one-mode approximations, the analytical solution (20) of the one-dimensional parabolic equation (13) was found for the spatial distribution of shear strain that is normal to the ice surface (see Fig. 2).

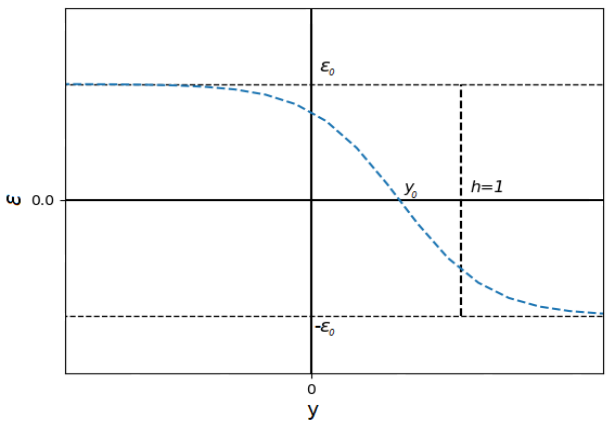

Fig. 2 – Coordinate dependence of the strains obtained as the analytical solution of (13)

3.2 Numerical Solution

In subsection 3.1 the system of nonlinear differential equations (1)-(3) within the framework of the one-mode and adiabatic approximations is reduced to the one-dimensional Ginzburg-Landau parabolic equation (13) where the synergetic potential is expressed by equation (14). In this subsection, we will solve the equation (13) numerically to study the evolution of the system.

To find the thermodynamic force $f(\varepsilon)$ the derivative of the synergetic potential (14) is taken with a negative sign

$$f(\varepsilon) = -\varepsilon\left[1 + g(3 - T_e)\right] - g(1-2g)\varepsilon^3. \tag{22}$$

Equation (13) has the form of the one-dimensional parabolic equation, which we will solve numerically using an explicit two-layer difference scheme, i.e. finite difference method [14-17]. In Fig. 3, a grid is shown that specifies the spatial partition for the numerical solution of equation (13).

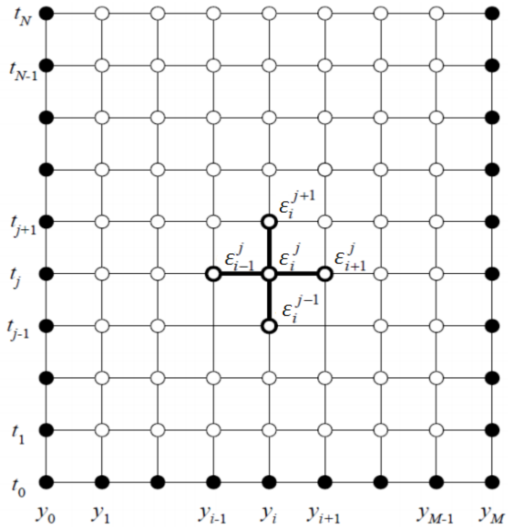

Fig. 3 – Grid that specifies the spatial partition for the numerical solution of equation (13) and shows the iterated nodes

The calculation will be performed on the y coordinate, which corresponds to the limits $[0; y_M]$, and in time from 0 to t_N. Also Fig. 3 depicts the points representing the necessary boundary and initial conditions, which will be determined based on the physical representation of the problem.

We approximate the derivatives for the explicit scheme, as shown in Fig. 3

$$\nabla^2\varepsilon \equiv \frac{\partial^2\varepsilon}{\partial y^2} = \frac{\varepsilon_{i+1}^j - 2\varepsilon_i^j + \varepsilon_{i-1}^j}{H^2}, \tag{23}$$

$$\frac{\partial\varepsilon}{\partial t} = \frac{\varepsilon_i^{j+1} - \varepsilon_i^j}{\Delta}, \tag{24}$$

where $H = y_M/M$ is the step in the coordinate, that is the distance between the nodes, M is the number of steps in the coordinate, $\Delta = t_N/N$ is the increment in time, N is the number of steps in time. The final relationship looks like this:

$$\varepsilon_i^{j+1} = \varepsilon_i^j + \frac{\Delta}{\tau_\varepsilon H^2}\left[\varepsilon_{i+1}^j - 2\varepsilon_i^j + \varepsilon_{i-1}^j + H^2 f\left(\varepsilon_i^j\right)\right], \tag{25}$$

where $f\left(\varepsilon_i^j\right)$ is a predetermined thermodynamic force (22).

Thus, the system (25) includes $N(M-1)$ equations and makes it possible to find ε_i^{j+1} with the known values in the previous time layer ε_i^j. To solve a certain problem, we fix a formal set of initial (at time $t = 0$) (26) and boundary (27) conditions

$$\varepsilon_i^0 = \phi_i, \; i = 0, 1, \ldots, M, \tag{26}$$

$$\varepsilon_0^j = \lambda_j, \quad \varepsilon_M^i = \eta_j, \; j = 1, 2, \ldots, N, \qquad (27)$$

which add $M+1$ and $2N$ equations, respectively. Together (25)-(27) represent $(M+1)(N+1)$ equations equal to the number of unknown grid nodes.

When the friction surfaces move, the stationary value ε_0 (17) is set, which we need to start the calculations. Consider the two most significant physical situations. The first is when the upper and lower surfaces move at equal speeds in opposite directions. That is, on the upper surface the shear strain is equal to ε_0, and on the opposite one $-\varepsilon_0$, since they define the speed. The strains are zero in the middle between the surfaces. Therefore, we set the following initial (at time $t=0$) and boundary conditions:

$$\varepsilon_i^0 = -\varepsilon_0 + \frac{2i\varepsilon_0}{M}, \; i = 0, 1, \ldots, M, \qquad (28)$$

$$\varepsilon_0^j = -\varepsilon_0, \quad \varepsilon_M^j = \varepsilon_0, \; j = 1, 2, \ldots, N. \qquad (29)$$

Consider the second situation when the upper surface is sheared at a fixed bottom. That is, on the upper surface the shear strain is equal to ε_0, and on the opposite one $\varepsilon = 0$. We obtain the following initial and boundary conditions, respectively:

$$\varepsilon_i^0 = \frac{i\varepsilon_0}{M}, \; i = 0, 1, \ldots, M, \qquad (30)$$

$$\varepsilon_0^j = 0, \quad \varepsilon_M^j = \varepsilon_0, \; j = 1, 2, \ldots, N. \qquad (31)$$

The coordinate y is measured in units of the thickness of the surface softened layer of ice h, its lower limit is 0, and the upper one is 1. For the solution we choose the steps in time $\Delta = 10^{-6}$ and coordinate $H = 0.01$, i.e. the y coordinate is divisible by $M = 100$ parts. Thus, a closed system of equations is obtained, which allows solving the problem.

To pass from shear strain ε to friction force we use the formula

$$F(t) = AG_\varepsilon \varepsilon(t), \qquad (32)$$

where A is the contact area and G_ε is the relaxed shear modulus of ice. We assume that the friction force is measured in units of AG_ε.

Fig. 4-Fig. 7 show the results of the solution of our chosen equations. Fig. 4 and Fig. 5 depict the results for the four time layers. The dashed line in both figures reflects the initial conditions (28) and (30), respectively, i.e. the initial friction force distribution on the zero time layer $t=0$. In Fig. 4, the time layers 500, 2500 and 50000 are displayed, which at the selected time step Δ give us the following moments of time: $t = 0.0005$, 0.0025, 0.05. The last curve does not change with the choice of a larger point in time, i.e. it corresponds to the stationary value of the distribution of friction force in the softened surface layer of ice.

In Fig. 5, the same time layers 500, 2500 and 50000 are shown, which are selected for Fig. 4, at the initial (30) and boundary (31) conditions. The last curve also does not change with the choice of a larger point in time, i.e. it corresponds to the steady-state value of the distribution of rubbing force in the softened surface layer of ice.

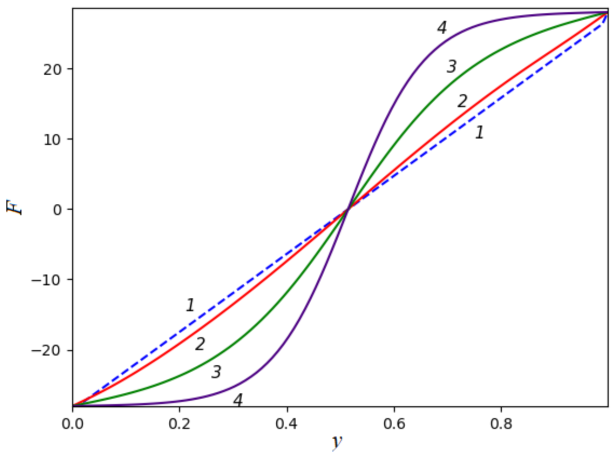

Fig. 4 – Coordinate dependence of the friction force obtained by the numerical solution at the parameters $T_e = 400$, $g = 0.25$, $\tau_\varepsilon = 0.3$, the initial (28) and boundary (29) conditions

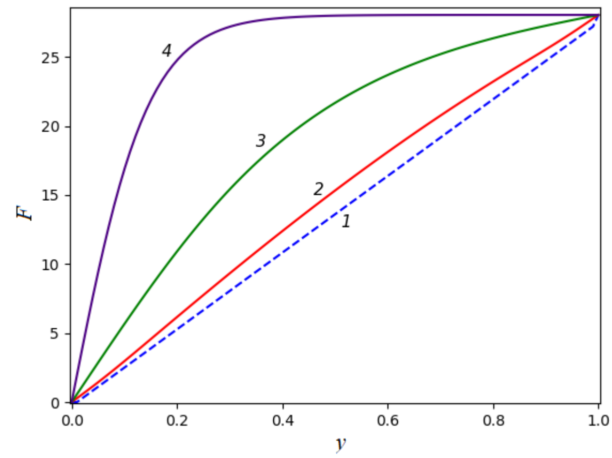

Fig. 5 – Coordinate dependence of the friction force obtained by the numerical solution at the parameters the same as in Fig. 4, the initial (30) and boundary (31) conditions

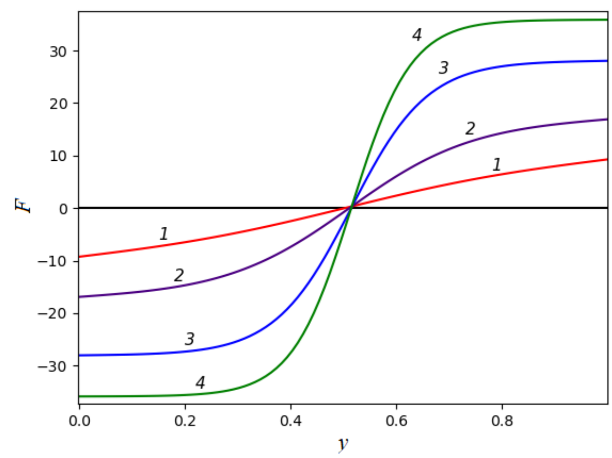

Fig. 6 – Coordinate dependence of stationary values of friction force at $T_e = 50$, 150, 400, 650, initial and boundary conditions of Fig. 4

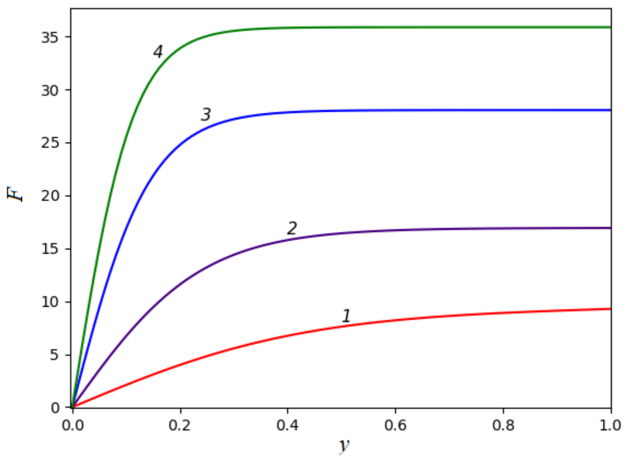

Fig. 7 – Coordinate dependence of stationary values of friction force at T_e = 50, 150, 400, 650, initial and boundary conditions of Fig. 5

Fig. 6 and Fig. 7 show the coordinate dependence of the friction force at different temperatures T_e and under different initial and boundary conditions. A time layer of 100000 is selected, which at a certain time step Δ gives us the corresponding time $t = 0.1$. The values of the constants used are the same as previously $g = 0.25$ and $\tau_\varepsilon = 0.3$.

It is seen that the increase in background ice temperature leads to a sharper change in the friction force along the thickness of the surface layer of ice, i.e. the relative shear velocity of the friction surfaces grows. This implies that the difference between the friction forces in the upper and lower coordinate layers increases a lot.

4. CONCLUSIONS

The analysis of spatial distributions of elastic and thermal fields is carried out over a surface film of ice softened at rubbing using analytical and numerical solutions. In the framework of adiabatic and one-mode approximations, an analytical solution of the one-dimensional parabolic equation for the spatial normal distribution of shear strain to the ice surface was found. It has kink soliton form. The dependence of temperature on the corresponding coordinate is determined.

Using an explicit two-layer difference scheme, a numerical study of a one-dimensional parabolic equation was performed. Distributions of current and stationary values of the friction force are constructed. Two cases are considered: 1) the upper and lower surfaces move with equal velocities in opposite directions; 2) the upper surface is sheared along the fixed lower one. The dependence of the stationary friction force on the coordinate along the axis perpendicular to the ice surface at different thermostat temperatures is analyzed. It is shown that the increase in time and temperature leads to a sharper change in the friction force along the thickness of the premelted surface layer of ice, i.e. the relative shear speed of the friction surfaces ascents.

ACKNOWLEDGEMENTS

This work was supported by the Ministry of Education and Science of Ukraine within the framework of the project “Atomistic and statistical representation of formation and friction of nanodimensional systems” (No 0118U003584).

REFERENCES

1. S.C. Buzzard, D.L. Feltham, D. Flocco, *J. Adv. Model. Earth Syst.* **10**, 262 (2018).
2. A.S. Shestov, A.V. Marchenko, *Cold Reg. Sci. Technol.* **122**, 71 (2016).
3. M.H.P. Ambaum, *Thermal Physics of the Atmosphere. Advancing Weather and Climate Science* (Chichester, UK: Wiley-Blackwell: 2010).
4. E. Ebert, J. Curry, *J.Geophys. Res.-Oceans* **98**(C6), 10085 (1993).
5. A.V. Khomenko, D.S. Troshchenko, L.S. Metlov, *Condens. Matter Phys.* **18** No 3, 33004 (2015).
6. L.S. Metlov, M.M. Myshlyaev, A.V. Khomenko, I.A. Lyashenko, *Tech. Phys. Lett.* **38** No 11, 972 (2012).
7. M. Lüthje, L. Pedersen, N. Reeh, W. Greuell, *J. Glaciol.* **52** (179), 608 (2006).
8. R. Arthern, D.G. Vaughan, A. Rankin, R. Mulvaney, E. Thomas, *J. Geophys. Res.-Earth* **115**, F03011 (2010).
9. S. Ligtenberg, M. Helsen, M. van den Broeke, *The Cryosphere* **5**, 809 (2011).
10. A.V. Khomenko, K.P. Khomenko, V.V. Falko, *Condens. Matter Phys.* **19** No 3, 33002 (2016).
11. A. Khomenko, *Tribol. Lett.* **66**, 82 (2018).
12. A. Khomenko, M. Khomenko, B.N.J. Persson, K. Khomenko, *Tribol. Lett.* **65**, 71 (2017).
13. O.I. Olemskoi, O.V. Yushchenko, T.I. Zhylenko, *Ukr. J. Phys.* **56**, 474 (2011).
14. I.A. Lyashenko, N.N. Manko, *J. Nano- Electron. Phys.* **5** No 3, 03040 (2013).
15. A. Pogrebnjak, A. Goncharov, *Metallofiz. Noveishie Tekhnol.* **38**, 1145 (2016).
16. O.Yu. Mazur, L.I. Stefanovich, *Phys. Solid State* **61**, 1420 (2019).
17. A.V. Khomenko, D.V. Boyko, M.V. Zakharov, K.P. Khomenko, Ya.V. Khyzhnya, *J. Nano- Electron. Phys.* **8** No1, 01028 (2016).

Нелінійна модель розм’якшення поверхні льоду при терті, що враховує просторову неоднорідність температури

О.В. Хоменко, Д.Т. Логвиненко, Я.В. Хижня

Сумський державний університет, вул. Римського-Корсакова, 2, 40007 Суми, Україна

Запропоновано нелінійну модель в’язкопружного середовища, яка представляє розм’якшення тонкого шару поверхні льоду при терті. Це перетворення описується на основі таких трьох основних рівнянь: рівняння Кельвіна-Фойгта для в'язкопружного середовища, релаксаційних рівнянь типу Ландау-

Халатнікова для зсувних напружень і теплопровідності. Показано, що дані рівняння формально збігаються з синергетичною системою Лоренца, де параметр порядку зводиться до деформації зсуву, напруження є спряженим полем, і температура відграє роль керувального параметра. В роботі здійснено подальший розвиток нелінійної моделі розм'якшення тонкого шару поверхні льоду при терті з урахуванням просторової неоднорідності температури в рівнянні теплопровідності. В рамках адіабатичного та одномодового наближень знайдено аналітичний солітонний розв'язок одновимірного параболічного рівняння для просторового нормального до поверхні льоду розподілу зсувної деформації. Завдяки числовому розв'язку одновимірного диференціального рівняння Гінзбурга-Ландау, отриманий та описаний розподіл сили тертя по розм'якшеному приповерхневому шару льоду. Розглядаються дві фізичні ситуації: 1) верхня і нижня поверхні рухаються з рівними за величиною швидкостями у протилежних напрямках; 2) верхня поверхня зсувається по нерухомій нижній. Побудовані координатні залежності сил тертя у різний час та показано еволюцію системи до стаціонарного стану. Показано, що зростання часу та температури термостату приводить до більш різкої зміни сили тертя по товщині розм'якшеного приповерхневого шару льоду, тобто збільшується відносна швидкість зсуву тертьових поверхонь.

Ключові слова: Тертя льоду, Фазовий перехід, Реологія, Пластичність, Сила тертя, Зсувні деформація та напруження.