# Design and Analysis of Waveguide Structure Sensor Based on Nanoparticles and Left-handed Material

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The use of left-handed materials in scientific applications has attracted attention of many researchers in the last few years. Nanoparticles are also new materials with small size in scale of nanometer which are rapidly attracting more interest and consideration especially in nanotechnology application. The main advantages of using both left-handed materials and nanoparticles in the proposed sensors are the possibility of a significant reduction in the structure size and the enhancement of the sensitivity sensor.

In this paper, we have investigated a three-layer planar waveguide sensor consisting of thin nanoparticles core based on left-handed material substrate and water cover which are employed for sensing applications. The interaction of TM and TE surface waves with the proposed planar waveguide structure will be studied for detection of the penetration depth, any changes in the refractive index of the analyte and the related sensitivity enhancement effect are analyzed. It is observed that the sensitivity of the proposed sensors is improved compared to that of the conventional three-layer waveguide sensors. High sensitivity has been observed which could be utilized for future sensor applications.

Keywords: Nanoparticles, Dispersion relation, Sensitivity, Left-handed materials.

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#### 1. INTRODUCTION

In the past decade, there is a growth of interest in optical sensors, especially their application in different fields, such as biochemical, biomedicines, environmental control, and other industries [1-6]. The principle of sensitivity is based on the evanescent tail of the electromagnetic field in the analyte medium, in which the propagation constant depends on the structure parameters such as thickness and optical properties of the waveguide structure [7]. However, the waveguide structure platforms are employing evanescent wave sensing techniques. So the theoretical and experimental designer must introduce the waveguide structure where there are appropriate materials of the film, substrate, and surrounding in order to achieve high sensitivity with fast response time, system compactness and low cost. Sensitivity of planar waveguide is to measure the effective index change due to change of the refractive index of both cladding and substrate layers [7]. Most sensing principles are described and classified into several groups: absorbance-based sensors, interferometric sensors, fluorescencebased sensors, fiber grating sensors, and resonance-based sensors [8].

Various optical waveguide sensors containing different artificial materials, such as nanoparticles [9, 10], graphene [10-13] or left-handed material (LHM) [14, 15] have been studied theoretically in various configuration structures. In the last few years, LHM has received more interest in optical waveguide sensors [6, 16]. These materials with negative permeability and permittivity have unique optical properties such as negative refraction, unusual nonlinearities, reversed Doppler effect and reversed Ceenekov radiation. Many novel applications are proposed based on metamaterials, such as cloak, concentrators, super absorbers, transparent devices directive antenna, and so on. In recent years, optical waveguides using LHM layer can be used as a sensor. The sharp resonant peaks in dispersion due to the LHM may enhance the evanescent wave, and the sensitivity of sensor will be increased [17, 18]. Nanoparticle materials have unique optical and chemical properties which make them suitable for designing new and improved sensing devices, especially electrochemical sensors and biosensors [8, 9]. This material exhibits higher extinction coefficient, sharper extinction band and higher ratio of scattering that can enhance the evanescent field in planar waveguide sensors. All previous studies on sensor structures had been investigated based on nanoparticles with a conventional material through a surface plasmon resonance (SPR) concept. In this paper, we discuss theoretically the propagation characteristics of TE and TM surface waves in a slab waveguide containing a nanoparticle and LHM. The sensitivity has been studied with the parameters of the proposed waveguide structure. The penetration depth in both cladding and substrate is also presented.

## 2. BASIC EQUATION

Fig. 1 shows the proposed schematic diagram of the three-layer waveguide sensor. The guiding film is nanoparticle material of thickness d and the effective refractive index  $n_2$  of nanoparticle material can be estimated as [10, 11]

$$n_{2} = \sqrt{\varepsilon_{2}} = \left[\phi_{p} n_{p}^{2} + (1 - \phi_{p}) n_{1}^{2}\right]^{\frac{1}{2}}, \qquad (1)$$

where  $\phi_p$  is the volume fraction of the particles in the nanoparticle material, while  $n_p$  and  $n_1$  are the refractive indexes of the particles and the water.

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Fig.  $1-\operatorname{Schematic}$  view of waveguide

The components of the electric and magnetic fields can easily be written for both TE and TM modes as:

$$\begin{split} \vec{H} &= \begin{bmatrix} 0, H_{y}(z), 0 \end{bmatrix} e^{i(kx - \omega t)}, \\ \vec{E} &= \begin{bmatrix} E_{x}(z), 0, E_{z}(z) \end{bmatrix} e^{i(kx - \omega t)} \text{ (TM surface wave), (2)} \\ \vec{E} &= \begin{bmatrix} 0, E_{y}(z), 0 \end{bmatrix} e^{i(kx - \omega t)}, \end{split}$$

$$\vec{H} = \left[H_x(z), 0, H_z(z)\right]e^{i(kx-\omega t)}$$
 (TE surface wave), (3)

where  $\omega$  is the angular frequency and k is the propagation wave number.

Using the basic Maxwell equation, one can get the Helmholtz equation for TM and TE surface waves as:

$$\frac{\partial^{2} H_{y}}{\partial z^{2}} - k_{o}^{2} \left( N^{2} - \varepsilon_{j} \right) H_{y} = 0 \quad \text{(TM surface wave)} \tag{4}$$

$$\frac{\partial^2 E_y}{\partial z^2} - k_o^2 \left( N^2 - \varepsilon_j \right) E_y = 0 \quad \text{(TE surface wave)} \tag{5}$$

where  $N = k/k_o$  is the effective wave number, j = 1, 2, 3 and  $k_o$  is the wave number in vacuum.

At first, we discuss the case of TM surface waves, the components of the magnetic field in three regions can be written as:

and of the electric field as:

$$E_{x1} = \frac{-k_1}{ik_o\varepsilon_1} A e^{-k_1} z ,$$

$$E_{x2} = \frac{k_1}{ik_o\varepsilon_2} \Big[ B\cosh(k_2 z) + C\sinh(k_2 z) \Big], \qquad (7)$$

$$E_{x2} = \frac{k_3}{i\varepsilon_3} D e^{k_1} z ,$$

where  $k_j = \sqrt{k^2 - k_o^2 \varepsilon_j \mu_j}$  is the transverse wave number in three layers j = 1, 2 and 3.

By applying electromagnetic boundary conditions at

the interfaces y = 0 and y = d, the dispersion relation of the TM surface waves is derived by

$$tanh(k_2d) = -\left[\frac{k_2\varepsilon_2(k_3\varepsilon_1 - k_1\varepsilon_3)}{k_1k_3\varepsilon_2^2 + k_2^2\varepsilon_1\varepsilon_3}\right].$$
(8)

In the same way, the dispersion relation of the TE surface waves cab be written as

$$tanh(k_2d) = -\left[\frac{k_2(k_3 + k_1\mu_3)}{k_1k_3 + k_2^2\mu_3}\right].$$
 (9)

The sensitivity of the waveguide sensor is defined as the rate of change of the effective refractive index with respect to the change of the cladding refractive index  $n_1$  [7, 8], i.e.

$$s = \frac{\partial N}{\partial n_1}$$
.

By carrying out the differentiation of the dispersion relations (7), (8) with respect to N, one can get:

$$S_1 = \frac{U_1 - U_2}{g_1 + g_2 + g_3}$$
 (TM surface wave) (10)

 $\operatorname{and}$ 

$$S_2 = \frac{G_1 - G_2}{m_1 + m_2 - m_3}$$
 (TE surface wave), (11)

where

$$\begin{split} U_{1} &= \left[k_{2}\varepsilon_{2}\left(k_{3}\varepsilon_{1}+k_{1}\varepsilon_{3}\right)\right] \left[2k_{2}^{2}n_{1}\varepsilon_{3}-\frac{n_{1}k_{3}\varepsilon_{2}^{2}}{k_{1}}\right],\\ U_{2} &= \left[k_{2}\varepsilon_{2}\left(2k_{3}n_{1}+\frac{n_{1}\varepsilon_{3}}{k_{1}}\right)\right] \left[k_{1}k_{3}\varepsilon_{2}^{2}-k_{2}^{2}\varepsilon_{1}\varepsilon_{2}\right],\\ g_{1} &= \left[k_{1}k_{3}\varepsilon_{2}^{2}+k_{2}^{2}\varepsilon_{1}\varepsilon_{3}\right] \left[\frac{N\varepsilon_{2}}{k_{2}}\left(k_{3}\varepsilon_{1}+k_{1}\varepsilon_{3}\right)+\frac{k_{2}N\varepsilon_{1}\varepsilon_{2}}{k_{3}}\right]\\ &+\frac{k_{2}N\varepsilon_{2}\varepsilon_{3}}{k_{1}}\right]\\ g_{2} &= \left[k_{1}k_{3}\varepsilon_{2}^{2}+k_{2}^{2}\varepsilon_{1}\varepsilon_{3}\right]^{2}\left(\frac{k_{o}Nd}{k_{2}}\right)sech^{2}\left(k_{o}k_{2}d\right),\\ g_{3} &= \left[k_{2}\varepsilon_{2}N\left(k_{3}\varepsilon_{1}+k_{1}\varepsilon_{3}\right)\right] \left[\frac{k_{3}\varepsilon_{2}^{2}}{k_{1}}+\frac{k_{1}\varepsilon_{2}^{2}}{k_{3}}+2\varepsilon_{1}\varepsilon_{3}\right],\\ G_{1} &= \left[k_{1}k_{3}+k_{2}^{2}\mu_{3}\right]\left(k_{2}\mu_{3}n_{1}\right),\\ G_{2} &= \left[k_{3}+k_{1}\mu_{3}\right]\left(k_{2}k_{3}n_{1}\right),\\ m_{1} &= \left[k_{1}k_{3}+k_{2}^{2}\mu_{3}\right]^{2}\left(\frac{k_{o}k_{1}Nd}{k_{2}}\right)sech^{2}\left(k_{o}k_{2}d\right),\\ \end{split}$$

$$m_2 = \frac{Nk_1}{k_2} \left[ k_1 k_3 + k_2^2 \mu_3 \right] \left[ k_3 + k_1 \mu_3 + \frac{k_2^2}{k_3} + \frac{k_2 \mu_3}{k_1} \right]$$

and

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$$m_{3} = \left[k_{1}k_{2}N\left(k_{3}+k_{1}\mu_{3}\right)\right]\left\lfloor\frac{k_{3}}{k_{1}}+\frac{k_{1}}{k_{3}}+2\mu_{3}\right\rfloor$$

#### 3. RESULTS AND DISCUSSION

In our study, we will discuss the effect of nanoparticle and LHM materials on the sensitivity of both TM and TE surface waves in our proposed waveguide structure sensor. Firstly, the dispersion relations (8), (9) will be solved by using Maple software to calculate the effective wave index N and plug into equations (10), (11) to figure out the sensitivity.

The penetration depth 
$$\left(rac{1}{2\,Im(N)}
ight)$$
 of the structure

has also been considered to measure how deep electromagnetic radiation can penetrate cladding of the waveguide sensor. In our analysis, we used the data parameters of nanoparticles  $_{np} = 1.46$ ,  $n_1 = 1.33$  for water and  $\mu_3 = -2$ ,  $\varepsilon_3 = -2$  of LHM.



**Fig. 2a** – The penetration depth  $\delta$  versus frequency for different values of  $\phi_p$ : 0.5 (dash-dotted), 0.56 (dotted), and 0.6 (solid) lines, d = 500 nm and  $\varepsilon_1 = 1.769$ 



**Fig. 2b** – The penetration depth  $\delta$  versus frequency for different values of  $\phi_p$ : 0.9 (dash-dotted), 0.95 (dotted), 1 (solid) lines, d = 40 nm and  $\varepsilon_1 = 1.769$ 

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In Fig. 2, the penetration depth  $\delta$  has been plotted upon changing the frequency for different values of  $\varphi_p$ . Fig. 2a (TM surface waves) illustrates that the penetration increases with increasing frequency, it means that the confinement gets lower with increasing frequency and more power penetration into the cladding layer and so the sensitivity would increase. These results are in contrast with the three layer without nanoparticle medium [13]. It is also noticed that, the penetration depth is almost linearly dependent on frequency. It has been seen that the penetration depth is independent on  $\phi_p$  up to frequency  $6 \times 10^{12}$  Hz, and then slowly dependent on  $\phi_p$  where the penetration depth increases with decreasing  $\phi_p$ . In TE surface waves, Fig. 2b, the penetration depth is more strongly dependent on the filling factor  $\phi_p$ , and also increases with increasing frequency. It can be seen that the penetration depth decreases gradually with increasing frequency until  $9.2 \times 10^{14}$  Hz which has the lowest value of  $\delta$  and then  $\delta$  increases with frequency. The penetration depth  $\delta$  with the frequency for different value of the guiding width is given in Fig. 3. From



**Fig. 3a** – The penetration depth  $\delta$  versus frequency for different values of film thickness *d*: 4 nm (dashdotted), 5 nm (dotted), 6 nm (solid) lines,  $\phi_p = 0.56$ and  $\varepsilon_1 = 1.769$ 



**Fig. 3b** – The penetration depth  $\delta$  versus frequency for different values of film thickness *d*: 35 nm (dashdotted), 40 nm (dotted), 45 nm (solid) lines,  $\phi_p = 0.95$ and  $\varepsilon_1 = 1.769$ 

Fig. 3a, it is reasonable that the penetration increases with frequency and gets higher penetration with lower thickness. This means that more power is now flowing in the cladding medium and the sensitivity would increase. Also, at lower frequency, the penetration is less dependent on thickness.

In Fig. 3b we got the same result as in Fig. 2b, however, it is noticed that the lowest value of the penetration depth can be shifted with the change of the frequency. It means that the penetration depth  $\delta$  is independent on thickness.

Fig. 4 investigates the dependence of the sensitivity on the frequency for different value of  $\phi_p$ . As shown in Fig. 4a, the sensitivity is more strongly dependent on lower frequency, i.e. the scattering is more rapidly increasing with higher wavelength, and so the evanescent waves are dramatically amplified and the sensitivity would be enhanced.



**Fig. 4a** – The sensitivity *S* versus frequency for different values of  $\phi_p$ : 0.5 (dash-dotted), 0.56 (dotted), 0.6 (solid) lines, d = 500 nm and  $\varepsilon_1 = 1.769$ 



**Fig. 4b** – The sensitivity *S* versus the frequency for different values of  $\phi_p$ : 0.5 (dash-dotted), 0.6 (dotted), 0.9 (solid) lines, d = 40 nm and  $\varepsilon_1 = 1.769$ 

It can also be seen that the sensitivity is highly dependent on the nanoparticle parameter and the sensitivity increases with decreasing filling factor  $\phi_p$  at lower frequency. In TE surface waves, Fig. 4b shows that sensitivity rapidly decreases with frequency and attains its minimum and then rapidly increases. Also this minimum is shifting to the region of higher frequencies by increasing  $\phi_p$ . This differs from TM mode, which remains constant at higher frequencies. It was also noticed that the sensitivity increases with the increase of the filling factor which differs from the results in TM surface waves.

The variation curves of sensitivity S with the different values of thickness d are shown in Fig. 5. In Fig. 5a, the sensitivity goes up with increasing nanoparticle thickness. This is also a unique property of nanoparticle material as well as in metamaterials [19].



**Fig. 5a** – The sensitivity S versus  $\phi_p$  for different frequency values:  $2 \cdot 10^{12}$  Hz (dash-dotted),  $4 \cdot 10^{12}$  Hz) (dotted), and  $6 \cdot 10^{12}$  Hz (solid) lines, d = 500 nm and  $\varepsilon_1 = 1.769$ 



**Fig. 5b** – The sensitivity *S* versus  $\phi_p$  for different frequency values:  $14 \cdot 10^{14}$  Hz (dash-dotted),  $15 \cdot 10^{14}$  Hz (dotted), and  $17 \cdot 10^{14}$  Hz (solid) lines, d = 40 nm and  $\varepsilon_1 = 1.769$ 

For TE mode, as shown in Fig. 5b, the sensitivity is no longer dependent on the thickness as in TM surface waves. So, we can fabricate sensors in any waveguide structure with the same sensitivity. Finally, Fig. 4b and Fig. 5b show the bistability behavior, i.e. for each filling factor  $\phi_p$  and film thickness of the proposed sensor as one value of the sensitivity is shown there are two values of the operating frequencies. DESIGN AND ANALYSIS OF WAVEGUIDE STRUCTURE SENSOR ...

In general, we have noticed that the nanoparticle material with metamaterials can amplify evanescent waves and the sensitivity of sensors with TM mode can be dramatically enhanced compared with the conventional three-layer TM wave waveguide sensor without nanoparticle material and metamaterials. But in the case of TE mode, we found that the proposed structure behaves as the conventional three-layer TM wave waveguide sensor [18].

## 4. CONCLUSIONS

It has been studied both TE and TM surface waves in in a three-layer waveguide optical sensor composed of core nanoparticles based on LHM substrate and water cover. The dispersion relation was solved to compute the penetration depth, and the sensitivity of the effective refractive index to any change in the analyte refractive index was investigated. In TE surface waves, it has been noticed that the penetration depth is more strongly dependent on the operating frequency and the filling factor than TM surface waves. We also noticed that, at lower frequency, the penetration depth is less dependent on

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guiding thickness. In general, nanoparticle material can be enhanced compared with the three-layer waveguide sensor without nanoparticle material. For TM surface, it has been noticed that the sensitivity is more strongly dependent on lower frequency and lower filling factor, but in TE surface waves it is different. It is also seen that the sensitivity is more strongly dependent on the nanoparticle thickness as in TM, but in TE it is no longer the thickness dependent.

Generally speaking, it is observed that nanoparticles and LHM increase the designing flexibility of sensors and improve their performance. Sensors using nanoparticles and LHM could lead to new revolution of sensing technology.

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### Проектування та аналіз хвилеводного датчика структури на основі наночастинок і лівогвинтового матеріалу

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Використання лівогвинтових матеріалів у наукових застосуваннях привернуло увагу багатьох дослідників за останні кілька років. Наночастинки – це також нові матеріали з невеликими розмірами в нанометровому масштабі, які швидко привертають до себе все більший інтерес і увагу, особливо в галузі нанотехнологій. Основними перевагами використання як лівогвинтових матеріалів, так і наночастинок у запропонованих датчиках є можливість значного зменшення розмірів структури та покращення датчика чутливості. У роботі ми дослідили тришаровий планарний хвилеводний датчик, що складається з ядра з тонких наночастинок на основі лівосторонніх матеріалів підкладки та водяного покриву, які використовуються для зондування. Взаємодія поверхневих хвиль TM і TE з запропоновановано планарною хвилеводною структурою буде вивчатися для виявлення глибини проникнення; будь-які зміни показника заломлення аналіту та пов'язаного з цим ефекту підвищення чутливості будуть проаналізовані. Спостерігається, що чутливість запропонованих датчиків покращується у порівнянні із звичайними тришаровими хвилеводними датчиками. Помічено високу чутливість, яка може бути використана для майбутніх застосувань датчиків.

Ключові слова: Наночастинки, Дисперсійне відношення, Чутливість, Лівогвинтові матеріали.