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A Method of Minority Charge Carriers Basic Parameters Determination in Solid Matter

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The new method of minority charge carriers basic parameters determination and the ratio of minority charge carriers conductivity to majority ones in solid matter based on magnetoresistance curve analyses within the framework of the phenomenological two-band model has been proposed. The criterion of the applying of the method has been found. As the examples of using this method the conductor, semiconductor and superconductor have been given. From the obtained temperature dependences of the aforementioned values in superconductor the conclusion about the deciding role of minority charge carriers in the emergence of superconductivity state has been made.

Keywords: Organic conductor, Semiconductor, Magnetoresistance, Hall-effect.

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1. INTRODUCTION

Up to now the researchers found the magnetoresistance measurements to be not informative concerning charge carriers basic parameters in solid matter. Among the galvanomagnetic effects just Hall-effect is presented as a tool for determining these parameters. However, we are going to show that magnetoresistance curve conceals in itself exact information about minority charge carriers mobility and concentration as well as their conductivity in relation to the conductivity of majority ones.

The aim of this paper is to develop the magnetoresistance method to determine basic parameters of the minority charge carriers in solid matter [1] and apply it to a different solids.

Then we will draw conclusions, which follow from the method.

2. CALCULATION

To achieve this aim we must consider the phenomenological model of galvanomagnetic phenomena for isotropic material with two types of charge carriers (having opposite or the same sign). It is known [2] that the transverse conductivity σ for this case depends upon the magnetic field induction in the following way:

$$\sigma = \frac{(\sigma_1+\sigma_2)^2 + {\sigma_1}^2{\sigma_2}^2(R_{H1}+R_{H2})^2B^2}{(\sigma_1+\sigma_2)+\sigma_1\sigma_2(\sigma_1{R_{H1}}^2+\sigma_2{R_{H2}}^2)B^2}.$$

Substitute here $\sigma_1 = en\mu_n$, $\sigma_2 = ep\mu_p$, $R_{H1} = -1/(en)$, $R_{H2} = 1/(ep)$, where e is electron charge, n, p, μ_n, μ_p are concentrations and mobilities of two types charge carriers (it can be electrons and holes or light and heavy holes). Note also that $\sigma = \rho/(\rho^2+{\rho_{xy}}^2)$ where ρ being transverse and ρ_{xy} Hall resistivity. Then, considering that $\rho_{xy} << \rho$ we have

$$\rho = \frac{1}{\sigma} = \frac{1}{e}\frac{n\mu_n + p\mu_p + \mu_n\mu_p(n\mu_p+p\mu_n)B^2}{(n\mu_n+p\mu_p)^2+{\mu_n}^2{\mu_p}^2(n-p)^2B^2}, \quad (1)$$

If we state now the condition $d^2\rho/dB^2 = 0$ we obtain the magnetic field position of ρ flex point B_f

$$B_f = \frac{1+ab}{\sqrt{3}(1-a)\mu_n}, \quad (2)$$

where $a = n/p$, $b = \mu_n/\mu_p$.

Now the equation (1) can be introduced by the form:

$$\rho = \rho_0\frac{1+\dfrac{\mu_n(a+b)}{\sqrt{3}B_f(1-a)b}B^2}{1+\dfrac{1}{3{B_f}^2}B^2},$$

where ρ_0 is resistivity at $B = 0$.

Let us write the condition for electrons as minority charge carriers. It is $a << 1$. Since, the minority charge carriers mobility is, as the rule, higher or compared with the majority ones, that is $b \geq 1$, then $a << b$ and the magnetic field dependence of ρ gets the form:

$$\rho = \rho_0\frac{1+\dfrac{\mu_n}{\sqrt{3}B_f}B^2}{1+\dfrac{1}{3{B_f}^2}B^2}. \quad (3)$$

For $B = B_f$ writing for convenience $\rho(B_f) = \rho_f$ we obtain:

$$\mu_n = \frac{\sqrt{3}}{B_f}\left(\frac{4}{3}\frac{\rho_f}{\rho_0}-1\right), \quad (4)$$

Thus, as can be seen from the last formula if the condition $n << p$ is fulfilled, the measuring of the transverse magnetoresistance provides the information about the minority charge carriers mobility. Is enough to find the magnetoresistance flex point B_f and to measure resistivity in this point ρ_f and in zero field ρ_0.

From equation (2) for $a<<1$, taking into account that $ab=\sigma_n/\sigma_p$, we get another useful formula

$$\frac{\sigma_n}{\sigma_p}=4\left(\frac{\rho_f}{\rho_0}-1\right), \quad (5)$$

Since in formula (4) and (7} the resistivities appeared in ratios, in practice we can substitute ρ_f/ρ_0 by corresponding ratio of potential differences U_f/U_0.

In the strong field limit, taking into account that $a<<1$ and $a<<b$, equation (1) results after the simple treatment in the formula for saturation resistivity:

$$\rho_\infty=\lim_{B\to\infty}\rho=\frac{1}{ep\mu_p}=\frac{1}{\sigma_p}. \quad (6)$$

From this equation, eq. (4) and (5) and taking into account that $\sigma_n=en\mu_n$ we obtain formula for determining the concentration of minority charge carriers on the base of experimentally measured values

$$n=\frac{4B_f}{\sqrt{3}e\rho_\infty}\frac{\frac{\rho_f}{\rho_0}-1}{\frac{4}{3}\frac{\rho_f}{\rho_0}-1}. \quad (7)$$

The latter formula expects also the possibility to avoid measuring in high magnetic field for determining n, so eq. (7) gets the form:

$$n=\frac{4\sqrt{3}B_f}{e\rho_0}\frac{\frac{\rho_f}{\rho_0}-1}{\left(4\frac{\rho_f}{\rho_0}-3\right)^2}.$$

Return to the equation (3). In the strong field limit we can neglect 1 in comparison with the summand containing B in the numerator and denominator of (3), getting another formula for determining μ_n

$$\mu_n=\frac{\rho_\infty}{\sqrt{3}\rho_0B_f}.$$

Combining the latter formula with (4) we get the relation between the experimentally measured values ρ_0,ρ_f and ρ_∞:

$$\rho_\infty=4\rho_f-3\rho_0, \quad (8)$$

which is the test relation for the semi classic behaviour (1) of a real experimental curve.

Now we give some examples illustrating how the introduced method works.

3. EXAMPLES OF CALCULATION

We have applied these results to the metal aluminium at $T=4.2\ K$. As *Al* is located in third group of periodic table of elements, its atom has 3 valent electrons $\left(3s^23p^1\right)$. Analysis of Hall-effect experimental data [3] proves that among this three electrons only two become free, the third one can tunnel through potential barrier to the neighboring atom because of the overlap of the wave functions. On this free place an electron from another neighboring atom can come etc. Thus this free place moving in chaotic manner within the crystal behave as a positive free particle with electron charge – the hole. The presence of hole in Al is confirmed both by Fermi surface calculation and Hall-effect experiments [4]. Dispersion low of free charge carriers in second Brillouin zone is pointed out by hole nature of these carriers and as in this zone there is approximately 1 of 3 valent electrons, then hole concentration must be approximately twice smaller than free electron concentration [4]. Only this is observed experimentally: the Hall coefficient depends on magnetic field and changes its sign from negative to positive in strong magnetic field, in which it is twice larger than in weak one. Such Hall-effect sign change is observed not only in *Al*, but at least in *Be*, *Mg*, *In* and *Pb* [4].

Fig. 1 shows the magnetic field dependence of resistivity for refined sample of this metal [3]. In fact, the presence of transverse magnetoresistivity is the evident of two types charge carriers existence. Calculations using formula (4), (5), (6) and (7) for different refined *Al* are introduced in the Table. 1.

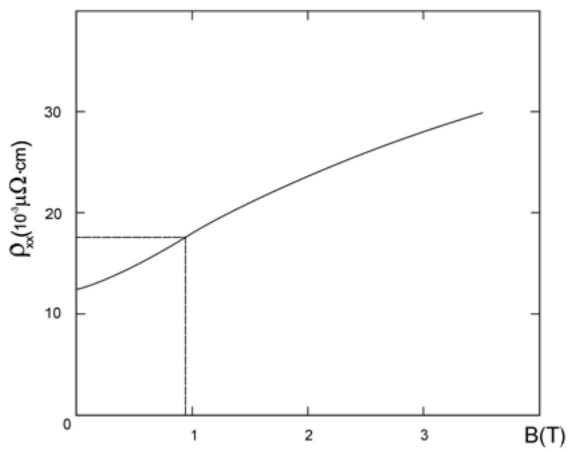

Fig. 1 – Magnetic field dependence of transverse resistivity for refined Al. Dashed lines indicate flex point B_f and resistivity in this point $\rho(B_f)$. The curve is obtained from experimental data of [4]

As it is seen from the Table. 1, refining influences only the mobility of charge carriers. Since the purification doesn't influence on the ratio $\frac{\sigma_n}{\sigma_p}$, we can conclude that hole mobility increases at the same rate as electron mobility.

As another example of conductor take an organic one $(BEDT-TTF)_2KHg(SCH)_4$.

The experimental data of [6] for this organic conductor with our calculations are shown in Table 2. The calculations after this experimental data result in the minority charge carriers mobility to be increased from 1.4 to 4.2 $\frac{m^2}{V\cdot s}$ when the temperature decreases from

Table 1 – Purification dependence of electron mobility μ_n, electron concentration n, electron conductivity σ_n and hole conductivity σ_p for Al

purification	$\mu_n\left(\frac{m^2}{V\cdot s}\right)$	$n(10^{28}\,m^{-3})$	$\sigma_n(10^9\Omega\cdot m)$	$\sigma_p(10^9\Omega\cdot m)$	σ_n/σ_p
refined	1.75	1.9	5.4	2.9	1.87
zone refining	14.3	2.0	45.9	23.3	1.97
superrefined	19.6	1.9	61.1	32.5	1.88
99.999+%	20.1	2.1	66.9	33.3	1.87

Table 2 – Temperature dependence of experimental data of [6] $(T, R_0, R_f, B_f, R_\infty^e)$ and quantities calculated by the formulas (8), (4) and (5) ($R_\infty, \mu_{\min}, \sigma_{\min}/\sigma_{maj}$ respectively), T – temperature, R_0 – zero field resistance, R_f – magnetoresistance flex point, B_f – magnetic field flex point, R_∞^e – experimental saturation magnetoresistance, R_∞ – calculated by the (8) saturation magnetoresistance, $\mu_{\min}$ – calculated by the (4) minority charge carriers mobility, $\sigma_{\min}/\sigma_{maj}$ – calculated by the (5) minority to majority conductivity ratio

T, K	R_0 (Ω)	R_f (Ω)	B_f (T)	R_∞^e (Ω)	R_∞ (Ω)	$\mu_{\min}\left(m^2/V\cdot S\right)$	$\sigma_{\min}/\sigma_{maj}$
0.6	0.025	0.17	3.3	0.59	0.60	4.2	23.0
2	0.030	0.15	3.1	0.52	0.51	3.2	16.2
4	0.036	0.12	2.3	0.35	0.37	2.6	9.4
6	0.041	0.07	1.6	0.15	0.16	1.4	2.9

6 to 0.6 K. In the same temperature range the minority charge carriers conductivity ration increased from 2.9 to 23 demonstrating rather rapid change of the conductivity tape. Please, pay, attention that theoretical value of saturation resistance R_∞ calculated by the formula (8) is in good agreement, with experimental one R_∞^e.

As the example of semiconductor we consider germanium doped indium antimonide (*InSb*). In order to measure the transverse magnetoresistance of a rectangular parallelepiped-like sample ($0.25\times1.2\times5.4\ mm$ size) we place it into pulse magnetic field, where the current $I = 1\ mA$ flowed normally to the magnetic field lines. The transverse voltage contacts were soldered at a distance $\frac{1}{3}l$ (where l – length of the sample) from the current contacts. The magnetoresistance experimental data at $T = 77\ K$ for this material are shown in Fig. 2.

The obtained value coincides with the value for light holes mobility given by different authors [2]. The ratio of corresponding conductivities is 0.07 what is in good agreement with the Hall-effect measurement results [6].

The most interesting example of applying of the equations is calculating the minority charge carriers mobility of superconductor in the critical temperature range.

It is known that high-temperature superconductors (cuprates) have planar structure and kinetic phenomena in these materials are connected with hybridized $O_{2p} - d_{x^2-y^2}$ orbitals [7]. P.W. Anderson [7] tells about that as a postulate (in the text – "dogma"). Until it is doped to 25 %, the Fermi surface is a simple hole surface around X [8]. Oxygen zone is hole-like and copper zone is electron-like (all the rest zones are located too far from Fermi level in order to take them in to account), although it is known opposite interpretation, when *O*-zone is electron-like and *Cu*-zone hole-like [9] However, in the both cases these two zones provide two types of free charge carriers, the majority of which are, as a rule, holes. It follows from the numerous Hall-effect experiments [9].

Fig. 3 shows the experimental dependences of transverse resistivity on the magnetic field inductance for several temperatures for high temperature multilayer superconductor $\left[YBa_2Cu_3O_7\left(72\ \overset{0}{A}\right)/\Pr Ba_2Cu_3O_7\left(12\ \overset{0}{A}\right)\right]_{25}$ near critical point $T_c = 89.5\ K$, namely for $T > T_c$. The fact of appearance of magnetoresistance at $T < 91\ K$ testiles the rise of a new sort of charge carriers. Indeed, the transverse resistance does not depend on magnetic 1eld in the materials with one type of charge carriers as it can be seen from equation (1) substituting there $p = 0$.

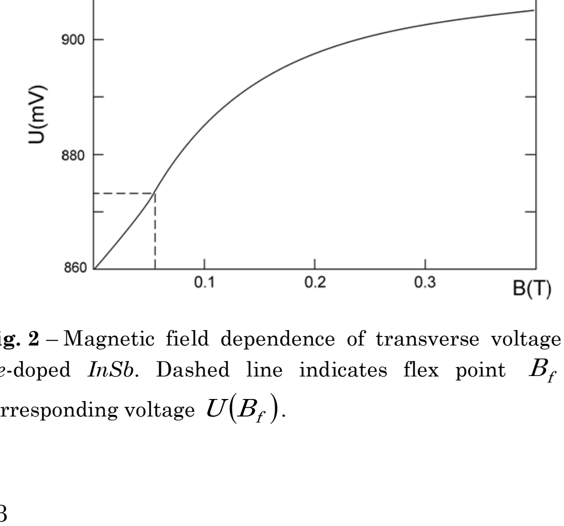

Fig. 2 – Magnetic field dependence of transverse voltage for *Ge*-doped *InSb*. Dashed line indicates flex point B_f and corresponding voltage $U(B_f)$.

Having determined for each curve the flex point B_f, zero 1eld resistivity ρ_0, flex point resistivity ρ_f and high 1eld limit resistivity ρ_∞, the basic parameters of the minority charge carriers can be calculated. The results of the calculations are shown in Table. 3 demonstrating the sharp increase of the negative sign charge carriers mobility, when the temperature approaches to superconductivity transition, which causes the same rate of their conductivity increasing. At the same time, majority charge carriers conductivity remains constant, what certainly shows their basic parameters to be constant. This abrupt conductivity change from hole to electron-like induces us to suppose that electrons being minority charge carriers in the normal state play majority role in the superconductive state.

Since one can 1nd out the similar magnetoresistivity behavior near T_c in other cuprate high temperature superconductors, doped by *Nd*, *Tm*, *Bi*, *Ca*, *Sr*, as well as in conventional ones [10 – 19], we can suppose that this temperature dependence of minority to majority charge carriers conductivity ratio is general, including the case when the minority charge carriers are positive, as it is for *Nd-Ce-Cu-O* [11].

Table. 3 – Temperature dependence of charge carriers basic parameters in layered superconductor $\left[YBa_2Cu_3O_7\left(72\overset{0}{A}\right)/\Pr Ba_2Cu_3O_7\left(12\overset{0}{A}\right)\right]_{25}$. The meanings of the symbols are the same as in Table. 1.

T_n (K)	$\mu_n\left(\frac{m^2}{V\cdot s}\right)$	$\mu_p\left(\frac{m^2}{V\cdot s}\right)$	p $\left(10^{27}\ m^{-3}\right)$	σ_n/σ_p	σ_p $\left(10^5\Omega\cdot m\right)$	σ_n $\left(10^5\Omega\cdot m\right)$	n $\left(10^{24}\ \Omega\cdot m\right)$	a	b
91	0.7	0.0018	2.2	0.23	6.5	1.5	1.3	0.0059	390
90.5	1.3	0.0018	2.2	0.67	6.5	4.4	2.1	0.0095	720
90.25	7.1	0.0018	2.2	2.7	6.5	17.6	1.5	0.0068	3900
90	40.9	0.0018	2.2	16.5	6.5	107	1.7	0.0077	22000

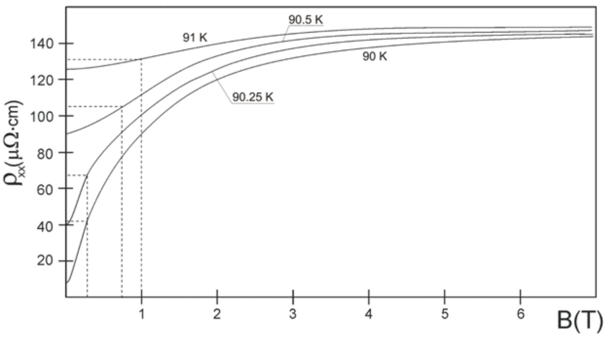

Fig. 3 – Magnetic field dependences of transverse resistivities for multilayer superconductor $\left[YBa_2Cu_3O_7\left(72\overset{0}{A}\right)/\Pr Ba_2Cu_3O_7\left(12\overset{0}{A}\right)\right]_{25}$ at diffrent temperatures near critical one $T = 89.5$ K. Magnetic field is perpendicular to the layers. Dashed lines indicate flex points B_f and resistivities in these points. Experimental data are obtained from temperature dependences of transverse resistivity at different magnetic fields from [10]

Note also, that applying of our equations, which origin from semiclassic analysis of galvanomagnetic phenomena is reasonable for this case, since experimental curves showed in the Fig. 3 are in good agreement with the test equation (8).

The introduced interpretation of magnetoresistance behavior in superconductors also solves the problem of Hall-effect anomaly near critical temperature [10-13], [15-20] which consists in sign change of Hall-effect in low magnetic field at the temperatures approaching T_c from the high temperature region. This phenomenon is common for both conventional [18-20] for example [14, 18, 19] and high temperature [10-12], [15-17], superconductors. The majority of the authors explain it as the vortex motion concept, some authors are disposed to the pining infuence, another ones suppose this phenomenon to be connected with the change of electron to hole conductivity ratio [11].

We explain this sign reversal by the great electron mobility obtained above. As we can see from the expression for the Hall constant in the weak magnetic field [1]

$$R_H = \frac{1}{e}\frac{p - nb^2}{(p + nb)^2}\ .$$

the sing of R_H will reverse negative when $b^2 > p/n$ that in its turn is provided by the great value of electron mobility μ_n.

Moreover, so-called ghost critical field appears in some superconductors that is Hall-effect maximum 1eld near critical temperature [13]. The prospect of our further research is to show that the maximum could appear in the solids with two types of holes.

4. CONCLUSIONS

We can conclude that the introduced new method of determination the minority charge carriers mobility can be applied for all solid materials (probably not only for solid ones) giving new opportunities for their studying. The most interesting result after using this method is found for a superconductor showing rapid increase of minority charge carriers mobility when the temperature approaches the critical one from the normal state temperature region. This rapid increase makes minority charge carriers responsible for appearance of superconductive state.

Визначення основних параметрів неосновних носіїв заряду в твердих тілах

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Запропоновано спосіб використання магнетоопору, як інструмент для визначення основних параметрів носіїв заряду та відношення провідності неосновних носіїв заряду до основних в твердих тілах на основі аналізу кривої магнетоопору в рамках феноменологічної двохзонної моделі. Встановлено критерії застосовності цієї моделі. В ролі прикладів застосування отриманих рівнянь приведено провідник, напівпровідник та надпровідник. Зі знайдених температурних залежностей згаданих вище величин в надпровіднику зроблено припущення про вирішальну роль неосновних носіїв заряду у виникненні надпровідного стану.

Ключові слова: Органічний провідник, Напівпровідник, Магнетоопір, Голл-ефект

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