

Short Communication

Collision of Two-dimensional Ultrashort Optical Pulses in the Medium with Carbon Nanotubes and the Order Parameter

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We study the influence of the order parameter on two-dimensional ultrashort optical pulses dynamics in their collision in the carbon nanotubes. The evolution of the electromagnetic field is modelled based on the wave equation supplemented by an equation determining the electric current in the system. The effect of the relaxation rate of the order parameter on the shape of the colliding ultrashort pulses is investigated.

Keywords: Ultrashort pulse, Carbon nanotubes, Collision, Order parameter.

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1. INTRODUCTION

Since the discovery of the carbon nanotubes (CNTs) by S. Iijima in 1991, they have been of great interest [1]. So media with carbon nanotubes cause increased attention of researchers primarily due to nonlinear properties [2, 3]. They are able to withstand extremely strong electromagnetic fields. And also, it is possible the stable propagation of localized ultrashort optical pulses in carbon nanotubes [4]. Media with a phase transition, i.e. with the order parameter (for example, all ferroelectrics and ferromagnetics), are attractive from the point of view of their practical applications [5, 6].

One of the unsolved problems in this area is the problem of the non-equilibrium dynamics of the order parameter in the presence of external variable fields. An important question for practical applications is the relaxation rate of the order parameter to the equilibrium value.

It was shown in [7] that collisions of pulses in nanotubes occur is elastic. But, taking into account the order parameter can lead to the breakdown of optical pulses. According to the foregoing, the problem of spectroscopy of the order parameter in media with the carbon nanotubes also arises. We also note that we work in a model with a scalar order parameter.

2. BASIC EQUATIONS

Let us consider, for definiteness, the dynamics of the scalar order parameter in a medium with CNTs. It is most simple to derive the motion equations using the approach developed in the paper [6]:

$$\frac{dP}{dt} = -\Gamma \frac{\delta\Phi}{\delta P}, \quad (1)$$

where Γ is the kinetic coefficient (hereafter it is equal to 1), P is the order parameter (for example, polarization, magnetization, etc.), Φ is the density of the free energy functional. Accordingly, the equation

below describes the dynamics of the most kind of the systems [6]. Further, taking into account the specifics of the problem, we assume that the scalar order parameter P is associated with an electric field directed along the CNT axis.

We choose Φ in the standard form.

$$\Phi = \Phi_0 + \alpha P^2 + \beta P^4 + \chi EP \quad (2)$$

Note that the electromagnetic field of the pulse and the field of the medium (3) act on the CNTs electrons:

$$E_s = \frac{\delta\Phi}{\delta P} \quad (3)$$

The electric field of the pulse is directed along the axis of the nanotubes. The dispersion law for the CNTs of zig-zag type can be written as [8]:

$$\varepsilon_s(p) = \pm \gamma \sqrt{1 + 4\cos(\alpha p)\cos\left(\frac{\pi s}{m}\right) + 4\cos^2\left(\frac{\pi s}{m}\right)} \quad (4)$$

$s = 1, 2, \dots, m$, type of the CNT $(m, 0)$, $\gamma = 2.7$ eV, $\alpha = 3b/2\hbar$, b is the coupling length C-C.

We consider the electric field E in the form:

$$E = -\frac{\partial A}{c \partial t}.$$

Let us write the expression for the density of current in carbon naotubes with similar calculations, which is in the paper [9]:

$$j_0 = -en_0 \sum_k D_k \sin\left(\frac{ke}{c} A(t)\right) \quad (5)$$

$$D_k = C_k(E_0) \sum_{s=1}^m \int_{-\pi/a}^{\pi/a} dp \frac{A_{ks} \cos(kp) \exp(-\varepsilon_s(p)/k_B T)}{1 + \exp(-\varepsilon_s(p)/k_B T)}$$

$$C_k(E_0) = \frac{w_0}{2\pi} \int_0^{2\pi/w_0} dt \cos\left(\frac{keE_0 \cos w_0 t}{w_0}\right)$$

here n_0 is the concentration of equilibrium electrons in CNTs, k_B is the Boltzmann's constant, T is the

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temperature, A_{ks} are the coefficients in a Fourier series for the charge carrier velocity.

Equation (5) in the dimensionless form can be represented as:

$$\Delta A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} + \text{sgn}(D_1) \sin(A + A_s) + \sum_{k=2}^{\infty} D_k \sin(kA + kA_s) / |D_1| = 0 \quad (6)$$

Here A_s corresponds to the electric field of the medium:

$$\begin{aligned} A(z, x, 0) &= \exp\left(-\left(\frac{x-x_0}{\gamma_x}\right)^2\right) \left(Q_1 \exp\left(-\left(\frac{z-z_{01}}{\gamma_z}\right)^2\right) + Q_2 \exp\left(-\left(\frac{z-z_{02}}{\gamma_z}\right)^2\right) \right) \\ \frac{dA(z, x, 0)}{dt} &= 2 \exp\left(-\left(\frac{x-x_0}{\gamma_x}\right)^2\right) \left[\frac{Q_1 v_{z1}}{\gamma_z^2} \exp\left(-\left(\frac{z-z_{01}}{\gamma_z}\right)^2\right) - \frac{Q_2 v_{z2}}{\gamma_z^2} \exp\left(-\left(\frac{z-z_{02}}{\gamma_z}\right)^2\right) \right] \end{aligned} \quad (7)$$

where Q_i is the amplitude of the i -th pulse, γ_z, γ_x determine the pulse width, z_{0i} is the initial coordinate of the i -th pulse; v_{zi} is the initial velocity of the i -th pulse along the z -axis, $i = 1, 2$.

The evolution of the two electromagnetic pulses collision is presented in the Fig. 1.

We observe the broadening of ultrashort optical pulses when they collide and propagate in the medium

$$E_s = \frac{1}{c} \frac{\partial A_s}{\partial t}$$

It should be noted, that Eq. (6) is a generalization of the well-known Sine-Gordon equation in the case when the generalized potential decomposes into a Fourier series. The investigated equation (6) is solved numerically based on the cross-type scheme [10]. The initial condition is chosen as Gaussian form:

with carbon nanotubes. The amplitude of the pulse electric field decreases. This behavior can be explained with the interaction of the current through the carbon nanotubes with the subsystem described by the order parameter.

In the case of the different pulse amplitudes, the dependence is shown in Fig. 2.

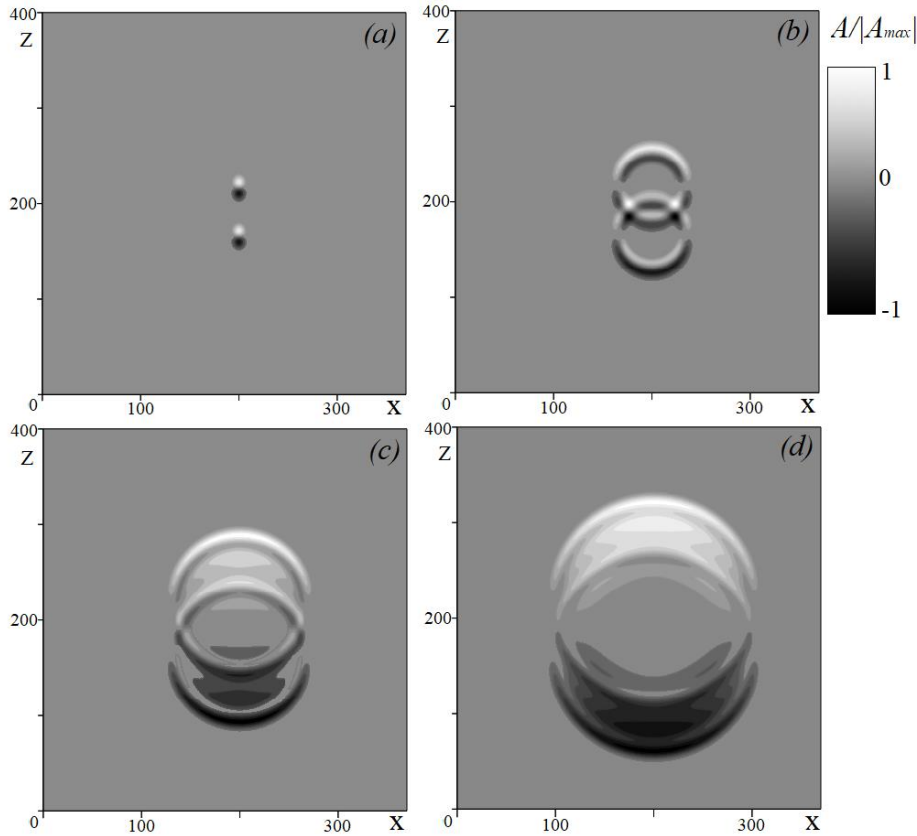


Fig. 1 – Vector-potential of the electric field at the different instances of time ($\alpha = 0.5, \beta = -1, \chi = 0.2, \Gamma = 0.1, Q_1 = Q_2$): (a) $t = 0$ s; (b) $t = 2 \cdot 10^{-13}$ s; (c) $t = 4 \cdot 10^{-13}$ s; (d) $t = 6 \cdot 10^{-13}$ s

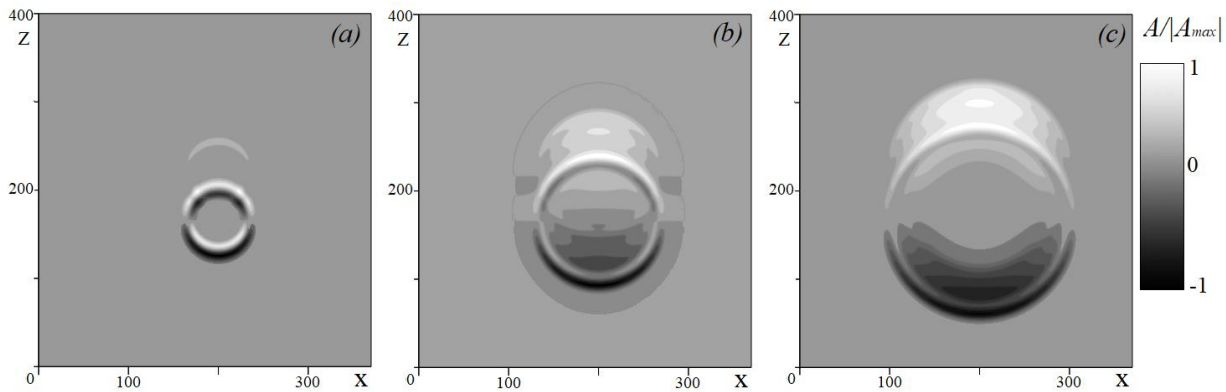


Fig. 2 – Vector-potential of the electric field at the different instances of time ($\alpha = 0.5$, $\beta = -1$, $\chi = 0.2$, $\Gamma = 0.1$, $Q_2/Q_1 = 5$): (a) $t = 2 \cdot 10^{-13}$ s; (b) $t = 4 \cdot 10^{-13}$ s; (c) $t = 6 \cdot 10^{-13}$ s; (d) cross-section of the electromagnetic pulse along the z -axis when $x = 200$ r.u.

It can be seen an absorption of a pulse with a smaller amplitude by a pulse with a larger amplitude. As a result, the pulse with smaller amplitude disappears. This fact can be related to the nonlinear interaction, which is caused the energy redistribution. This interaction can be useful in light control devices using the light and can be a base for an analog comparator of the pulse amplitudes.

The effect of the relaxation rate of the order parameter on the process of ultrashort optical pulses interaction is shown in Fig. 3.

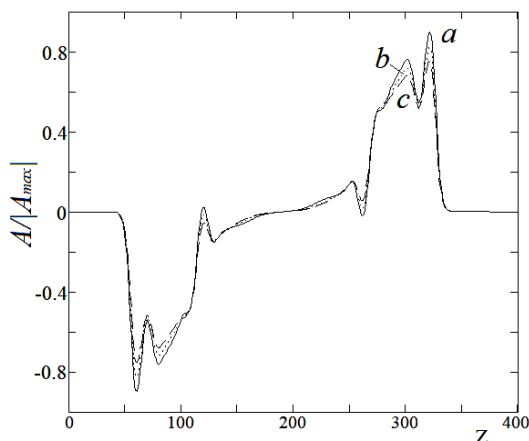


Fig. 3 – The cross-section along the z -axis at the $x = 200$ r.u. for vector-potential of the electric field at the different values of the relaxation rate Γ (at the time $t = 6 \cdot 10^{-13}$ s, $\alpha = 0.5$, $\beta = -1$): (a) $\Gamma = 0$; (b) $\Gamma = 0.05$; (c) $\Gamma = 0.1$

According to Fig. 3 we can conclude, that when the relaxation rate of the order parameter increases, the pulse amplitude decreases. We note also the change of

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the "tail" shape, which following the basic momentum when the parameter Γ increases. This, in turn, proves the mechanism of decreasing the pulse amplitude, which is associated with the relaxation dynamics of the order parameter.

It should be noted, although the pulses experience a certain broadening according to the dispersion, the bulk of the pulse energy remains concentrated in a limited region of space. And, in this sense the propagation of the pulse is stable.

3. CONCLUSION

Key results of this work may be summarized as follows:

1. An effective equation, which describes the collision of two-dimensional ultrashort optical pulses in the CNT with taking into account the order parameter is obtained.
2. After the collision, the pulses pass through each other. At the same time, they are divided into several peaks, which continue to propagate stably with a damped amplitude due to the relaxation dynamics of the order parameter.
3. The possibility of performing the spectroscopy of the order parameter based on the ultrashort pulses is shown.

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