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Effects of Three-Dimensional Noncommutative Theories on Bound States Schrödinger Molecular under New Modified Kratzer-type Interactions

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The new exact energy eigenvalues for new modified Kratzer-type potentials (NMKP) are calculated for a few typical (N_2 , CO , NO , CH) molecules through the Bopp's shift method in the global quantum group (GQG) of noncommutative three-dimensional real space-phase symmetries (NC: 3D-RSP) in the framework of two infinitesimal parameters θ and $\bar{\theta}$ due to (space-phase) noncommutativity, by means of the solution of the noncommutative Schrödinger equation (NCSE). The perturbation property of the spin-orbital Hamiltonian operator and new Zeeman effect of 3D system are investigated and the corresponding energy eigenvalues $E_{\text{kp}}(n,j,l,s)$ and $E_{\text{nc-kp}}(n,j,l,s,m)$ are easily calculated from standard perturbation theory. We have shown also that, the GQG of (NC: 3D-RSP) reduce to new sub-group symmetry of NC three-dimensional real space (NC: 3D-RS) under new three-dimensional NMKP interactions

Keywords: Three dimensional Schrödinger equation, Modified Kratzer-type Potential, Noncommutative space-phase, Star product and Bopp's shift method.

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1. INTRODUCTION

The modified Kratzer-type potentials have the general features of the true interaction energy, inter atomic and dynamical properties in solid-state physics and play an important role in the history of molecular structures, molecular physics and interactions, furthermore, this potential offered one of the most important exactly models of atomic and molecular physics and quantum chemistry, on another hand, the MKP can be describe the interaction between two atoms and have attracted a great of interest for some decades in the history of quantum chemistry, the interactions of atoms and molecules like (N_2 , CO , NO , CH) can be described in terms of a MKP using the time-independent Schrödinger wave equation [1 – 5]. And in view of what has been mentioned, we would like to study the results of the interactions of this potentials in a large space of quantum mechanics, currently known by the noncommutative quantum mechanics or extended quantum mechanics, which known firstly by Heisenberg and was formalized by Snyder at 1947, suggest by the physical recent results in string theory [6]. Motivated by these, over the past few years, theoretical physicists have shown a great deal of interest in solving Schrödinger equation for various potentials in NC space-phase to obtaining profound interpretations at microscopic scale [7 – 12] and in particularly, our previously works [13 – 15]. The notions of noncommutativity of space and phase based essentially on Seiberg-Witten map, the Bopp's shift method and the star product, which modified the ordinary product $(fg)(x,p)$ to the new form $(f*g)(x,p)$ on the first order of two infinitesimal antisymmetric parameters $2\left(\theta^{\mu\nu},\bar{\theta}^{\mu\nu}\right)\equiv\varepsilon^{k\mu\nu}\left(\theta_k,\bar{\theta}_k\right)$ as (Throughout this paper the atomic units i.e. $c=\hbar=1$ are employed) [7 – 11]:

$$\delta(f*g)(x,p)=-\frac{i}{2}\left(\theta^{\mu\nu}\partial_{\mu}^{x}f\,\partial_{\nu}^{x}g+\bar{\theta}^{\mu\nu}\partial_{\mu}^{p}f\,\partial_{\nu}^{p}g\right)(x,p),\qquad(1)$$

where $\delta(f*g)(x,p)=(f*g-fg)(x,p)$ and $(\theta^{\mu\nu},\bar{\theta}^{\mu\nu})$ denotes the two antisymmetric constants tensors. The above equation presents the noncommutativity effects of space and phase, then, on based to eq.(1) we can be obtaining the following new non nulls commutators for noncommutative coordinate $\hat{x}_{\mu}$ and the momenta $\hat{p}_{\mu}$ in GQG of (NC: 3D-RSP) symmetries as follows [12 – 15]:

$$[\hat{x}_{\mu},\hat{x}_{\nu}]_{*}=i\theta_{\mu\nu}\text{and }[\hat{p}_{\mu},\hat{p}_{\nu}]_{*}=i\bar{\theta}_{\mu\nu}.\qquad(2)$$

On the other hand, although the three-dimensional NMKP has attracted wide attention, this is not the case for the two dimensional NMKP, the two dimensional NMKP has been studied for example in [13], thus, the purpose of the present work is extend our work in ref. [13] from (NC: 2D-RSP) model to (NC: 3D-RSP) model on base to the main references [4, 5] to find out what will happen for three-dimensional nonrelativistic spectrum if effects of noncommutativity of both space and phase are considered for NMKP and to discover the new spectrum of energy and a possibility to obtain new applications in different fields of matters sciences. However, the solutions of modified radial

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Schrodinger equation for any angular momentum quantum number l, with NMKP, using Bopp's shift method in symmetries (NC: 3D-RSP) which is the aim of this paper, has not yet been reported. The present paper consists of five sections. The rest content of this study is regulated as follows: In the second and the third sections, we have briefly review the SE with 3D-MKP and we shall briefly give the fundamental concepts of the Bopp's shift method and then we derive the deformed potential $V_{kp}(\hat{r})$ and NC spin-orbital Hamiltonian operator $H_{\text{so-kp}}(r,\theta,\bar{\theta})$ for NMKP, in the next step, we apply the perturbation theory to find the modified spectrum $E_{\text{kp}}(n,j,l,s)$ for n^{th} excited stats and then we end this section by deduce the spectrum $E_{\text{mag-kp}}(n,r_e,\eta,m)$ produced automatically by the external magnetic field. In section 4, we resume the global spectrum for NMKP and we conclude the corresponding global NC Hamiltonian operator $\hat{H}_{nc-kp}$ in GQG of (NC: 3D-RSP) symmetries. Finally, section five is devoted to a brief summary and conclusion.

2. REVIEW THE SPECTRUM OF 3D-MKP IN ORDINARY QUANTUM MECHANICS

Let us present a brief review the ordinary energy eigenvalues for MKP in order to understand the parallels between this and noncommutative theories and to gives a guides us to our new energy eigenvalues, the radial part $R_{nl}(r)$ of $\Psi_{nlm}(r,\theta,\phi)$ for three-dimensional SE satisfied the following equation [4, 5]:

$$\frac{d^2R_{nl}(r)}{dr^2}+\frac{2}{r}\frac{dR_{nl}(r)}{dr}+ +2\mu\left(E_{nl}-V(r)-\frac{l(l+1)}{2\mu r^2}\right)R_{nl}(r)=0, \tag{3}$$

where n and l are denotes the principal quantum and orbital angular momentum quantum number , respectively, while the MKP is given by:

$$V(r)=D_e\left(\frac{r-r_e}{r}\right)^2\equiv-\frac{A}{r}+\frac{B}{r^2}+D_e \tag{4}$$

with $A=2D_e r_e$, $B=D_e r_e^2$ and μ represents the reduced mass of the two interacting particles. Nevertheless, the above potential can be consider as a particular case from the general form of the following Mie-type potential $V_{mt}(r)$ [1 – 3]:

$$V_{mt}(r)=-D_e\left[\frac{a}{b-a}\left(\frac{r_e}{r}\right)^b-\frac{b}{b-a}\left(\frac{r_e}{r}\right)^a\right] \tag{5}$$

when $a=2$ and $b=1$ are substituted into eq. (5), we obtain eq. (5). According to the references [4, 5], the complete orthonormalized wave function $\Psi_{nlm}(r,\theta,\phi)$ and energy eigenvalues E_{nl} for MKP are given by:

$$\Psi_{nlm}(r,\theta,\phi)=\left(\frac{8\mu D_e r_e}{2n+\eta+1}\right)^{3/2}\bullet \bullet\left[\frac{n!}{(2n+\eta+1)(n+\eta)!}\right]^{1/2} r^{-\frac{1}{2}(1-\eta)}\exp(-r/2)L_n^{\eta}(r)Y_{lm}(\theta,\phi) \tag{6}$$

and

$$E_{nl}=D_e-\frac{1}{2\mu}\bullet \bullet\left[(4\mu D_e r_e)^2\left(1+2n\sqrt{1+4\left(2\mu D_e r_e{}^2+l(l+1)\right)}\right)^{-2}\right] \tag{7}$$

where $L_n^k(r)$ stands for the associated Laguerre functions, while the factor $\eta=\sqrt{1+8\mu\left(D_e r_e{}^2+l(l+1)/2\mu\right)}$.

3. THEORETICAL FRAMEWORK

3.1 Theoretical Overview of Bopp's Shift Method in Three-dimensional Space-phase

In order to obtain modified Schrödinger equation (MSE) which play a major role in (NC: 3D-RSP) symmetries, we replace ordinary Hamiltonian operator $\hat{H}(p_i,x_i)$, ordinary spinor $\Psi(\vec{r})$ and ordinary energy E_{nl} and ordinary product by NC Hamiltonian operator $\hat{H}(\hat{p}_i,\hat{x}_i)$, new spinor $\hat{\Psi}(\vec{\hat{r}})$ and new energy E_{nc-kp} and new star product $(*)$, respectively. Allow us to writing the new 3-D MSE for NMKP as follows [11 - 14]:

$$\hat{H}(\hat{p}_i,\hat{x}_i)*\hat{\Psi}(\vec{\hat{r}})=E_{nc-kp}\hat{\Psi}(\vec{\hat{r}}) \tag{8}$$

The new Hamiltonian operator $\hat{H}(\hat{p}_i,\hat{x}_i)$ acts on a suitable by star product on the wave function of the system $\hat{\Psi}(\vec{\hat{r}})$ to give us the energy eigenvalues of the system E_{nc-kp} energy in (NC: 3D-RSP) symmetries. It is important to notice that, the new Hamiltonian operator $\hat{H}(\hat{p}_i,\hat{x}_i)$ can be expressed in three general varieties: both NC space and NC phase (NC: 3D-RSP), only NC space (NC: 3D-RS) and only NC phase (NC: 3D-RP) as, respectively:

$$\hat{H}_{nc-kp}\left(\hat{p}_i,\hat{x}_i\right)\equiv\hat{H}\left(\hat{p}_i=p_i+\frac{\bar{\theta}_{ij}}{2}x_j;\hat{x}_i=x_i-\frac{\theta_{ij}}{2}p_j\right)\quad\text{for (NC : 3D - RSP)}$$
$$\hat{H}_{nc-kp}\left(\hat{p}_i,\hat{x}_i\right)\equiv\hat{H}\left(\hat{p}_i=p_i;\hat{x}_i=x_i-\frac{\theta_{ij}}{2}p_j\right)\quad\text{for (NC : 3D - RS)}\qquad(9)$$
$$\hat{H}_{nc-kp}\left(\hat{p}_i,\hat{x}_i\right)\equiv\hat{H}\left(\hat{p}_i=p_i+\frac{\bar{\theta}_{ij}}{2}x_j;\hat{x}_i=x_i\right)\quad\text{for (NC : 3D - RP)}$$

To find the analytical solutions of the eq. (8) we must apply the Bopp's shift method instead of solving the MSE directly with star product; we treated by using directly the two commutators, in addition to usual commutators on quantum mechanics [12 – 15]:

$$\left[\hat{x}_\mu,\hat{x}_\nu\right]=i\theta_{\mu\nu}\text{ and }\left[\hat{p}_\mu,\hat{p}_\nu\right]=i\bar{\theta}_{\mu\nu}\qquad(10)$$

It is well known, that the two new operators ($\hat{x}_\mu$ and $\hat{p}_\mu$) are given by the following Darboux transformations [8 – 13]:

$$\hat{x}_\mu=x_\mu-\frac{\theta_{\mu\nu}}{2}p_\nu\text{ and }\hat{p}_\mu=p_\mu+\frac{\bar{\theta}_{\mu\nu}}{2}x_\nu\qquad(11)$$

The two variables $\left(x_\mu,p_\mu\right)$ satisfy the usual canonical commutation relations in quantum mechanics. In recently work, we are interest with the first variety in eq. (9). We may go a step further and consider the Bopp's method (modified by a shift), which allows us to reducing the above MSE to new ordinary form, in addition two fundamental translations of space and phase which are presenting in eq. (8):

$$H_{nc-kp}\left(\hat{p}_i,\hat{x}_i\right)\psi\left(\vec{r}\right)=E_{nc-kp}\psi\left(\vec{r}\right)\qquad(12)$$

The new modified Hamiltonian $H_{nc-kp}\left(\hat{p}_i,\hat{x}_i\right)$ that appears above is given by:

$$H_{nc-kp}\left(\hat{p}_\mu,\hat{x}_\mu\right)=\frac{\hat{p}^2}{2\mu}+V_{kp}\left(\hat{r}\right)\qquad(13)$$

The new potential $V_{kp}\left(\hat{r}\right)$ in the GQG of (NC: 3D-RSP) can be written as:

$$V_{kp}\left(\hat{r}\right)=-\frac{D_e r_e}{\hat{r}}+\frac{D_e r_e^{\,2}}{\hat{r}^2}+D_e\qquad(14)$$

According to our references [13 – 15], we can write the two operators $\hat{r}^2$ and $\hat{p}^2$ in GQG of (NC: 3D-RSP) as follows:

$$\hat{r}^2=r^2-\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}+O\left(\theta\right)\text{ and }\hat{p}^2=p^2+\vec{\mathbf{L}}\bar{\vec{\vec{\boldsymbol{\theta}}}}+O\left(\bar{\theta}\right),\qquad(15)$$

where $\mathbf{L}\Theta\equiv L_x\Theta_{12}+L_y\Theta_{23}+L_z\Theta_{13}$ and $\vec{\mathbf{L}}\bar{\vec{\vec{\boldsymbol{\theta}}}}\equiv L_x\bar{\theta}_{12}+L_y\bar{\theta}_{23}+L_z\bar{\theta}_{13}$. After straightforward calculations one can obtains the important two terms ($\frac{1}{\hat{r}}$ and $\frac{1}{\hat{r}^2}$), which will be used to determine the NMKP $V_{kp}\left(\hat{r}\right)$ in GQG of (NC: 3D-RSP) symmetries as follows:

$$\frac{1}{\hat{r}}=\frac{1}{r}+\frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{2r^3}+O\left(\theta^2\right)\text{ and }\frac{1}{\hat{r}^2}=\frac{1}{r^2}+\frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{r^4}+O\left(\theta^2\right).\qquad(16)$$

Substituting, eq. (16) into eq. (14), one gets the NMKP $V_{kp}\left(\hat{r}\right)$ in GQG of (NC: 3D-RSP) symmetries as follows:

$$V_{kp}\left(\hat{r}\right)=\left(-\frac{D_e r_e}{r}+\frac{D_e r_e^{\,2}}{r^2}+D_e\right)-\left(\frac{D_e r_e}{2r^3}-\frac{D_e r_e^{\,2}}{r^4}\right)\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}.\qquad(17)$$

It is clear that, the first three terms in above equation represent the ordinary MKP while the rest terms are produced by the deformations of space-phase noncommutativity. Now simultaneously transforming $V_{kp}\left(\hat{r}\right)$ and $\frac{\hat{\vec{p}}^2}{2\mu}$ gives the global perturbative potential operators $H_{\text{pert-kp}}\left(r,\theta,\bar{\theta}\right)$ for NMKP in GQG of (NC: 3D-RSP) symmetries:

$$H_{\text{pert-kp}}\left(r,\theta,\bar{\theta}\right)=-\left(\frac{D_e r_e}{2r^3}-\frac{D_e r_e^{\,2}}{r^4}\right)\times\times\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}+\frac{\vec{\mathbf{L}}\bar{\vec{\vec{\boldsymbol{\theta}}}}}{2\mu}+O\left(\theta,\bar{\theta}\right).\qquad(18)$$

The above operator can be considering of the sum of $V_{pert-kp}\left(r,\theta,\bar{\theta}\right)$ and $\frac{\vec{\mathbf{L}}\bar{\vec{\vec{\boldsymbol{\theta}}}}}{2\mu}$ .Since we are only interested in the corrections of order θ and $\bar{\theta}$, we can disregard the second term in $H_{\text{pert-kp}}\left(r,\theta,\bar{\theta}\right)$.

3.2 Three-dimensional Spin-orbital Hamiltonian Operators for NMKP in GQG of (NC: 3D-RSP)

In this sub-section we apply the same strategy, which we have seen in our previously works [13 – 15], under such particular choice, one can easily reproduce both $\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}$ and $\vec{\mathbf{L}}\bar{\vec{\vec{\boldsymbol{\theta}}}}$ to the new physical forms $\alpha\theta\vec{S}\vec{L}$ and $\alpha\bar{\theta}\vec{S}\vec{L}$, respectively, to obtain the new forms of $H_{\text{pert-kp}}(r,\theta,\bar{\theta})$ for NMKP as follows:

$$H_{\text{so-kp}}(r,\theta,\bar{\theta}) \equiv H_{\text{pert-kp}}(r,\theta,\bar{\theta}) = = -\alpha\left\{\frac{A\theta}{2r^3} - \frac{B\theta}{r^4} + \frac{\bar{\theta}}{2\mu}\right\}\vec{\mathbf{L}}\vec{S}. \quad (19)$$

Here $\vec{S}$ denote the spin of molecular and α is real constant, thus, the spin-orbital interactions $H_{\text{pert-kp}}(r,\theta,\bar{\theta})$ appear automatically because of the new properties of space-phase. Now, it is possible to rewrite the above equation as follows:

$$H_{\text{pert-kp}}(r,\theta,\bar{\theta}) = -\frac{\alpha}{2}\left\{\frac{A\theta}{2r^3} - \frac{B\theta}{r^4} + \frac{\bar{\theta}}{2\mu}\right\}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right). \quad (20)$$

We have replaced the coupling $\vec{\mathbf{L}}\vec{S}$ by new physical values $\frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$. As it well known, the eigenvalues j of the total operator $\vec{J} = \vec{L} + \vec{S}$ can be obtains from the interval $|l - s| \le j \le |l + s|$, which allow us to obtaining the eigenvalues $k(j,l,s) \equiv j(j+1) + l(l+1) - s(s+1)$ of the operator $\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$. After straightforward calculation, one can show that, the radial function $R_{nl}(r)$ satisfying the following differential equation, in GQG of (NC: 3D-RSP) symmetries for NMKP:

$$\frac{d^2R_{nl}(r)}{dr^2} + \frac{2}{r}\frac{dR_{nl}(r)}{dr} + 2\mu\left(\begin{matrix} E_{nl} + \frac{D_e r_e}{\hat{r}} - \frac{D_e r_e^2}{\hat{r}^2} - D_e + \\ \left(\frac{D_e r_e}{2r^3} - \frac{D_e r_e^2}{r^4}\right)\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}} + \frac{\vec{\mathbf{L}}\vec{\vec{\boldsymbol{\theta}}}}{2\mu} - \frac{l(l+1)}{2\mu r^2}\end{matrix}\right)R_{nl}(r) = 0 . \quad (21)$$

In the next parts of this article we consider the term $H_{\text{pert-kp}}(r,\theta,\bar{\theta})$, as an infinitesimal part compared of the principal part of Hamiltonian operator $H_{kp}(p,x)$ for 3D-MKP in ordinary quantum mechanics, this allows to apply standard perturbation theory to obtaining the nonrelativistic energy corrections $E_{kp}(n,j,l,s)$ of molecular at first order of two parameters θ and $\bar{\theta}$.

3.3 The Exact Spin-orbital Spectrum for NMKP in GQG of (NC: 3D-RSP) Symmetries

In order to find the differences in the energy spectrum $E_{\text{kp}}(n,j,l,s)$, we use perturbation theory up to first order in θ and $\bar{\theta}$ and through the structure constants which specified the dimensionality of NMKP of molecular, which is sufficient to obtain differences in the energy, thus, we have the following results:

$$E_{\text{kp}}(n_r,j,l,s) = \left(\frac{8\mu D_e r_e}{2n+\eta+1}\right)^3\left[\frac{n!}{(2n+\eta+1)(n+\eta)!}\right]k(j,l,s) \int_0^{+\infty} r^{\eta+1}\exp(-r)\left[L_n^{\eta}(r)\right]^2\left\{\theta\left(-\frac{D_e r_e}{2r^3} + \frac{D_e r_e^2}{r^4}\right) + \frac{\bar{\theta}}{2\mu}\right\}dr. \quad (22)$$

If we introduce the following factors $T_1(n,r_e,\eta)$, $T_2(n,r_e,\eta)$ and $T_3(n,r_e,\eta)$ as:

$$T_1(n,r_e,\eta) \equiv -\frac{D_e r_e}{2}\int_0^{+\infty} r^{(\eta-1)-1}\exp(-r)\left(L_n^{\eta}(r)\right)^2 dr,$$

$$T_2(n,r_e,\eta) \equiv D_e r_e^2\int_0^{+\infty} r^{(\eta-2)-1}\exp(-r)\left(L_n^{\eta}(r)\right)^2 dr, \quad (23)$$

$$T_3(n,r_e,\eta) \equiv -D_e r_e\int_0^{+\infty} r^{(\eta+2)-1}\exp(-r)\left(L_n^{\eta}(r)\right)^2 dr.$$

Then, the nonrelativistic energy levels $E_{\text{kp}}(n,j,l,s)$ at first order of two parameters θ and $\bar{\theta}$ for molecular will expressed as a function of the previously factors as:

$$E_{\text{kp}}(n,j,l,s) = \left(\frac{8\mu D_e r_e}{2n+\eta+1}\right)^3\left[\frac{n!}{(2n+\eta+1)(n+\eta)!}\right] \cdot k(j,l,s)\left\{\theta\sum_{i=1}^{2}T_i(n,r_e,\eta) + \frac{\bar{\theta}}{2\mu}T_3(n,r_e,\eta)\right\}. \quad (24)$$

It is very important to calculate the three terms $T_i(n,r_e,\eta)\left(i = \overline{1,3}\right)$, to achieve this goal; we apply the following special integral of hypergeometric function [16]:

$$\int_0^{+\infty} t^{\alpha-1}\exp(-\delta t)L_m^{\lambda}(\delta t)L_n^{\beta}(\delta t)dt = \frac{\delta^{-\alpha}\Gamma(n-\alpha+\beta+1)\Gamma(m+\lambda+1)}{m!n!\Gamma(1-\alpha+\beta)\Gamma(1+\lambda)} \times {}_3F_2(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1), \quad (25)$$

where ${}_3F_2\left(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1\right)$ denote to the hypergeometric function, it's a particular case from the generalized hypergeometric series ${}_qF_p\left(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1\right)$ for p = 3 and q = 2, which gives the three factors $T_1\left(n,r_e,\eta\right)$, $T_2\left(n,r_e,\eta\right)$ and $T_3\left(n,r_e,\eta\right)$ after straightforward calculations, as follows:

$$\begin{aligned} T_1\left(n,r_e,\eta\right) &= -\frac{D_e r_e}{2}\frac{\Gamma(n+2)\Gamma(n+\eta+1)}{n!^2\Gamma(2)\Gamma(1+\eta)}\times {}_3F_2\left(-n,\eta-1,-1;-n+\eta+1,\eta+1;1\right),\\ T_2\left(n,r_e,\eta\right) &= D_e r_e^2\frac{\Gamma(n+3)\Gamma(n+\eta+1)}{n!^2\Gamma(3)\Gamma(1+\eta)}\times {}_3F_2\left(-n,\eta-2,-2;-n+\eta-2,\eta+1;1\right),\\ T_3\left(n,r_e,\eta\right) &= -D_e r_e\frac{\Gamma(n-1)\Gamma(n+\eta+1)}{n!^2\Gamma(-1)\Gamma(1+\eta)}\times {}_3F_2\left(-n,\eta+2,2;-n+\eta+2,\eta+1;1\right). \end{aligned} \tag{26}$$

We have $\Gamma(-1)=(-2)!=\infty$, then, the term $T_3\left(n,r_e,v\right)$ will be zero. Further, the substitution of eq. (28) into eq. (26), enables us to obtain the first quantum correction $E_{kp}\left(n,j,l,s\right)$ of energy levels of all bound states in three dimension as:

$$\begin{aligned} E_{\text{kp}}\left(n,j,l,s\right) &= \left(\frac{8\mu D_e r_e}{2n+\eta+1}\right)^3\left[\frac{n!}{(2n+\eta+1)(n+\eta)!}\right]\times\\ &\times k\left(j,l,s\right)\theta T_{nc-skp}\left(n_r,r_e,v\right) \end{aligned} \tag{27}$$

with $T_{nc-skp}\left(n,r_e,\eta\right)\equiv T_1\left(n,r_e,\eta\right)+T_2\left(n,r_e,\eta\right)$.Thus, the GQG of (NC: 3D-RSP) reduce to new sub-group symmetry (NC: 3D-RS) for NMKP.

3.4 The Exact Magnetic Spectrum for NMKP in GQG of (NC: 3D-RSP) Symmetries

On the other hand, it's possible to found another automatically symmetry for NMKP related to the influence of an external uniform magnetic field $\vec{\aleph}$, if we make the following transformations to ensure that previous calculations are not reputed:

$$\left(\theta,\bar{\theta}\right)\rightarrow\left(\chi,\ \bar{\sigma}\right)\aleph. \tag{28}$$

Here χ and $\bar{\sigma}$ are two infinitesimal real proportional's constants and further insight can be gained when we choose the magnetic field $\vec{\aleph}=\aleph\vec{k}$, then we can make the following translation:

$$\begin{aligned} &\frac{\alpha}{2}\left\{\theta\left(-\frac{D_e r_e}{2r^3}+\frac{D_e r_e^2}{r^4}\right)+\frac{\bar{\theta}}{2\mu}\right\}L_z\rightarrow\\ &\left(\chi\left(-\frac{D_e r_e}{2r^3}+\frac{D_e r_e^2}{r^4}\right)+\frac{\bar{\sigma}}{2\mu}\right)\aleph L_z. \end{aligned} \tag{29}$$

Allow us to introduce the modified magnetic Hamiltonian operator $\hat{H}_{m-kp}$ in global (NC: 3D-RSP) as:

$$\hat{H}_{m-kp}=\left(\chi\left(-\frac{D_e r_e}{2r^3}+\frac{D_e r_e^2}{r^4}\right)+\frac{\bar{\sigma}}{2\mu}\right)\left(\vec{\aleph}\vec{J}-\vec{S}\vec{\aleph}\right) \tag{30}$$

Here $\left(-\vec{S}\vec{\aleph}\right)$ denote to the ordinary Hamiltonian of Zeeman Effect. To obtain the exact NC magnetic modifications of energy $E_{\text{m-kp}}\left(n,r_e,\eta,m\right)$ for NMKP, we replace both $k\left(j,l,s\right)$ and θ in the eq. (27) by the discreet quantum number $m\left(-l\le m\le +l\right)$ and new infinitesimal parameter χ, respectively:

$$\begin{aligned} E_{\text{m-kp}}\left(n,r_e,\eta,m\right) &= \chi\left(\frac{8\mu D_e r_e}{2n+\eta+1}\right)^3\times\\ &\times\left[\frac{n!}{(2n+\eta+1)(n+\eta)!}\right]\aleph m T_{nc-skp}\left(n,r_e,\eta\right) \end{aligned} \tag{31}$$

However, very little has been achieved in the solution of MSE for studied potential NMKP.

4. RESULTS AND DISCUSSION OF GLOBAL SPECTRUM FOR NMKP IN GLOBAL (NC: 3D-RSP) SYMMETRIES

We have solved the modified radial Schrödinger equation and obtained the differences in the energy eigenvalues $E_{\text{kp}}\left(n,j,l,s\right)$ and $E_{\text{mag-kp}}\left(n,r_e,\eta,m\right)$ for the NMKP in Eqs. (27) and (31) which are produced automatically by the effects of spin-orbital interaction $H_{\text{pert-kp}}\left(r,\theta,\bar{\theta}\right)$ and new Zeeman effect $\hat{H}_{m-kp}$, respectively, in the following, we summarize obtained results of the modified energy levels $E_{\text{nc-kp}}\left(n,j,l,s,m\right)$ of molecular moving in NMKP as provided in subsections 3.3 and 3.4, according to three equations (7), (27) and (31) the explicit form for $E_{\text{nc- kp}}\left(n,j,l,s,m\right)$ is then:

$$\begin{aligned} E_{\text{nc -kp}}\left(n,j,l,s,m\right) &= D_e-\frac{1}{2\mu}\left[\left(4\mu D_e r_e\right)^2\left(1+2n\sqrt{1+4\left(2\mu D_e r_e^2+l\left(l+1\right)\right)}\right)^{-2}\right]\\ &+\left(\frac{8\mu D_e r_e}{2n+\eta+1}\right)^3\left[\frac{n!}{(2n+\eta+1)(n+\eta)!}\right]\left\{\theta k\left(j,l,s\right)+\chi\aleph m\right\}T_{nc-skp}\left(n,r_e,\eta\right) \end{aligned} \tag{32}$$

On the other hand, the total energy $E_{\text{nc-kp}}(n,j,l,s,m)$ is the sum of the principal part of energy $E_{n_r,l}$ and the two corrections energy $E_{\text{kp}}(n,j,l,s)$ and $E_{\text{mag-kp}}(n,r_e,\eta,m)$, this is one of the main motivations for the topic of this work. It's clear, that the obtained eigenvalues of energies are reals, which allow us to consider the NC diagonal Hamiltonian $\hat{H}_{nc-kp}$ as a Hermitian operator, $\left(\hat{H}_{nc-kp}=\left(\hat{H}_{nc-kp}\right)^{+}\right)$ and regarding the previous obtained results (eq. (21) and eq. (33)), we can re-write, up to first order in θ and $\bar{\theta}$, as:

$$\hat{H}_{nc-kp}=\left(-\frac{\Delta}{2\mu}-\frac{D_e r_e}{r}+\frac{D_e r_e^2}{r^2}+D_e\right)+\alpha\left(-\frac{D_e r_e\theta}{2r^3}+\frac{D_e r_e^2\theta}{r^4}+\frac{\bar{\theta}}{2\mu}\right)\vec{\mathbf{L}}\vec{S}+\left(\chi\left(-\frac{D_e r_e}{2r^3}+\frac{D_e r_e^2}{r^4}\right)+\frac{\bar{\sigma}}{2\mu}\right)\left(\overline{\aleph}\overline{J}-\overline{S}\overline{\aleph}\right), \tag{33}$$

which is the equation of a molecular under the influence of MKP. It should be pointed out that this treatment considers only first order terms in either θ or $\bar{\theta}$, it's worth to note that the first part presents the Hamiltonian operator in the ordinary quantum mechanics for MKP while the second and the third parts are respectively present the spin-orbital and new Zeeman Hamiltonians operators which are induced automatically by the NC properties of space and phase.

5. CONCLUSION

In this paper, we have studied the new bound state solutions of the MSE under NMKP new interactions in the case of GQG of (NC: 3D-RSP) via the Bopp's method and standard perturbation theory, we briefly summarize what has been achieved in this reach work and comment on the outlook on future work that can follow from this paper:

We have reviewed the 3-D nonrelativistic MKP for molecular and the Bopp's method.

-We have solved the MSE in 3D space-phase for its new bound states with NMKP plus the new part $H_{\text{pert-kp}}(r,\theta,\bar{\theta})$.

Our approach allows us to re-derive new Hamiltonian operators $\hat{H}_{nc-kp}$ (which contains two new perturbative terms: the first one is spin-orbital interaction $\hat{H}_{so-kp}$ while the other is new Zeeman effect $\hat{H}_{m-kp}$) and corresponding new energies eigenvalues $E_{\text{nc-kp}}(n,j,l,s,m)$.

We hope to get some interesting applications to this new potential in the study of different fields of matter sciences, because our results are not only interesting for the pure theoretical physicists but also for experimental physicists (solid-state physics, the history of molecular structures molecular physics (N_2, CO, NO, CH ,) and interactions).

Our results obtained are in exact agreement with those obtained in previously working [13].

t should be noted that the results obtained in this research will be identical with corresponding results in ordinary quantum mechanics when the two parameters $(\theta,\bar{\theta})$ are reduced to $(0,0)$.

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