# Mathematical Modeling of Physical Processes of Electromagnetic Field Transformation in Elastic Oscillations Field in Microthick Layers of Metals

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The results of the mathematical studies on the modeling of high-frequency electromagnetic field conversion in the field of elastic oscillations process in microthick surface layers or electrically conductive ferromagnetic material thin films placed in a magnetic field are given, taking into account the coherence of elastic, electric and magnetic properties of the metal. It is shown that in practical calculations, especially in the case of high-frequency oscillations, it is necessary to take into account thickness of skin layer in which electromagnetic field transforms into acoustic field.

**Keywords:** Mathematical model, Boundary-value problem, Ferromagnet, Microthick layer of metal, Electrically conductive material, Elastic oscillations, Electromagnetic field, Electromagnetic-acoustic transformation.

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## INTRODUCTION

Ultrasonic waves are effectively used to solve various problems of solid state physics. Excitation and reception of ultrasonic waves in metals is carried out by contact [1] and non-contact [2-4] methods. With the contact method, excitation and reception of ultrasonic waves are traditionally carried out by means of piezoelectric transducers (PET). However, in a number of cases, the use of PET becomes fundamentally unacceptable [5]. The non-contact method of excitation of ultrasonic vibrations is realized by the influence of fields of various types on the study sample. To implement a non-contact method, electromagnetic fields are most often used [2-5]. Excitation of ultrasonic waves under the influence of electromagnetic radiation occurs in the surface layer corresponding to thickness of skin layer. For high-frequency range, the thickness of ferromagnetic metals skin layer can be reduced to 1 μm or less. The area of ultrasonic waves reception has similar thickness of the layer taking part in the transformation. In the mathematical description of the electromagnetic method of excitation of elastic vibrations in metals, the main attention was paid, as a rule, to the ponderomotive action of the electromagnetic field. In the case of ferromagnetic metals, the situation is different. In general, the physical theory of magnetostriction phenomena was created to solve problems of technical magnetization [6]. This theory explains the nature of the magnetostrictive effects at the level of the crystal lattice and is practically not applicable for the quantitative description of the magnetostrictive mechanism of the formation of deformations in ferromagnetic metals and ferrodielectrics (ferrites) when they interact with electromagnetic field. This issue becomes especially important when examining the interaction between an electromagnetic field and a thin surface layer of metal or a film.

Shulga N.A. [7] through their works have begun the application of the component of phenomenological theory of magnetostrictive phenomena. They recorded and used the generalized Hooke's law for elastic media with magnetostrictive effects, but there is no second equation of the physical state, which has the meaning of the law of magnetic polarization of a ferromagnet considering its magnetostrictive properties. At the same time, the connection between the elastic, magnetic and electromagnetic fields, which exist in the volume of the deformable ferromagnet layer, is not established.

At the same time, the paper [8] is well known, in which an adiabatic version of the phenomenological theory of magnetostrictive phenomena is derived, in which nonlinearity of the elastic and magnetic properties of a ferromagnet is taken into account. Therefore, it is advisable to use this theory for mathematical modeling of the electromagnetic method of excitation of elastic vibrations in metals of a ferromagnetic group, the improvement of which will allow constructing highly effective electromagnetic-acoustic transducers of various purposes. At the same time it is important to take into account the influence of micron thicknesses of metal or electromagnetic into elastic oscillations field transformation area, which makes the research to be of current interest in the field of microelectronic engineering.

## 1. PROBLEM STATEMENT

Let us consider a conductive polycrystalline ferromagnet inside which using external devices a permanent magnetic field with intensity $\vec{H}^0(x_k)$ and a time-varying magnetic field $\vec{H}^*(x_k,t)=\vec{H}^*(x_k)e^{i\omega t}$ are created, where $x_k$ - coordinates of a point in the right-handed Cartesian (physical) coordinate system; $i=\sqrt{-1}$; $\omega$ - circular frequency; $t$ - time. In the volume V of ferromagnet and on its surface S a system of force factors appears, which is the source of elastic vibrations of the material particles of the metal. If there is a strong inequality $\left|\vec{H}^0(x_k)\right| >> \left|\vec{H}^*(x_k)\right|$, then the desired displacement vector of material particles is $\vec{u}(x_k,t)=\vec{u}(x_k)e^{i\omega t}$. Amplitude value $u_n(x_k)$ of the $n$-th component of the displacement vector satisfies the equation of steady harmonic vibrations (the second

 

Newton's law in differential form), which is written in the form:

$$\sigma_{mn,m} + \omega^2 \rho_0 u_n - L_n = 0 \forall x_k \in V, \qquad (1.1)$$

where $\sigma_{mn}$ - time-varying amplitude value under the law $e^{i\omega t}$ of tensor of the resulting elastic stresses in a ferromagnetic metal; a comma between indices means the operation of expression differentiation, which is written before a comma, by the coordinate which index is put after the comma; $\rho_0$ - ferromagnet density. The symbol $L_n$ denotes $n$-th component of the amplitude value of the vector of the volume density of Lorentz forces. Neglecting the currents that are caused by the motion of the material particles of the deformed ferromagnet, the component $L_n = \varepsilon_{nkm} J_k B_m^0$, where $\varepsilon_{nkm}$ - component of the Levi-Civita tensor that is equal to plus one, when the indices $n$, $k$, $m$ form permutations of numbers 1, 2, 3 with an even number of derangements, equal to minus one, when the indices $n$, $k$, $m$ form permutations of the numbers 1, 2, 3 with an odd number of derangements and equal to zero, when any two of the three indices are equal to each other; $J_k$ - $k$-th component of the amplitude value of the vector of the eddy current density, which varies in time according to the law $e^{i\omega t}$, and which is determined by the rotation of an alternating magnetic field in the volume of metallic ferromagnet; $B_m^0 = \mu_{mn}^{\varepsilon} H_n^0$ - $m$-th component of the magnetic induction vector of the constant bias field; $\mu_{mn}^{\varepsilon}$ - component of the magnetic permeability tensor of a magnetized ferromagnet, which is experimentally determined in the regime of constancy (equal to zero) of elastic deformations (symbol $\varepsilon$).

The amplitude value of the $n$-th component of the eddy current density vector $J_n(x_k)$ is determined by the Ohm's law in differential form, that is $J_n(x_k) = r_0 E_n(x_k)$, where $r_0$ - electric conductivity of a ferromagnet; $E_n(x_k)$ - amplitude value of the $n$-th component of the intensity vector of an alternating electric field in the volume of the deformable ferromagnet.

The electric and magnetic states of a dynamically-deformed ferromagnet are determined by the Maxwell equations, which, neglecting the motion of material particles and displacement currents, are written in the following form:

$$\varepsilon_{nsp} H_{p,s} = r_0 E_n, \qquad (1.2)$$

$$\varepsilon_{jqn} E_{n,q} = -i\omega B_j, \qquad (1.3)$$

where $H_p = H_p^* + h_p$; $h_p$ - amplitude value of the $p$-th component of the intensity vector of the internal magnetic field, which arises in the deformable ferromagnet due to the motion of the domain walls; $B_j$ - amplitude value of the $j$ - th component of the vector of magnetic induction of an alternating magnetic field in the volume of the deformable ferromagnet.

The connecting link between the fundamental equations (1.1) of mechanics and the equations of electrodynamics (1.2) and (1.3) are the equations of the physical state. If at any point of the deformable ferromagnet there is a strong inequality $\left|\vec{H}^0(x_k)\right| >> \left|\vec{H}(x_k)\right|$, where $\vec{H}(x_k) = \vec{H}^*(x_k) + \vec{h}(x_k)$, then from the general nonlinear relations follows the linear approximation, which can be written in the form:

$$\sigma_{mn} = c_{mnk\ell}^H u_{k,\ell} - m_{pkmn} H_p^0 \left(H_k^* + h_k\right), \qquad (1.4)$$

$$B_j = m_{pjqs} H_p^0 u_{s,q} + \mu_{jm}^{\varepsilon} \left(H_m^* + h_m\right). \qquad (1.5)$$

where $c_{mnk\ell}^H$ - modulus of elasticity of a ferromagnet, which is experimentally determined in the regime of constancy (equal to zero) of the magnetic field (symbol $H$) or, in other words, the elastic modulus of the demagnetized ferromagnet; $m_{pkmn}$ - magnetostrictive constant, whose numerical value depends on the magnitude and direction of the constant bias field. Obviously, in the case of a polycrystalline, non-textured metal, the material constants $c_{mnk\ell}^H$ и $m_{pkmn}$ are components of isotropic tensors of the fourth rank and are defined by the following relations

$$c_{mnk\ell}^H = \lambda \delta_{mn} \delta_{k\ell} + G\left(\delta_{mk}\delta_{n\ell} + \delta_{m\ell}\delta_{nk}\right),$$
$$m_{pkmn} = m_2 \delta_{pk} \delta_{mn} + \frac{m_1 - m_2}{2}\left(\delta_{pm}\delta_{kn} + \delta_{pn}\delta_{km}\right),$$

where $\lambda$ and $G$ - Lame's constants (moduli of elasticity); $\delta_{mn}$, …, $\delta_{km}$ - Kronecker symbols; $m_1$ and $m_2$ - linearly independent, experimentally determined constants. From the above definition of magnetostrictive constants it follows that the tensor matrix $m_{\beta\gamma}$ ($\beta$ and $\gamma$ - Voigt indexes) has the form:

$$\left\|m_{\beta\gamma}\right\| = \left\| \begin{array}{cccccc} m_1 & m_2 & m_2 & 0 & 0 & 0 \\ m_2 & m_1 & m_2 & 0 & 0 & 0 \\ m_2 & m_2 & m_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (m_1 - m_2)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & (m_1 - m_2)/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & (m_1 - m_2)/2 \end{array} \right\|.$$

Substituing relations (1.4) and (1.5) into equations (1.1) and Maxwell's equations (1.2) и (1.3), allows us to write the following system of differential equations:

$$c^H_{mnk\ell}u_{k,\ell m} + \omega^2\rho_0 u_n - m_{pkmn}\left(H^0_p h_k\right)_{,m} - f^*_n = 0\,\forall\, x_k \in V \,, \tag{1.6}$$

$$\varepsilon_{jqn}\varepsilon_{nsp}h_{p,sq} + i\omega r_0\mu^\varepsilon_{jm}h_m + i\omega r_0 m_{pjqs}H^0_p u_{q,s} = 0\,\forall\, x_k \in V \,, \tag{1.7}$$

where $f^*_n = L_n + m_{pkmn}\left(H^0_p H^*_k\right)_{,m}$ - the resulting value of the amplitude of the $n$-th component of the volume density vector's forces generated by external sources; when writing the equations (1.7) it was taken into account that $\varepsilon_{jqn}\varepsilon_{nsp}H^*_{p,sq} + i\omega r_0\mu^\varepsilon_{jm}H^*_m = 0$ by the definition of the vector $\vec{H}^*(x_k)$ as an intensity vector of alternating magnetic field in the volume of conducting ferromagnet.

The uniqueness of the solution of equations (1.6) and (1.7) is determined by the boundary conditions. Assuming that the ferromagnetic sample is in vacuum, we write condition:

$$n_j\left(\sigma_{jk} - \sigma^*_{jk}\right) = 0\,\forall\, x_k \in S \,, \tag{1.8}$$

which, in its essence, is the third Newton's law in a differential form. The symbol $n_j$ denotes the $j$-th component of the vector of the outer unit normal to the surface $S$ at the point with the coordinates $x_k$. The amplitude value of the component of the elastic stress tensor is $\sigma_{jk} = G\left(u_{j,k} + u_{k,j}\right) + \delta_{jk}\lambda u_{n,n}$. The amplitude value of the surface density of forces, which are created by external sources $\sigma^*_{jk} = m_{pnjk}H^0_p\left(H^*_n + h_n\right) + M^*_{jk}$, where $M^*_{jk} = \left(H^*_j + h_j\right)B^0_k + \delta_{jk}\left[\left(\vec{H}^* + \vec{h}\right)\cdot \vec{B}^0\right]/2$ - amplitude value of the Maxwell tensor of tension.

Many ferromagnets have relatively small values of magnetic permeabilities and therefore, in contrast to piezoceramic materials, it is necessary to take into account the emission of the energy of the electromagnetic field in the surrounding space of ferromagnet. In this case, the boundary conditions for the amplitude values of the components of the intensity vector $\vec{h}(x_k)$ of the internal magnetic field are written as follows:

$$\varepsilon_{kpq}n_p\left(h_q - \tilde{H}_q\right) = 0\,\forall\, x_k \in S \,, \tag{1.9}$$

$$n_p\left(B_p - \mu_0\tilde{H}_p\right) = 0\,\forall\, x_k \in S \,, \tag{1.10}$$

where $n_p$ - $p$-th component of the unit normal vector; $\tilde{H}_q$ - amplitude value of the $q$-th component of the intensity vector $\tilde{\vec{H}}(x_k)$ of the stray magnetic field, which varies with time according to the law $e^{i\omega t}$; $B_p = m_{qpmn}H^0_q u_{m,n} + \mu^\varepsilon_{pn}h_n$ - amplitude value of the $p$-th component of the magnetic induction vector in the volume of the deformed ferromagnet; $\mu_0 = 4\pi\cdot 10^{-7}\ H/m$ - vacuum permeability.

The vector $\tilde{\vec{H}}(x_k)$ is defined by the Maxwell equations in vacuum, that is, it is the general solution of equation:

$$rot\,rot\,\tilde{\vec{H}} - \omega^2\mu_0\chi_0\tilde{\vec{H}} = 0\,\forall\, x_k \notin V \,, \tag{1.11}$$

And satisfies the conditions of physical realizability of the source of the field, that is, the limiting conditions

$$\lim_{R\to\infty}\left[\tilde{H}_k \,,\, \tilde{H}_{k,m}\right] = 0 \,, \tag{1.12}$$

where $R$ - distance from the surface $S$ of the ferromagnetic sample. The symbol $\chi_0$ in equation (1.11) denotes the dielectric constant $\chi_0 = 8{,}85\cdot 10^{-12}\ F/m$.

Thus, a precise determination of the parameters of the stress-strain state of a magnetized ferromagnet assumes a joint solution of a system of six differential equations (1.6) and (1.7). The uniqueness of the solution of this system of equations is provided by the boundary conditions (1.8) - (1.10), in which the components of the intensity vector of the stray magnetic field are determined as a result of the solution of the complementary boundary value problem (1.11), (1.12).

## 2. MODEL EXAMPLE AND QUANTITATIVE ESTIMATION OF THE CONNECTION OF ELASTIC AND MAGNETIC FIELDS IN THE VOLUME OF DEFORMED FERROMAGNET LAYER

Suppose that at the end of a semi-infinite prismatic layer of a rectangular cross-section (Figure 1) harmoniously varying forces produced by external devices are operated. The external forces are distributed uniformly on the surface $x_2 = 0$ with surface density $\sigma^*_2 = \sigma_0 e^{i\omega t}$. The layer is magnetized by a constant longitudinal magnetic field, the intensity of which $H^0_2$ is constant throughout the entire volume of the layer.

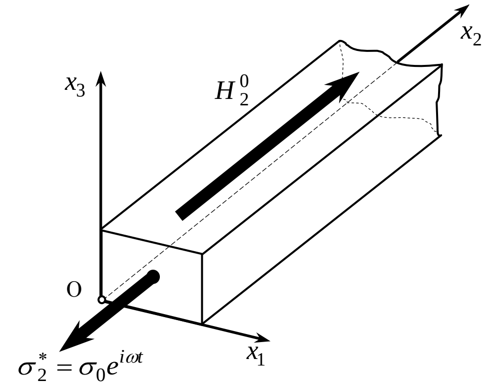


**Fig. 1** – To the definition of the dynamic stress-strain state of a longitudinally magnetized layer

It is obvious that external forces excite harmonic elastic waves that spread in the positive direction of the coordinate axis $Ox_2$. The displacement vector of the material particles of the layer can be described as follows $\vec{u}(x_k,t)=\vec{u}(x_k)e^{i\omega t}$, where $\vec{u}(x_k)$ - spatially-developed amplitude. The components of the displacement vector satisfy the equations of steady-state harmonic vibrations (1.1).

Since torsion and bending in the ferromagnetic state are absent in the sense of the problem statement, the tangential stresses are zero, and the amplitude values of the normal resultant stresses are determined as follows:

$$\sigma_{11}=(2G+\lambda)\varepsilon_{11}+\lambda(\varepsilon_{22}+\varepsilon_{33})-m_2H_2^0h_2, \quad (2.1)$$

$$\sigma_{22}=\lambda\varepsilon_{11}+(2G+\lambda)\varepsilon_{22}+\lambda\varepsilon_{33}-m_1H_2^0h_2, \quad (2.2)$$

$$\sigma_{33}=\lambda(\varepsilon_{11}+\varepsilon_{22})+(2G+\lambda)\varepsilon_{33}-m_2H_2^0h_2, \quad (2.3)$$

where $\varepsilon_{11}$, $\varepsilon_{22}$ и $\varepsilon_{33}$ - compression-extension deformations along the coordinate axes $Ox_1$, $Ox_2$ and $Ox_3$ respectively; $h_2$ - longitudinal component of the intensity vector of the internal magnetic field in the volume of the deformable ferromagnet.

The magnetic state of the deformable ferromagnetic layer is described by the law of magnetic polarization of a ferromagnet layer with allowance for its magnetostrictive properties (1.5), from which the calculated relationships for the amplitude values of the components of the magnetic induction vector are:

$$B_1=\mu_1^{\varepsilon}h_1;\; B_2=m_2H_2^0(\varepsilon_{11}+\varepsilon_{33})+m_1H_2^0\varepsilon_{22}+\mu_2^{\varepsilon}h_2;\; B_3=\mu_3^{\varepsilon}h_3. \quad (2.4)$$

Consider the frequency range in which the length of the elastic wave is many times greater than the largest cross-sectional dimension of the layer. For the variable magnetic field frequencies traditionally used in physical studies, as well as the ranges of magnetic permeabilities and electrical conductivities of materials, the physical transformation of energies occurs for the layer thicknesses of 0.1 ... 100 μm. Therefore the length of an elastic wave is a measure and scale of the spatial inhomogeneity of the stress-strain state of the rod. Taking this circumstance into account, it can be stated that in the region of established frequencies the stress-strain state practically does not change within the cross-sectional area of the layer.

Since on the lateral surfaces perpendicular to the axes $Ox_1$ and $Ox_3$, normal stresses $\sigma_{11}$ and $\sigma_{33}$ of the layer being in the vacuum shall be zero (Newton's third law in differential form), and the change in the stress-strain state in the cross-sectional plane is absent, then it can be asserted that $\sigma_{11}=\sigma_{33}=0\,\forall x_k\in V$, where $V$ - layer volume.

According to the assumptions about the nature of the strained-deformed state in the plane of the cross-section, we assume that the magnetic induction also remains practically unchanged in the limits of the cross-sectional area of the layer. From this assumption it follows that $B_1=B_3=0\,\forall x_k\in V$.

If we turn to the Maxwell equation $rot\,\vec{E}=-i\omega\vec{B}$ and calculate the divergence of its left and right sides, we obtain the condition for the absence of magnetic charges, that is $div\,\vec{B}=0$. For the one-dimensional problem under consideration, this condition is equivalent to the assertion that

$$B_{2,2}=0. \quad (2.5)$$

Substituting the zeros into the left-hand sides of equations (2.1) and (2.3), we define deformations $\varepsilon_{11}$ and $\varepsilon_{33}$ through deformation $\varepsilon_{22}$:

$$\varepsilon_{11}=\varepsilon_{33}=\frac{m_2H_2^0h_2}{2\left(\lambda^H+G^H\right)}-\nu\varepsilon_{22}, \quad (2.6)$$

where $\nu=\lambda/[2(\lambda+G)]$ - Poisson's ratio of the demagnetized ferromagnet.

Substituting the deformations $\varepsilon_{11}$ and $\varepsilon_{33}$ from (2.6) into the equations of the physical state (2.2) and (2.4), we obtain the calculated relations of the following form:

$$\sigma_{22}=E\varepsilon_{22}-m_1^*H_2^0h_2=Eu_{2,2}-m_1^*H_2^0h_2, \quad (2.7)$$

$$B_2=m_1^*H_2^0\varepsilon_{22}+\mu_2^*h_2=m_1^*H_2^0u_{2,2}+\mu_2^*h_2, \quad (2.8)$$

where $E=G(2G+3\lambda)/(G+\lambda)$ - Young's modulus of the demagnetized ferromagnet; $m_1^*=m_1-\dfrac{m_2\lambda}{(\lambda+G)}$ and $\mu_2^*=\mu_2^{\varepsilon}+\left(m_2H_2^0\right)^2/(\lambda+G)$ - magnetostrictive constant and magnetic permeability for uniaxial tension-and-compression mode of a prismatic layer.

It is quite clear that the system of equations (1.6) is reduced to one equation for the longitudinal component $u_2$ of the displacement vector of the material particles of the layer, which can be written in the following form:

$$Eu_{2,22}-m_1^*H_2^0h_{2,2}+\rho_0\omega^2u_2=0. \quad (2.9)$$

Substituting (2.7) and (2.8) into condition (2.5), we find that

$$h_{2,2}=-\frac{m_1^*H_2^0}{\mu_2^*}u_{2,2}. \quad (2.10)$$

Eliminating the derivative $h_{2,2}$ from the equation (2.9) with the help of the relation (2.10), we obtain an ordinary differential equation of the following form:

$$E(1+\Delta E)u_{2,22}+\rho_0\omega^2u_2=0, \quad (2.11)$$

where $\Delta E=\left(m_1^*H_2^0\right)^2/\left(\mu_2^*E\right)$ - increase in the mechanical rigidity of the pre-magnetized ferromagnet due to the corresponding (coherent) action of elastic forces and

the forces of magnetic interaction between the poles of domains in the deformable ferromagnet.
The solution of the equation (2.11) is obvious

$$u_2(x_2) = U_0 e^{-i\gamma x_2},$$

where $U_0$ - constant to be determined; $\gamma = \omega/v$ - wave number of longitudinal nondispersive waves; $v = \sqrt{E(1+\Delta E)/\rho_0}$ - propagation velocity of longitudinal nondispersive waves (bar velocity) in a pre-magnetized ferromagnet. In cross section $x_2 = 0$ The third Newton's law must be satisfied, from which the constant $U_0$ is defined

$$U_0 = \frac{i\sigma_0}{\gamma E(1+\Delta E)}.$$

The relative change $\Delta E$ of the Young's modulus, or, as they say, $\Delta E$ -effect, is the result of the interaction (connection) of elastic and magnetic fields in the volume of the deformed, pre-magnetized ferromagnet. The consequence of coherence or correspondent action of elastic forces and magnetic forces is the emergence of new material constants $m_1^*$ и $\mu_2^*$. For a ferromagnet with elastic moduli $E = 2\cdot10^{11}\,Pa$ and $\nu = 0,3$ ($G = 7,69\cdot10^{10}\,Pa$, $\lambda = 11,54\cdot10^{10}\,Pa$) and constants $m_1 = 0,2\ H/m$, $m_2 = -0,1\ H/m$ (the order of the magnetostrictive constants corresponds to the experimental data) with magnetic permeability $\mu_2^\varepsilon = 30\mu_0 = 3,77\cdot10^{-5}\ H/m$. The connection between elastic and magnetic fields at magnetic bias field $H_2^0 = 1\ \kappa A/m$ appears as follows: $\Delta E = 8,95\cdot10^{-3}$, $m_1^* = 0,26\ H/m$, $\mu_2^* = 3,775\cdot10^{-5}\ H/m$. In this case, the bar speed increases by $\Delta v = 4,5\cdot10^{-3}$. If the magnetostrictive constants are of great value, such as $m_1 = 1,0\ H/m$ and $m_2 = -0,5\ H/m$, then for the same values of the remaining parameters we get the following values of material constants: $\Delta E = 0,217$, $m_1^* = 1,30\ H/m$, $\mu_2^* = 3,90\cdot10^{-5}\ H/m$ and $\Delta v = 0,103$.

In all probability, the value $\Delta E \cong 0,3$ is the maximum value of $\Delta E$ -effect, which can be observed in deformable, pre-magnetized ferromagnets.

It is obvious that in the case of ferromagnets with low values of the magnetostrictive consonant, one can neglect the effects that appear due to the connection between the elastic and magnetic fields.

## 3. METHOD OF SEQUENTIAL APPROXIMATIONS IN MATHEMATICAL MODELING OF ELECTROMAGNETIC FIELD TRANSFORMATION IN THE FIELD OF ELASTIC OSCILLATIONS IN A CONDUCTING PRE-MAGNETIZED FERROMAGNET LAYER.

At moderate values ($m_1 \le 0,5$) of magnetostriction constants, the corrections to the elastic moduli do not exceed 5%, which entails a variation of the propagation velocities of elastic waves not exceeding 3%. These errors are commensurable, or less, with systematic errors in the experimental determination of the numerical values of the material constants of ferromagnetic metals. This circumstance makes it possible to neglect the coupling effect of elastic and magnetic fields in dynamically deformed, pre-magnetized ferromagnets and write equations (1.6) in the following for

$$(\lambda + 2G)\,grad\,div\vec{u} - G\,rot\,rot\,\vec{u} + \omega^2\rho_0\vec{u} - \vec{f}^* = 0\,\forall x_k \in V, \tag{3.1}$$

where the amplitude value of the time-varying under the law $e^{i\omega t}$ vector of the volume density of external forces $\vec{f}^* = \vec{L} + \vec{e}_n m_{pkmn}\left(H_p^0 H_k^*\right)_{,m}$ ($\vec{e}_n$ - unit vector (vector with unit length) of the coordinate axis $Ox_n$) is a value that is known by the meaning of the problem statement.

The boundary conditions (1.8), which ensure the uniqueness of the solution of equation (3.1), retain their form, that is:

$$n_j\left(\sigma_{jk} - \sigma_{jk}^*\right) = 0\,\forall x_k \in S, \tag{3.2}$$

but the meaning of the component $\sigma_{jk}^*$ changes significantly. The surface density $\sigma_{jk}^* = m_{pnjk}H_p^0 H_n^* + H_j^* B_k^0 + \delta_{jk}\vec{H}^*\vec{B}^0/2$ becomes the known value as of the problem statement.

The boundary value problem (3.1), (3.2) has the meaning of the problem of excitation of harmonic vibrations in an isotropic solid by a system of volume and surface loads.

The results of the solution of the boundary value problem (3.1), (3.2) determine in the zeroth approximation the stress-strain state of a conductive, pre-magnetized ferromagnet layer. We denote the amplitude values of the components of the displacement vector of the material particles of the ferromagnet, obtained as a result of solving the boundary value problem (3.1), (3.2) by the symbol $u_n^{(0)}(x_k)$. From the known values $u_q^{(0)}(x_k)$ the deformations in equations $u_{q,s}^{(0)}(x_k)$ are determined, after which the zero approximations $h_p^{(0)}(x_k)$ to the exact amplitude values of the components of the intensity vector of the internal magnetic field are determined.

The values $h_p^{(0)}(x_k)$ are determined as a result of solving the next boundary value problem

$$\varepsilon_{jqn}\varepsilon_{nsp}h_{p,sq}^{(0)} + i\omega r_0\mu_{jm}^\varepsilon h_m^{(0)} + i\omega r_0 m_{pjqs}H_p^0 u_{q,s}^{(0)} = 0\,\forall x_k \in V, \tag{3.3}$$

$$\varepsilon_{kpq}n_p\left(h_q^{(0)} - \tilde{H}_q\right) = 0\,\forall x_k \in S, \tag{3.4}$$

$$n_p\left(B_p^{(0)} - \mu_0 \tilde{H}_p\right) = 0 \,\forall\, x_k \in S\,, \tag{3.5}$$

where $B_p^{(0)} = m_{qpmn} H_q^0 u_{m,n}^{(0)} + \mu_{pn}^{\varepsilon} h_n^{(0)}$; Amplitude values of the components of the vector $\tilde{\vec{H}}(x_k)$ of the stray magnetic field are determined exactly as a result of the solution of the complementary boundary value problem (1.11), (1.12). The first approximation $u_n^{(1)}(x_k)$ to the exact value of the components $u_n(x_k)$ is defined as follows:

$$u_n^{(1)}(x_k) = u_n^{(0)}(x_k) + \Delta u_n^{(1)}(x_k), \tag{3.6}$$

where $\Delta u_n^{(1)}(x_k)$ - first order correction.

Substituting the definition (3.6) and the found zero approximation $\vec{h}^{(0)}(x_k)$ to the exact amplitude value of the intensity vector of the internal magnetic field into equations (1.6) and the boundary conditions (1.8), we obtain:

$$(\lambda + 2G)\, grad\, div\, \Delta\vec{u}^{(1)} - G\, rot\, rot\, \Delta\vec{u}^{(1)} + \omega^2 \rho_0 \Delta\vec{u}^{(1)} - \vec{f}^{(0)} = 0 \,\forall\, x_k \in V\,, \tag{3.7}$$

$$n_j\left(\sigma_{jk}^{(1)} - \sigma_{jk}^{(0)}\right) = 0 \,\forall\, x_k \in S\,, \tag{3.8}$$

where $f_n^{(0)} = m_{pkmn}\left(H_p^0 h_k^{(0)}\right)_{,m}$;

$\sigma_{jk}^{(1)} = G\left(\Delta u_{j,k}^{(1)} + \Delta u_{k,j}^{(1)}\right) + \lambda \delta_{jk} \Delta u_{n,n}^{(1)}$;

$\sigma_{jk}^{(0)} = m_{pnjk} H_p^0 h_n^{(0)} + h_j^{(0)} B_k^0 + \delta_{jk}\, \vec{h}^{(0)} \vec{B}^0 / 2$.

The solution of the boundary value problem (3.7), (3.8) allows us to determine the correction of the first approximation $\Delta u_n^{(1)}(x_k)$ to the exact amplitude value $n$-th of the displacement vector's component of the material particles of the ferromagnetic metal layer. It should be emphasized that the solution of the boundary value problem (3.7), (3.8) has exactly the same construction as the general solution of the boundary value problem (3.1), (3.2). The solution of the problem (3.7), (3.8) is written by replacing the values $H_m^*(x_k)$ while solving the problem (3.1), (3.2) to the values $h_m^{(0)}(x_k)$.

The first approximation $h_p^{(1)}(x_k)$ to the exact value $h_p(x_k)$ is determined by the expression

$$h_p^{(1)}(x_k) = h_p^{(0)}(x_k) + \Delta h_p^{(1)}(x_k), \tag{3.9}$$

where $\Delta h_p^{(1)}(x_k)$ - first-order correction.

After substituting the expressions (3.6) and (3.9) into (3.3) and the boundary conditions (3.4), (3.5), we obtain:

$$\varepsilon_{jqn} \varepsilon_{nsp} \Delta h_{p,sq}^{(1)} + i\omega r_0 \mu_{jm}^{\varepsilon} \Delta h_m^{(1)} + i\omega r_0 m_{pjqs} H_p^0 \Delta u_{q,s}^{(1)} = 0 \,\forall\, x_k \in V\,, \tag{3.10}$$

$$\varepsilon_{kpq} n_p \Delta h_q^{(1)} = 0 \,\forall\, x_k \in S\,,\ n_p \Delta B_p^{(1)} = 0 \,\forall\, x_k \in S\,, \tag{3.11}$$

where $\Delta B_p^{(1)} = m_{qpmn} H_q^0 \Delta u_{m,n}^{(1)} + \mu_{pn}^{\varepsilon} \Delta h_n^{(1)}$.

The boundary value problem (3.10), (3.11) allows us to determine the corrections $\Delta h_p^{(1)}(x_k)$ as functions of the corrections $\Delta u_n^{(1)}(x_k)$.

After determining the corrections $\Delta h_p^{(1)}(x_k)$, assuming that the second approximation to the exact value of the displacement vector $\vec{u}(x_k)$ is $\vec{u}^{(2)}(x_k) = \vec{u}^{(1)}(x_k) + \Delta\vec{u}^{(2)}(x_k)$, the boundary value problem (3.7), (3.8) is solved, and corrections $\Delta u_n^{(2)}(x_k)$ are determined. By them, as a result of solving the boundary value problem (3.10), (3.11), corrections $\Delta h_p^{(2)}(x_k)$ are determined. Computational procedures can be repeated as many times as you like.

A small parameter of the described above procedure of successive approximations is the square of the coefficient of the magnetomechanical coupling, the numerical value of which does not exceed the value of the $\Delta E$ -effect. Where in $\left|\Delta\vec{u}^{(1)}\right| / \left|\vec{u}^{(0)}\right| \le \Delta E$, $\left|\Delta\vec{u}^{(2)}\right| / \left|\vec{u}^{(0)}\right| \le (\Delta E)^2$, … , $\left|\Delta\vec{u}^{(m)}\right| / \left|\vec{u}^{(0)}\right| \le (\Delta E)^m$. Similar estimates are valid for corrections to the zero approximation to the exact value of the vector $\vec{h}(x_k)$.

If the magnetostrictive properties of the ferromagnet ensure the $\Delta E$ -effect at the level of less than 10%, then it can be assumed that the zero approximations $\vec{u}^{(0)}(x_k)$ and $\vec{h}^{(0)}(x_k)$ to the exact values of the displacement vector of material particles and the intensity vector of the internal magnetic field provide estimates of the physical state of the deformed, pre-magnetized ferromagnet, which, in terms of technical applications, are quite satisfactory both in a quantitative sense and in terms of qualitative (physical) content.

## 4. CONCLUSIONS

1. A boundary problem is formulated for the electromagnetic excitation of elastic oscillations in the micro-thick layers (films) of ferromagnetic group metals, in which the linear approximation of the phenomenological theory of magnetostriction phenomena is used.

2. A mathematical model of electromagnetic energy into acoustic one transformation process in a thin metal layer is developed, taking into account the coherence of the elastic and magnetic fields in the volume of the dynamically deformable micro-thick layer of an electrically conductive ferromagnet.

An increase in the mechanical rigidity of a pre-magnetized ferromagnet due to the coupling action of elastic forces and magnetic interaction forces between the poles of domains in a deformable thin layer of ferromagnet (*ΔE* effect) is estimated. The boundaries are determined for which the *ΔE* effect can be ignored in practical calculations.

3. On the basis of numerical values of the ΔE effect estimates, a method of successive approximations is proposed for solving the boundary problem of the conversion of a high-frequency electromagnetic field into the field of elastic waves in the micro-thick layers of ferromagnetic group metals.

4. It is shown that the developed mathematical model is adequate in wide range of frequencies of the applied high-frequency electromagnetic field. For the conventionally used in physical studies variable magnetic field frequencies, as well as ranges of magnetic permeabilities and electrical conductivities of materials, the physical transformation of energies occurs in the layer thicknesses of 0.1 ... 100 μm. The length of the excited elastic wave will be much larger than the thickness of the layer's section, therefore providing a metal layer stress-strain state, which practically does not change within the area of its cross section.

## Математическое моделирование физических процессов преобразования электромагнитного поля в поле упругих колебаний в микротолщинных слоях металлов

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Приведены результаты математических исследований по моделированию процесса преобразования высокочастотного электромагнитного поля в поле упругих колебаний в микротолщинных поверхностных слоях или тонких пленках из электропроводного ферромагнитного материала, помещенного в магнитное поле, с учетом связности упругих, электрических и магнитных свойств металла. Показано, что при практических расчётах, особенно в случае высокочастотных колебаний, необходимо учитывать толщину скин-слоя, в котором происходит преобразование электромагнитного поля в акустическое.

**Ключевые слова**: Математическая модель, Граничная задача, Ферромагнетик, Микротолщинный слой металла, Электропроводный материал, Упругие колебания, Электромагнитное поле, Электромагнитно-акустическое преобразование.

## Математичне моделювання фізичних процесів перетворення електромагнітного поля в поле пружних коливань в мікротовщинних шарах металів

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Наведено результати математичних досліджень з моделювання процесу перетворення високочастотного електромагнітного поля в поле пружних коливань в мікротовщинних поверхневих шарах або тонких плівках з електропровідного феромагнітного матеріалу, розміщеного в магнітному полі, з урахуванням зв'язності пружних, електричних і магнітних властивостей металу. Показано, що при практичних розрахунках, особливо в разі високочастотних коливань, необхідно враховувати товщину скін-шару, в якому відбувається перетворення електромагнітного поля в акустичне.

**Ключові слова**: Математична модель, Гранична задача, Феромагнетик, Мікротовщинний шар металу, Електропровідний матеріал, Пружні коливання, Електромагнітне поле, Електромагнітно-акустичне перетворення.

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