JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. 8 No 4(2), 04076(9pp) (2016)

ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том 8 № 4(2), 04076(9cc) (2016)

A new Nonrelativistic Investigation for Interactions in One-electron Atoms with Modified Vibrational-Rotational Analysis of Supersingular plus Quadratic Potential: Extended Quantum Mechanics

Abdelmadjid Maireche*

Laboratory of Physics and Material Chemistry, Physics department, Sciences Faculty, University of M'sila-M'sila, Algeria

(Received 04 October 2016; revised manuscript received 19 December 2016; published online 23 December 2016)

In our recent work, three-dimensional modified time-independent Schrödinger equation (MSE) of modified vibrational-rotational analysis of supersingular plus quadratic potential (v.r.a.s.q.) potential was solved using Boopp's shift method instead to apply star product, in the framework of both noncommutativity three dimensional real space and phase (NC: 3D-RSP). Furthermore, the exact correction for ground state and first excited state are found straightforwardly for interactions in one-electron atoms has been solved using standard perturbation theory. Furthermore, the obtained corrections of energies are depended on infinitesimal parameters (Θ,χ) and $(\bar{\theta},\bar{\sigma})$ which are induced by position-position and momentum-momentum noncommutativity, respectively, in addition to the discreet atomic quantum numbers: $j=l\pm 1/2, s=\pm 1/2, l$ and m. Moreover, the usual states in ordinary quantum mechanics for vibrational-rotational analysis of supersingular plus quadratic potential are canceled and has been replaced by new degenerated $2(2l+1)$ sub-states in the extended new quantum symmetries of (NC: 3D-RSP).

Keywords: The vibrational-rotational analysis of supersingular plus quadratic potential, Noncommutative space and phase, Star product and Boopp's shift method.

DOI: 10.21272/jnep.8(4(2)).04076

PACS numbers: 03.65.Ge, 03.65. – w, 03.65.Ca

1. INTRODUCTION

Recently, one of the interesting problems of the nonrelativistic and nonrelativistic quantum mechanics is to find exact solutions to the Dirac, Klein-Gordon and Schrödinger equations for certain potentials of the physical interest. It is known that the nonrelativistic symmetries of the Schrödinger equation and Schrödinger-like equation, for certain shape of potentials, had been discovered many decades ago and the exact solutions of the wave equation and bound states in the case of ordinary commutative space with central and non-central potentials are very important for describing atoms, nuclei, various methods have been applied to solve the ordinary Schrödinger equation by means of asymptotic iteration method, improved AIM, Laplace integral transform, factorization method, proper quantization rule and exact quantization rule, Nikiforov-Uvarov method, supersymmetry quantum mechanics in two, three and D-dimensional spaces [1-32]. The ordinary nonrelativistic quantum mechanics based to the ordinary canonical commutations relations (CCRs) in both Schrödinger (time-independent operators) and Heisenberg pictures (time dependent operators), as $(c=\hbar=1)$:

$$\begin{gathered}[x_i,p_j]=[x_i(t),p_j(t)]=i\delta_{ij}\\ [x_i,x_j]=[p_i,p_j]=[x_i(t),x_j(t)]=[p_i(t),p_j(t)]=0\end{gathered} \quad (1)$$

Where the two operators $(x_i(t),p_i(t))$ in Heisenberg picture are related to the corresponding two operators (x_i,p_i) in Schrödinger picture from the two projections relations:

$$\begin{gathered}x_i(t)=\exp(iH_{vrasq}(t-t_0))x_i\exp(-iH_{vrasq}(t-t_0))\\ p_i(t)=\exp(iH_{vrasq}(t-t_0))p_i\exp(-iH_{vrasq}(t-t_0))\end{gathered} \quad (2)$$

Here H_{vrasq} denote to the ordinary quantum Hamiltonian operator for studied potential which composed from two terms, the first one is the kinetic energy while the second is the interaction potential. Recently, much considerable effort has been expanded to obtain the solutions of Schrödinger, Dirac and Klein-Gordon equations in the extended quantum mechanics or noncommutative quantum mechanics, to search an a profound interpretation in microscopic scales, which based to new noncommutative canonical commutations relations (NNCCRs) in both Schrödinger and Heisenberg pictures, as follows [34-66]:

$$\begin{gathered}[\hat{x}_i \overset{*}{,} \hat{p}_j]=[\hat{x}_i(t)\overset{*}{,}\hat{p}_j(t)]=i\delta_{ij},[\hat{x}_i\overset{*}{,}\hat{x}_j]=\\ =[\hat{x}_i(t)\overset{*}{,}\hat{x}_j(t)]=i\theta_{ij} \\ \text{and } [\hat{p}_i\overset{*}{,}\hat{p}_j]=[\hat{p}_i(t)\overset{*}{,}\hat{p}_j(t)]=i\bar{\theta}_{ij}\end{gathered} \quad (3)$$

Where the two new operators $(\hat{x}_i(t),\hat{p}_i(t))$ in Heisenberg picture are related to the corresponding two new

* abmaireche@gmail.com

operators $(\hat{x}_i, \hat{p}_i)$ in Schrödinger picture from the two projections relations:

$$\hat{x}_i(t) = \exp(iH_{nc}(t-t_0)) * \hat{x}_i * \exp(-iH_{nc}(t-t_0)) \\ \text{and } \hat{p}_i(t) = \exp(iH_{nc}(t-t_0)) * \hat{p}_i * \exp(-iH_{nc}(t-t_0)) \quad (4)$$

Here H_{nc} denote to the new quantum Hamiltonian operator in the (NC: 3D-RSP) symmetries. The very small two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$ (compared to the energy) are elements of two antisymmetric real matrixes and $(*)$ denote to the new star product, which is generalized between two arbitrary functions $\hat{f}(\hat{x},\hat{p})$ and $\hat{g}(\hat{x},\hat{p})$ to $\hat{f}(\hat{x},\hat{p})\hat{g}(\hat{x},\hat{p}) \equiv (f*g)(x,p)$ instead of the usual product $(fg)(x,p)$ in ordinary three dimensional spaces [33-65]:

$$\hat{f}(\hat{x},\hat{p})\hat{g}(\hat{x},\hat{p}) \equiv (f*g)(x,p) \\ \equiv \left(fg - \frac{i}{2}\theta^{\mu\nu}\partial_\mu^x f \partial_\nu^x g - \frac{i}{2}\bar{\theta}^{\mu\nu}\partial_\mu^p f \partial_\nu^p g\right)(x,p)\bigg|_{(x^\mu = x^\nu, p^\mu = p^\nu)} + \quad (5) \\ + O\left(\theta^2, \bar{\theta}^2\right)$$

where $\hat{f}(\hat{x},\hat{p})$ and $\hat{g}(\hat{x},\hat{p})$ are the new function in (NC: 3D-RSP), the two covariant derivatives $(\partial_\mu^x f(x,p), \partial_\mu^p f(x,p))$ are denotes to the $\left(\frac{\partial f(x,p)}{\partial x^\mu}, \frac{\partial f(x,p)}{\partial p^\mu}\right)$, respectively, the two following terms $-\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x f(x,p)\partial_\nu^x g(x,p)$ and $-\frac{i}{2}\bar{\theta}^{\mu\nu}\partial_\mu^p f(x,p)\partial_\nu^p g(x,p)$ are induced by (space-space) and (phase-phase) noncommutativity properties, respectively, and $O\left(\theta^2, \bar{\theta}^2\right)$ stands for the second and higher order terms of θ and $\bar{\theta}$, a Boopp's shift method can be used, instead of solving any quantum systems by using directly star product procedure [34-65]:

$$\left[\hat{x}_i, \hat{x}_j\right] = \left[\hat{x}_i(t), \hat{x}_j(t)\right] = i\theta_{ij} \\ \text{and } \left[\hat{p}_i, \hat{p}_j\right] = \left[\hat{p}_i(t), \hat{p}_j(t)\right] = i\bar{\theta}_{ij} \quad (6)$$

The 6 generalized positions and momentum coordinates in the noncommutative three dimensions quantum mechanics $(\hat{x},\hat{y},\hat{z})$ and $(\hat{p}_x,\hat{p}_y,\hat{p}_z)$ are depended with corresponding 6 usual generalized positions and momentum coordinates in the usual three dimensions quantum mechanics (x,y,z) and (p_x,p_y,p_z) by the following four relations, respectively, as follows [34-53]:

$$\begin{cases} x \to \hat{x} = x - \frac{\theta_{12}}{2}p_y - \frac{\theta_{13}}{2}p_z, \\ y \to \hat{y} = y - \frac{\theta_{21}}{2}p_x - \frac{\theta_{23}}{2}p_z \\ z \to \hat{z} = z - \frac{\theta_{31}}{2}p_x - \frac{\theta_{32}}{2}p_y \end{cases} \quad (7)$$

and

$$\begin{cases} p_x \to \hat{p}_x = p_x - \frac{\bar{\theta}_{12}}{2}y - \frac{\bar{\theta}_{13}}{2}z, \\ p_y \to \hat{p}_y = p_y - \frac{\bar{\theta}_{21}}{2}x - \frac{\bar{\theta}_{23}}{2}z \\ p_z \to \hat{p}_z = p_z - \frac{\bar{\theta}_{31}}{2}x - \frac{\bar{\theta}_{32}}{2}y \end{cases} \quad (8)$$

The non-vanish 9 commutators in (NC-3D: RSP) can be determined, as follows:

$$\left[\hat{x},\hat{p}_x\right] = \left[\hat{y},\hat{p}_y\right] = \left[\hat{z},\hat{p}_z\right] = i, \\ \left[\hat{x},\hat{y}\right] = i\theta_{12}, \left[\hat{x},\hat{z}\right] = i\theta_{13}, \left[\hat{y},\hat{z}\right] = i\theta_{23} \quad (9) \\ \left[\hat{p}_x,\hat{p}_y\right] = i\bar{\theta}_{12}, \left[\hat{p}_y,\hat{p}_z\right] = i\bar{\theta}_{23}, \left[\hat{p}_x,\hat{p}_z\right] = i\bar{\theta}_{13}$$

and

$$\left[\hat{x}(t),\hat{p}_x(t)\right] = \left[\hat{y},(t),\hat{p}_y(t)\right] = \left[\hat{z}(t),\hat{p}_z(t)\right] = i, \\ \left[\hat{x}(t),\hat{y}(t)\right] = i\theta_{12}, \left[\hat{x}(t),\hat{z}(t)\right] = i\theta_{13}, \left[\hat{y}(t),\hat{z}(t)\right] = i\theta_{23} \\ \left[\hat{p}_x(t),\hat{p}_y(t)\right] = i\bar{\theta}_{12}, \left[\hat{p}_y(t),\hat{p}_z(t)\right] = \quad (10) \\ = i\bar{\theta}_{23}, \left[\hat{p}_x(t),\hat{p}_z(t)\right] = i\bar{\theta}_{13}$$

Which allow us to getting the two operators ($\hat{r}^2$ and $\hat{p}^2$) in (NC-3D: RSP), respectively, as follows [34-53]:

$$\hat{r}^2 = r^2 - \vec{\mathbf{L}}\Theta \text{ and } \hat{p}^2 = p^2 + \vec{\mathbf{L}}\bar{\bar{\mathbf{\theta}}} \quad (11)$$

Where the two couplings $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\bar{\bar{\mathbf{\theta}}}$ are given by, respectively ($\Theta_{ij} = \frac{\theta_{ij}}{2}$):

$$\mathbf{L}\Theta \equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13} \quad \text{and} \\ \vec{\mathbf{L}}\bar{\bar{\mathbf{\theta}}} \equiv L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13} \quad (12)$$

It is-well known, that, in quantum mechanics, the three components (L_x, L_y and L_z) are determined, in Cartesian coordinates:

$$L_x = -i\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right), \\ L_y = -i\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) \quad (13) \\ L_z = -i\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

The vibrational-rotational analysis of supersingular plus quadratic potential is extensively used to describe the bound state of the interaction systems, and has been applied for both classical and modern physics, it plays a basic role in chemical and molecular physics [31], the bound state solutions of the non-relativistic Schrödinger equation, with the modified vibrational-rotational analysis of supersingular plus quadratic potential has not been obtained yet. This is the priority for this work. The modified vibrational-rotational analysis of supersingular plus quadratic potential $V_{nc-vrasq}\left(r,A,Z,\Theta,\bar{\theta}\right)$ used in this frame work takes the form:

$$V_{nc-vrasq}\left(r,A,Z,\Theta,\bar{\theta}\right)=\frac{A}{r^4}+r^2+\frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2m_0}+\left(1-\frac{2A}{r^6}\right)\vec{\mathbf{L}}\vec{\Theta} \quad (14)$$

We wish to discuss a model describing a Hydrogen atom or a Hydrogen-like ion interacted with modified vibrational-rotational analysis of supersingular plus quadratic potential in the extended quantum mechanics, we want to calculate of energy levels of above potential in (NC: 3D-RSP) symmetries using the generalization Boopp's shift method based on mentioned formalisms on above equations to discover the new symmetries and a possibility to obtain another applications to this potential in different fields. It is worth to mention that, the noncommutative idea was introduced firstly by H. Snyder [33]. In the recent years, the problem of finding exact solutions of the non-relativistic modified Schrödinger equation in noncommutative spaces and phases for a number of special potential has been a line of great interest [34-66]. This paper has been divided into six sections: In next section, we briefly review the Schrödinger equation with vibrational-rotational analysis of supersingular plus quadratic potential on based to ref. [31]. The Section 3, devoted to studying the three deformed Schrödinger equation by applying both Boopp's shift method to the vibrational-rotational analysis of supersingular plus quadratic potential. In the fourth section, by applying standard perturbation theory we find the quantum spectrum of the lowest excited states in (NC-3D: RSP) for spin-orbital interaction corresponding the ground state and first excited state. In the next section, we derive the magnetic spectrum for studied potential. In the fifth section, we resume the global spectrum and corresponding noncommutative Hamiltonian for vibrational-rotational analysis of supersingular plus quadratic potential. The concluding remarks are given in section 6.

2. REVIEW THE EIGNENFUNCTIONS AND THE ENERGY EIGENVALUES FOR VIBRATIONAL-ROTATIONAL ANALYSIS OF SUPERSINGULAR PLUS QUADRATIC POTENTIAL IN ORDINARY THREE DIMENSIONAL SPACES

Here we will firstly present the shortcuts, which give the solutions of time independent Schrödinger equation for a fermionic particle like electron of rest mass m_0 and its energy E moving in (v.r.a.s.q.) potential [31]:

$$V\left(r\right)=\frac{A}{r^4}+r^2 \quad (15)$$

where A play the role of positive constant coefficient. The (v.r.a.s.q.) potential plays a basic role in chemical and molecular physics since it can be used to calculate the molecular vibration-rotation energy spectrum of linear and non-linear systems. The above potential is the sum of quadratic $\left(r^2\right)$ and vibrational-rotational structure of supersingular potential $\left(\frac{A}{r^4}\right)$, if we insert this potential into the non-relativistic Schrödinger equation; we obtain the following equation in three dimensional spaces as follows [32]:

$$\left\{\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)+\frac{l(l+1)}{r^2}-\frac{A}{r^4}+r^2\right\}\varphi_{n\lambda}\left(r\right)=\mathrm{E}^0_{n\lambda}\varphi_{n\lambda}\left(r\right) \quad (16)$$

here $\varphi_{n\lambda}\left(r\right)$ is the radial function. As it is montionated in ref. [31], the solution of above second order differential equation in the coordinate basis turns out that there is a family of solutions;

$$\varphi_{n\lambda}\left(r\right)=N_{n\lambda}e^{-\omega r^2/2}r^{\lambda}M\left(-n,\lambda+3/2;\omega r^2\right) \quad (17)$$

Where $n=n_r+l+1$ is the principal quantum number, λ is the centrifugal parameter which defined from the relation $\lambda\left(\lambda+1\right)=l\left(l+1\right)+A$, the harmonic variational parameter $\omega=\sqrt{\dfrac{12\lambda^2+8\lambda+1}{8\lambda^2+4\lambda+4l\left(l+1\right)+1}}$ while $M\left(-n,\lambda+3/2;\omega r^2\right)$ is the confluent hypergeometric functions reducing to a polynomial of degree $n=0,1,2,...$ respectively. The normalized generalized eigenfunctions $\Psi\left(r,\theta,\phi\right)$ expressed in terms of the radial functions and spherical harmonic functions read as [31]:

$$\Psi\left(r,\theta,\phi\right)=N_{n\lambda}e^{-\omega^2/2}r^{\lambda}M\left(-n,\lambda+3/2;\omega r^2\right)Y^l_l\left(\theta,\phi\right) \quad (18)$$

here $N_{n\lambda}$ is a normalized constant and the corresponding eigenvalues $\mathrm{E}^0_{n\lambda}$ is determined from relation [31]:

$$\mathrm{E}^0_{n\lambda}=\left(4n+2\lambda+3\right)\omega \quad (19)$$

3. THEORETICAL FRAMEWORK

This section is devoted to review the main formalism of non relativistic (MSE) for modified (v.r.a.s.q.) potential in (NC-3D: RSP) symmetries; to achieve this subject, we apply the essentials following steps [34-55]:

1. Ordinary three dimensional Hamiltonian operator $\hat{H}_{nc-vrasq}\left(p_i,x_i\right)$ will be replace by new three dimensional Hamiltonian operator $\hat{H}_{nc-vraq}\left(\hat{p}_i,\hat{x}_i\right)$, in (NC-3D: RSP),

2. Ordinary complex wave function $\Psi\left(\vec{r}\right)$ will be replacing by new complex wave function $\hat{\Psi}\left(\vec{\hat{r}}\right)$,

3. Ordinary energies $\mathrm{E}^0_{n\lambda}$, in three dimensional spaces will be replace by new values $\mathrm{E}_{nc-vrasq}$, in (NC-3D: RSP) symmetries.

4. And the last step corresponds to replace the ordinary old product by new star product $(*)$, which allow us to constructing the modified three dimensional Schrödinger equation in (NC-3D: RSP) symmetries for modified (v.r.a.s.q.) potential:

$$\hat{H}_{nc-vraq}(\hat{p}_i,\hat{x}_i)*\hat{\Psi}(\vec{\hat{r}}) = \mathrm{E}_{nc-vraq}\hat{\Psi}(\vec{\hat{r}}) \quad (20)$$

The Boopp's shift method allow us to rewrite the Schrödinger equation in the following form for modified (v.r.a.s.q.) potential in (NC-3D: RSP) symmetries

$$H_{nc-vraq}(\hat{p}_i,\hat{x}_i)\Psi(\vec{r}) = \mathrm{E}_{nc-vraq}\Psi(\vec{r}) \quad (21)$$

Where the new operator of Hamiltonian $H_{nc-vrasq}(\hat{p}_i,\hat{x}_i)$ can be expressed in three general varieties: both noncommutative space and noncommutative phase (NC-3D: RSP), only noncommutative space (NC-3D: RS) and only noncommutative phase (NC: 3D-RP) as, respectively:

$$H_{nc-vrasq}(\hat{p}_i,\hat{x}_i) \equiv H\left(\hat{p}_i = p_i - \frac{1}{2}\bar{\theta}_{ij}x_j; \hat{x}_i = x_i - \frac{1}{2}\theta_{ij}p_j\right) \quad (22)$$

for NC-3D: RSP

$$H_{nc-vrasq}(\hat{p}_i,\hat{x}_i) \equiv H\left(\hat{p}_i = p_i; x_j; \hat{x}_i = x_i - \frac{1}{2}\theta_{ij}p_j\right) \quad (23)$$

for NC-3D: RS

$$H_{nc-vrasq}(\hat{p}_i,\hat{x}_i) \equiv H\left(\hat{p}_i = p_i - \frac{1}{2}\bar{\theta}_{ij}x_j; \hat{x}_i = x_i\right) \quad (24)$$

for NC-3D: RP

In our recent work, we are interest with the first variety which presented by eq. (22), after straightforward calculations, we can obtain the five important terms, which will be use to determine the (v.r.a.s.q.) potential in (NC: 3D-RSP), as:

$$\frac{A}{\hat{r}^4} = \frac{A}{r^4} - \frac{2A\vec{L}\vec{\Theta}}{r^6} \quad \text{and} \quad \frac{\hat{p}^2}{2m_0} = \frac{p^2}{2m_0} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2m_0} \quad (25)$$

Which allow us to obtaining the global potential operator $H_{nc-vrasa}(\hat{p}_i,\hat{x}_i)(\hat{p}_i,\hat{x}_i)$ for modified (v.r.a.s.q.) potential in (NC: 3D-RSP), as:

$$H_{nc-vrasq}(\hat{p}_i,\hat{x}_i)(\hat{p}_i,\hat{x}_i) = \frac{A}{r^4} + r^2 + \\ + \frac{p^2}{2m_0} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2m_0} + \left(1 - \frac{2A}{r^6}\right)\vec{\mathbf{L}}\vec{\Theta} \quad (26)$$

It's clearly, that the three first terms are given the ordinary inverse-square potential and kinetic energy in three dimensional spaces, while the rest terms are proportional's with infinitesimals parameter $(\Theta,\bar{\theta})$, thus, we can considered as a perturbations terms, we noted by $\hat{H}_{vrasq-so-pert}(r,A,Z,\Theta,\bar{\theta})$ for (NC: 3D-RSP) symmetries as:

$$\hat{H}_{vrasq-so-pert}(r,A,Z,\Theta,\bar{\theta}) = \\ = \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2m_0} + \left(1 - \frac{2A}{r^6}\right)\vec{\mathbf{L}}\vec{\Theta} \quad (27)$$

4. THE EXACT SPIN-ORBITAL HAMILTONIAN AND THE CORRESPONDING SPECTRUM FOR MODIFIED (V.R.A.S.Q.) POTENTIAL FOR EXCITED n^{th} STATES FOR ONE-ELECTRON ATOMS IN (NC: 3D- RSP) SYMMETRIES:

4.1 The exact spin-orbital Hamiltonian for modified (v.r.a.s.q.) potential for one-electron atoms in (NC: 3D- RSP) symmetries:

In this article, we consider a fermionic particle of mass m_0, charge e and spin $\vec{S} = \overrightarrow{1/2}$ that moves in modified (v.r.a.s.q.) potential presented by eq. (26), the perturbative terms $\hat{H}_{vrasq-so-pert}(r,A,Z,\Theta,\bar{\theta})$ can be rewritten to the equivalent physical form:

$$\hat{H}_{vrasq-so-pert}(r,A,Z,\Theta,\bar{\theta}) = \left\{\frac{\bar{\theta}}{2m_0} + \Theta\left(1 - \frac{2A}{r^6}\right)\right\}\vec{S}\vec{L} \quad (28)$$

We have choses the infinitesimal two vector $(\vec{\Theta},\vec{\bar{\boldsymbol{\theta}}})$ parallel to spin operator $\vec{S}$, which allow us to replace $\vec{\mathbf{L}}\Theta$ by spin –orbital coupling $\vec{S}\vec{L}$, however, the local equivalent potential $\hat{H}_{vrasq-so-pert}(r,A,Z,\Theta,\bar{\theta})$ can be rewritten to the following new equivalent form for modified (v.r.a.s.q.) potential:

$$\hat{H}_{vrasq-so-pert}(r,A,Z,\Theta,\bar{\theta}) = \\ = \frac{1}{2}\left\{\frac{\bar{\theta}}{2m_0} + \Theta\left(1 - \frac{2A}{r^6}\right)\right\}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right) \quad (29)$$

To the best of our knowledge, we just replace the coupling spin-orbital $\vec{S}\vec{L}$ by the expression $\frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$, in quantum mechanics. The set ($H_{nc-vrasq}(\hat{p}_i,\hat{x}_i)(\hat{p}_i,\hat{x}_i)$, J^2, L^2, S^2 and J_z) forms a complete of conserved physics quantities and the eigenvalues of the spin orbital coupling operator are:

$$p_{\pm}(j = l \pm 1/2, l, s = 1/2) \equiv \\ \equiv \frac{1}{2}\begin{cases} \left(l + \frac{1}{2}\right)(l + \frac{1}{2} + 1) + l(l+1) - \frac{3}{4} \\ \equiv p_+ \quad \text{for} \quad j = l + \frac{1}{2} \Rightarrow \text{polarization} - \text{up} \\ \left(l - \frac{1}{2}\right)(l - \frac{1}{2} + 1) + l(l+1) - \frac{3}{4} \\ \equiv p_- \quad \text{for} \quad j = l + \frac{1}{2} \Rightarrow \text{polarization} - \text{down} \end{cases} \quad (30)$$

Which allows us to form a diagonal (3×3) matrixes, with non null elements are $[(\hat{H}_{so-vrasq})_{11}, (\hat{H}_{so-vrasq})_{22}, (\hat{H}_{so-vraq})_{33}]$ for (v.r.a.s.q.) potential in (NC: 3D-RSP) symmetries, as:

$$\left(\hat{H}_{so-vrasq}\right)_{11} = p_{+}\left\{\frac{\bar{\theta}}{2m_0} + \Theta\left(1 - \frac{2A}{r^6} - \right)\right\}$$

if $j = l + \frac{1}{2} \Rightarrow$ spin up

$$\left(\hat{H}_{so-vrasq}\right)_{22} = p_{-}\left\{\frac{\bar{\theta}}{2m_0} - \Theta\left(1 - \frac{2A}{r^6}\right)\right\} \tag{31}$$

if $j = l - \frac{1}{2} \Rightarrow$ spin down

$$\left(\hat{H}_{so-vrasq}\right)_{33} = 0$$

Substituting eq. (28) into eq. (21) and then, the radial part of the (MSE), satisfying the following important equation:

$$\left\{\begin{array}{l}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right) + \frac{l(l+1)}{r^2} - \frac{A}{r^4} + r^2 \\ -\left\{\frac{\bar{\theta}}{2m_0} + \Theta\left(1 - \frac{2A}{r^6}\right)\right\}\vec{S}\vec{L}\end{array}\right\}\varphi_{n\lambda}(r) = \tag{32}$$

$$= \mathrm{E}_{nc-vrasq}\varphi_{n\lambda}(r)$$

It is clearly that the above equation including eq. (28) which represent the perturbative terms of Hamiltonian operator and in order to find the eigenvalues for modified (v.r.a.s.q.) potential, in (NC: 3D-RSP), we must solve the modified time independent radial equation (32) by applying standard perturbation theory in next sub-section.

4.2 The exact spin-orbital spectrum for modified (v.r.a.s.q.) potential for ground state and first excited state for one-electron atoms in (NC: 3D-RSP) symmetries:

The main goal of this sub section, is to study the modifications to the energy levels ($E_{nc-per:u}(n,\Theta,\bar{\theta})$, $E_{nc-per:D}(n,\Theta,\bar{\theta})$) for spin up and spin down, respectively, at first order of parameters (Θ, $\bar{\theta}$), for excited states n^{th}, obtained by applying the standard perturbation theory, using wave function which presented by eq. (19) and the perturbative terms of Hamiltonian operator which presented by eq. (31) in (NC-3D: RSP) symmetries, as:

$$\frac{E_{nc-per:u}(n,\Theta,\bar{\theta})}{\left|N_{n_r\lambda}\right|^2} \equiv$$

$$p_{+}\int e^{-\omega r^2} r^{2\lambda}\left[M\left(-n,\lambda+3/2;\omega r^2\right)\right]^2\left\{\frac{\bar{\theta}}{2m_0} + \left(1 - \frac{2A}{r^6}\right)\right\}r^2 dr$$

$$\frac{E_{nc-per:D}(n,\Theta,\bar{\theta})}{\left|N_{n_r l}\right|^2} \equiv \tag{33}$$

$$p_{-}\int e^{-\omega r^2} r^{2\lambda}\left[M\left(-n,\lambda+3/2;\omega r^2\right)\right]^2\left\{\frac{\bar{\theta}}{2m_0} + \left(1 - \frac{2A}{r^6}\right)\right\}r^2 dr$$

As it is montionated in ref. [31] and in order to have a better understanding of the effects of the presence of the supersingular components of the potential in such systems, it is necessary to investigate the ground state and first excited states in (*NC-3D: RSP*) symmetries, to achieve this goal we search the modifications to the energy levels ($E_{nc-per:u}(n=0,\Theta,\bar{\theta})$, $E_{nc-per:D}(n=0,\Theta,\bar{\theta})$) and ($E_{nc-per:u}(n=1,\Theta,\bar{\theta})$ $E_{nc-per:D}(n=1,\Theta,\bar{\theta})$) corresponding the ground state and first excited state, eq. (33) leads to obtain the following modifications:

$$\frac{E_{nc-per:u}(n=0,\Theta,\bar{\theta})}{\left|N_{0\lambda}\right|^2} \equiv$$

$$p_{+}\left(\Theta\sum_{i=1}^{2}T_{i-3}(n=0) + \frac{\bar{\theta}}{2m_0}T_{nc-p}(n=0)\right)$$

$$\frac{E_{nc-per:D}(n=0,\Theta,\bar{\theta})}{\left|N_{0\lambda}\right|^2} \equiv \tag{34}$$

$$p_{-}\left(\Theta\sum_{i=1}^{2}T_{i-3}(n=0) + \frac{\bar{\theta}}{2m_0}T_{nc-p}(n=0)\right)$$

and

$$\frac{E_{nc-per:u}(n=1,\Theta,\bar{\theta})}{\left|N_{1\lambda}\right|^2} \equiv$$

$$p_{+}\left(\Theta\sum_{i=1}^{2}T_{i-3}(n=1) + \frac{\bar{\theta}}{2m_0}T_{nc-p}(n=1)\right)$$

$$\frac{E_{nc-per:D}(n=1,\Theta,\bar{\theta})}{\left|N_{1\lambda}\right|^2} \equiv \tag{35}$$

$$p_{-}\left(\Theta\sum_{i=1}^{2}T_{i-3}(n=1) + \frac{\bar{\theta}}{2m_0}T_{nc-p}(n=1)\right)$$

Where, the 6- terms: ($T_{i-3}(n=0)$, $T_{i-3}(n=1)$ $i=1,2$), $T_{nc-p}(n=0)$ and $T_{nc-p}(n=1)$ are given by:

$$T_{1-3}(n=0) = -2A\int r^{2\lambda-4}e^{-\omega r^2}dr$$

$$T_{2-3}(n=0) = T_{nc-p}(n=0) = \int r^{2\lambda+2}e^{-\omega r^2}dr \tag{36}$$

and

$$T_{1-3}(n=1) = T_{1-3}(n=0) -$$

$$-4A\int r^{2\lambda-3}e^{-\omega r^2}dr - 2A\int r^{2\lambda-2}e^{-\omega r^2}dr$$

$$T_{2-3}(n=1) = T_{nc-p}(n=1) \tag{37}$$

$$= T_{nc-p}(n=0) + 2\int r^{2\lambda+3}e^{-\omega r^2}dr + \int r^{2\lambda+4}e^{-\omega r^2}dr$$

Moreover, we use the following form of special integral [67]:

$$\int_0^{+\infty} x^m \exp\left(-\beta x^n\right)dx = \frac{\Gamma\left(\frac{m+1}{n},\beta x^n\right)}{n\beta^{\frac{m+1}{n}}} \tag{38}$$

where $\Gamma\left(\frac{m+1}{n},\beta x^n\right)$ is incomplete Gamma function, after straightforward calculations, we can obtain the explicitly results:

$$T_{1-3}(n=0)=-2A\frac{\Gamma\left(\frac{2\lambda-3}{2},\omega r^2\right)}{2\omega^{\frac{2\lambda-3}{2}}}$$
$$T_{2-3}(n=0)=T_{nc-p}(n=0)=\frac{\Gamma\left(\frac{2\lambda+3}{2},\omega r^2\right)}{2\omega^{\frac{2\lambda+3}{2}}} \tag{39}$$

and

$$T_{1-3}(n_r=1)=T_{1-3}(n=0)-$$
$$4A\frac{\Gamma(\lambda-1,\omega r^2)}{2\omega^{\lambda-1}}-2A\frac{\Gamma\left(\frac{2\lambda-1}{2},\omega r^2\right)}{2\omega^{\frac{2\lambda-1}{2}}}$$
$$T_{2-3}(n_r=1)=T_{nc-p}(n_r=1)$$
$$=T_{nc-p}(n=0)+2\frac{\Gamma(\lambda+2,\omega r^2)}{2\omega^{\lambda+2}}+\frac{\Gamma\left(\frac{2\lambda+5}{2},\omega r^2\right)}{2\omega^{\frac{2\lambda+5}{2}}} \tag{40}$$

Substituting eqs. (39) and (40) into eqs. (34) and (35), respectively, lead us to the following relations:

$$E_{nc-per:u}(n=0,\Theta,\bar{\theta})\equiv|N_{0\lambda}|^2$$
$$p_+\left(\Theta T_{nc-s}(n=0)+\frac{\bar{\theta}}{2m_0}T_{nc-p}(n=0)\right)$$
$$E_{nc-per:D}(n=0,\Theta,\bar{\theta})\equiv|N_{0\lambda}|^2$$
$$p_-\left(\Theta T_{nc-s}(n=0)+\frac{\bar{\theta}}{2m_0}T_{nc-p}(n=0)\right) \tag{41}$$

and

$$E_{nc-per:u}(n=1,\Theta,\bar{\theta})\equiv|N_{1\lambda}|^2$$
$$p_+\left(\Theta T_{nc-s}(n=1)+\frac{\bar{\theta}}{2m_0}T_{nc-p}(n=1)\right)$$
$$E_{nc-per:D}(n=1,\Theta,\bar{\theta})\equiv|N_{1\lambda}|^2$$
$$p_-\left(\Theta T_{nc-s}(n=1)+\frac{\bar{\theta}}{2m_0}T_{nc-p}(n=1)\right) \tag{42}$$

Where, the two factors $T_{nc-s}(n=0)$ and $T_{nc-s}(n=1)$ are given by, respectively:

$$T_{nc-s}(n=0)\equiv T_{1-3}(n=0)+T_{2-3}(n=0)$$
$$T_{nc-s}(n=1)\equiv T_{1-3}(n=1)+T_{2-3}(n=1) \tag{43}$$

4.3 The exact magnetic spectrum for modified (v.r.a.s.q.) potential for ground state and first excited state for one-electron atoms in (NC: 3D-RSP) symmetries:

Having obtained the exact modifications to the energy levels ($E_{nc-per:u}(n=0,\Theta,\bar{\theta})$, $E_{nc-per:D}(n=0,\Theta,\bar{\theta})$) and ($E_{nc-per:u}(n=1,\Theta,\bar{\theta})$, $E_{nc-per:D}(n=1,\Theta,\bar{\theta})$) corresponding the ground states and first excited states, produced with spin-orbital induced Hamiltonians operator $\hat{H}_{vrsaq-so-pert}(r,A,Z,\Theta,\bar{\theta})$, we now consider another interested physically meaningful phenomena, which produced from the perturbative terms of inverse-square potential related to the influence of an external uniform magnetic field, it's sufficient to apply the following three replacements to describing these phenomena:

$$\frac{\vec{\mathbf{L}}\vec{\vec{\theta}}}{2m_0}+\left(1-\frac{2A}{r^6}\right)\vec{\mathbf{L}}\vec{\Theta}\rightarrow\left\{\frac{\bar{\sigma}}{2m_0}+\chi\left(1-\frac{2A}{r^6}\right)\right\}\vec{B}\vec{L} \tag{44}$$

and

$$\theta\rightarrow\chi B,\Theta\rightarrow\chi B\quad\text{and }\bar{\theta}\rightarrow\bar{\sigma}B \tag{45}$$

Here χ and $\bar{\sigma}$ are infinitesimal real proportional's constants, and we choose the uniform magnetic field parallel to the (Oz) axes, which allow us to introduce the modified new magnetic Hamiltonians $\hat{H}_{vrsaq-m-pert}(r,A,Z,\chi,\bar{\sigma})$ in (NC: 3D-RSP) symmetries, as:

$$\hat{H}_{vrsaq-m-pert}(r,A,Z,\chi,\bar{\sigma})=$$
$$=\left(\frac{\bar{\sigma}}{2m_0}+\chi\left(1-\frac{2A}{r^6}\right)\right)\left(\vec{B}\vec{J}-\vec{S}\vec{B}\right) \tag{46}$$

Here $\left(\vec{B}\vec{J}-\vec{S}\vec{B}\right)$ is the new modified Hamiltonian of Zeeman Effect and $\left(-\vec{S}\vec{B}\right)$ denote to the ordinary Hamiltonian of Zeeman Effect in commutative space. To obtain the exact noncommutative magnetic modifications of energy $E_{nc-per:m}(n=0,\Theta,\bar{\theta})$ and $E_{nc-per:m}(n=1,\Theta,\bar{\theta})$ corresponding the ground states and first excited states, produced with spin-orbital induced Hamiltonians operator $\hat{H}_{vrsaq-m-pert}(r,A,Z,\Theta,\bar{\theta})$ for modified (v.r.a.s.q.) potential, we make the following three simultaneously replacements, this to avoid repetition in the previous calculations:

$$p_+(p_-)\rightarrow m,(\theta,\Theta)\rightarrow(\chi,\chi)\quad\text{and}\quad\bar{\theta}\rightarrow\bar{\sigma}B \tag{47}$$

Into two eqs. (41) and (42) to obtain the new modifications values $E_{nc-per:m}(n=0,\chi,\bar{\sigma})$ and $E_{nc-per:m}(n=1,\chi,\bar{\sigma})$, respectively, as:

$$E_{nc-per:m}(n=0,\chi,\bar{\sigma})\equiv|N_{0l}|^2$$
$$m\left(\chi T_{nc-s}(n=0)+\frac{\bar{\sigma}}{2m_0}T_{nc-p}(n=0)\right)$$
$$E_{nc-per:m}(n=1,\chi,\bar{\sigma})\equiv|N_{1l}|^2$$
$$p_+\left(\chi T_{nc-s}(n=1)+\frac{\bar{\sigma}}{2m_0}T_{nc-p}(n=1)\right) \tag{48}$$

It is known that the angular momentum quantum number m can be takes $(2l+1)$ values and satisfying $-l \le m \le +l$.

5. RESULTS OF EXACT MODIFIED GLOBAL SPECTRUM OF THE LOWEST EXCITATIONS STATES FOR MODIFIED (V.R.A.S.Q.) POTENTIAL FOR ONE-ELECTRON ATOMS IN (NC:3D- RSP) SYMMETRIES:

It is useful to resume the energies levels ($E_{nc-vrasq:u}\left(n=0,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$, $E_{nc-vrasq:D}\left(n=0,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$) and ($E_{nc-vrasq:u}\left(n=1,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$, $E_{nc-vrasq:D}\left(n=1,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$) of the (MSE) of a fermionic particle with spin up and spin down for the ground state and first excited state, respectively, for modified (v.r.a.s.q.) potential on based to the obtained new results (41), (42) and (48), in addition to the original results (19) of energies corresponding ordinary commutative space, we obtain the detailed energy behaviours of the system as:

$$E_{nc-vrasq:u}\left(n=0,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)=\mathrm{E}_{0\lambda}^{0}+ +\left|N_{0\lambda}\right|^2\begin{pmatrix}\left(p_+\Theta+m\chi\right)T_{nc-s}\left(n=0\right)+\\+\dfrac{1}{2m_0}\left(\bar{\theta}p_+\Theta+\bar{\sigma}\chi\right)T_{nc-p}\left(n=0\right)\end{pmatrix} \quad (49)$$

$$E_{nc-vrasq:D}\left(n=0,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)=\mathrm{E}_{0\lambda}^{0}+ +\left|N_{0\lambda}\right|^2\begin{pmatrix}\left(p_-\Theta+m\chi\right)T_{nc-s}\left(n=0\right)\\+\dfrac{1}{2m_0}\left(\bar{\theta}p_-\Theta+\bar{\sigma}\chi\right)T_{nc-p}\left(n=0\right)\end{pmatrix} \quad (50)$$

$$E_{nc-vrasq:u}\left(n=1,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)=\mathrm{E}_{1\lambda}^{0}+ +\left|N_{1\lambda}\right|^2\begin{pmatrix}\left(p_+\Theta+m\chi\right)T_{nc-s}\left(n=1\right)\\+\dfrac{1}{2m_0}\left(\bar{\theta}p_+\Theta+\bar{\sigma}\chi\right)T_{nc-p}\left(n=1\right)\end{pmatrix} \quad (51)$$

$$E_{nc-vrasq:D}\left(n=1,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)=\mathrm{E}_{1\lambda}^{0}+ +\left|N_{1l}\right|^2\begin{pmatrix}\left(p_-\Theta+m\chi\right)T_{nc-s}\left(n=1\right)\\+\dfrac{1}{2m_0}\left(\bar{\theta}p_-\Theta+\bar{\sigma}\chi\right)T_{nc-p}\left(n=1\right)\end{pmatrix} \quad (52)$$

Where $\mathrm{E}_{0\lambda}^{0}$ and $\mathrm{E}_{1\lambda}^{0}$ are given by, respectively:

$$\begin{aligned}\mathrm{E}_{0\lambda}^{0}&=\left(2\lambda+3\right)\omega\\ \mathrm{E}_{1\lambda}^{0}&=\left(2\lambda+7\right)\omega\end{aligned} \quad (53)$$

In this way, one can obtain the complete energy spectra for (v.r.a.s.q.) potential in (NC: 3D-RSP) symmetries. Know the following accompanying constraint relations:

1. The original spectrum contain two possible values of energies in ordinary two–three dimensional space which presented by eq. (19),
2. The quantum number m satisfied the interval: $-l \le m \le +l$, thus we have $(2l+1)$ values for this quantum number,
3. It is known that for a fermionic particle with spin $s=1/2$ we have also two values for global momentum $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$ corresponding spin up and spin down, respectively.

Allow us to deduce the important original results: every state in usually three dimensional space will be replace by $2\left(2l+1\right)$ sub-states and then the degenerated state can be take $2\sum_{i=0}^{n-1}\left(2l+1\right)\equiv 2n^2$ values in (NC: 3D-RSP) symmetries . It's clearly, that the obtained eigenvalues of energies are real and then the noncommutative diagonal Hamiltonian operators $\hat{H}_{nc-vrsacq}\left(n_r,A,Z,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$ are Hermitian, furthermore it's possible to writing the elements $\left(\hat{H}_{nc-vrsacq}\right)_{11}$, $\left(\hat{H}_{nc-vrsacq}\right)_{22}$ and $\left(\hat{H}_{nc-vrsacq}\right)_{33}$ as follows:

$$\left(\hat{H}_{nc-vrasaq}\right)_{11}=\underbrace{-\frac{1}{2m_0}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\right]}_{1\text{(the kinetic -energy)}}+ + \underbrace{\frac{A}{r^4}+r^2}_{2\text{((v.r.s.a.q) potential)}} + \underbrace{\mathrm{p}_+\left[\frac{\bar{\theta}}{2m_0}-\Theta\left(1-\frac{2A}{r^6}\right)\right]}_{3\text{(sPerturbative spin-orbital terms)}} + \underbrace{\left\{\frac{\bar{\sigma}}{2m_0}-\chi\left(1-\frac{2A}{r^6}\right)\right\}\overline{B}\overline{L}}_{4\text{(modified Zeeman Effect)}}$$

for $\mathrm{j}=\ell+1/2 \Rightarrow$ spin up

(54)

$$\left(\hat{H}_{nc-vrscaq}\right)_{22}=\underbrace{-\frac{1}{2m_0}\left[\begin{matrix}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\\+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\end{matrix}\right]}_{1\text{(the kinetic -energy)}} + \underbrace{\frac{A}{r^4}+r^2}_{2\text{((v.r.s.a.q) potential)}} \underbrace{\mathrm{p}_-\left[\frac{\bar{\theta}}{2m_0}-\Theta\left(1-\frac{2A}{r^6}\right)\right]}_{3\text{(Perturbative spin-orbital terms)}} + \underbrace{\left\{\frac{\bar{\sigma}}{2m_0}-\chi\left(1-\frac{2A}{r^6}\right)\right\}\overline{B}\overline{L}}_{4\text{(modified Zeeman Effect)}}$$

for $\mathrm{j}=\ell-1/2 \Rightarrow$ spin down

(55)

$$\left(\hat{H}_{nc-vrscaq}\right)_{33}=\underbrace{-\frac{1}{2m_0}\left[\begin{matrix}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)+\\+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2(\sin\theta)^2}\frac{\partial^2}{\partial\phi^2}\end{matrix}\right]}_{1\text{(the kinetic -energy)}} + \underbrace{\frac{A}{r^4}+r^2}_{2\text{((v.r.s.c) potential)}}$$

(56)

The first term in the modified Hamiltonian operator $\hat{H}_{nc-vrsaq}\left(n_r,A,Z,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$ represents the kinetic energy $\hat{H}_{0vrsaq}$ of the fermionic particle or the free Hamiltonian, the second term represents the potential energy $\hat{H}_{vrsaq-\mathrm{int}}I_{3*3}$ in ordinary quantum mechanics, the third term $\hat{H}_{vrasq-so-pert}\left(r,A,Z,\Theta,\bar{\theta}\right)$ represents the induced spin-orbital parts and the last tem $\hat{H}_{vrasq-m-pert}\left(r,A,Z,\chi,\bar{\sigma}\right)$ is induced automatically by external uniform magnetic field, the last two terms have been produced automatically from the position-position and momentum-momentum noncommutativity properties. On the other hand, the above obtain results allow us to constructing the diagonal anisotropic matrixes $\left[\left(\hat{H}_{nc-vrsacq}\right)_{11} \neq \left(\hat{H}_{nc-vrsacq}\right)_{22}\right] \neq \left(\hat{H}_{nc-vrsacq}\right)_{22}$ of the Hamiltonian operator $\hat{H}_{nc-vrsaq}\left(n_r,A,Z,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$ for modified (v.r.a.s.q.) potential in (NC: 3D-RSP) symmetries is given below:

$$\hat{H}_{nc-vrsaq}\left(n_r,A,Z,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)=\hat{H}_{0vvrsaq}I_{3*3}+\hat{H}_{vrasq-\text{int}}I_{3*3}+ +\hat{H}_{vrsaq-so-pert}\left(r,A,Z,\Theta,\bar{\theta}\right)+\hat{H}_{vrsaq-m-pert}\left(r,A,Z,\chi,\bar{\sigma}\right) \quad (57)$$

Which allows us to obtain the original results for this investigation: the obtained Hamiltonian operator $\hat{H}_{nc-vrsaq}\left(n_r,A,Z,\Theta,\bar{\theta},\chi,\bar{\sigma}\right)$ which determined by eq. (57) Can be describing atom which has two permanent dipoles: the first is electric dipole moment and the second is magnetic moment in external stationary electromagnetic field.

CONCLUSION

In this work, we have solved the (MSE) for the modified vibrational-rotational analysis of supersingular plus quadratic potential in the framework of Boopp's shift method and standard perturbation theory methods in spherical coordinates. It is found that the modified energy eigenvalues depend on the dimensionality of the problem and new atomic quantum numbers ($j=l\pm 1/2, s=\pm 1/2, l$ and the angular momentum quantum number in addition to four infinitesimals parameters ($\Theta,\bar{\theta},\chi,\bar{\sigma}$) in the symmetries of (NC: 3D-RSP). Furthermore, we found that the obtained new energy spectra degenerate and every old state in ordinary quantum mechanics will be replaced by $2(2l+1)$ sub-states and the degenerated state can be taking $2n^2$ values. Consequently, this research has thrown up many questions in need of further profound investigation and will serve as a base for future profound studies.

AKNOWLEDGEMENTS

This work was supported with search laboratory of: Physique et Chimie des matériaux, in University of M'sila, Algeria.

REFERENCES

1. S.-H. Dong, G.-H. Sun, *Physica Scripta* **70,** № 2-3, 94 (2004).
2. J.C. Pekeris, *Phys. Rev.* **45** No 2, 98 (1934).
3. L. Buragohain, S.A.S. Ahmed, *Lat. Am. J. Phys. Educ.* **4**, No 1, 79 (2010).
4. A. Niknam, A.A. Rajab, M. Solaimani, *J Theor App Phys*, **10**, 53–59 (2015)
5. S.M. Ikhdair, R. Sever, *CEJP*. **5** No 4, 516 (2007).
6. M.M. Nieto, *Am. J. Phys.* **47**, 1067 (1979).
7. S.M. Ikhdair, R. Sever, *J. Mol. Struc.-Theochem.* **806**, 155 (2007).
8. A.S. Ahmed, L. Buragohain, *Physica Scripta*. **80**, 1 (2009).
9. S.K. Bose, N. *Cimento B*. **113** 299(1996).
10. G.P. Flesses, A. Watt, *J. Phys. A: Math. Gen.* **14**, L315 (1981).
11. M. Yildiz, *Act. Phys. Polonica. A*, **123** No 1, 25 (2013).
12. S.H. Dong, *Physica Scripta* **64**, No 4, 273 (2001).
13. S.H. Dong, Z.Q. Ma, *J. Phys. A* **31**, No 49, 9855 (1998).
14. S.H. Dong, *Int J Theor Phys*. **40**, No 2, 559 (2001).
15. A. Akder, et al., *J. Theor. Appl. Phys*. **7**, 17 (2013).
16. H. Egrifes, D. Demirhan, F. Buyukkılıc, *Phys. Lett. A* **275**, No 4, 229 (2000).
17. S.-H. Dong, G.-H. San, *Found. Phys. Lett.* **16** No 4, 357 (2003).
18. C. Onate, K. Oyewumi, B. Falaye, *Few-Body Systems* **55**, No 1, 61 (2014).
19. S.M. Ikhdair, *J. Modern Phys*. **3**, No 2, 170 (2012).
20. Z.-T. Yi, Z.-N. Wu, *Act. Physica Polonica A* **116** 2, 141 (2009).
21. H. Hassanabadi, M. Hamzavi, S. Zarrinkamar, A.A. Rajabi, *Int. J. Phys. Sci.* **6** №3, 583 (2011).
22. S.-H. Dong, Z.-Q. Ma, G. Esposito, *Found. Phys. Lett.* **12** No 5, 465 (1999).
23. D. Agboola, *Act. Phys. Polonika A* **120**, 371 (2011).
24. D. Shi-Hai Dong, *Int. J. Theor. Phys.* **39**, 1119 (2000).
25. B.I. Ita, *Int. J. Recent Adv. Phys.* **2**, No 4, 25 (2013).
26. B.I. Ita, A.I. Ikeuda, *J. Atom. Mol. Phys.* **2013**, 4 (2013)
27. B.I. Ita, A.I. Ikeuba, A.N. Ikot, *Commun. Theor. Phys.* **61**, 149 (2014).
28. A. Ghoshal, Y.K. Ho, (2009), *J. Phys. B: At. Mol. Opt. Phys.* **42**, No 7, 075002 (2009).
29. S.M. Kuchin, N.V. Maksimenko, *Univ. J. Phys. Appl.* **7** No 3, 295 (2013).
30. H. Egrifes, D. Demirhan, F. Buyukkilic, *Phys. Lett. A* **275**, 229 (2000).
31. V.C. Aguilera-Navarro, E. Ley-Koo, S. Mateos-Cortés, *Int. J. Theor. Phys.* **40** No 10, 157 (2001).
32. V.C. Aguilera-Navarro, E. Ley-Koo, S. Mateos-Cortés, *Int J. Theor. Phys*, **36** No 01, 1809 (1997).
33. H. Snyder, *Phys. Rev.* **71**, 38 (1946).
34. A. Maireche, *Afr. Rev Phys*. **10**, 177 (2015).
35. A. Maireche, *Afr. Rev Phys*. **10**, 97 (2015).
36. A. Maireche, *Afr. Rev Phys*. **9**, 479 (2014).
37. A. Maireche, *Afr. Rev Phys*. **9**, 185 (2014).
38. A. Maireche, *Afr. Rev Phys*. **11**, 111 (2016).
39. A. Maireche, *Afr. Rev Phys*. **10**, 373 (2016).
40. A. Maireche, *Int. Lett. Chem. Phys. Astronomy* **56**, 1 (2015).
41. A. Maireche, *Int. Lett. Chem. Phys. Astronomy* **58**, 164 (2015).
42. A. Maireche, *Int. Lett. Chem. Phys. Astronomy* **60**, 11 (2015).
43. A. Maireche, *Int. Lett. Chem. Phys. Astronomy* **61**, 38 (2015).
44. A. Maireche, *Int. Front. Sci. Lett.* **9** 33 (2016).
45. A. Maireche, *J. Nano- Electron. Phys.* **7** No 2, 02003 (2015).
46. A. Maireche, *J. Nano- Electron. Phys.* **7** No 3, 03047 (2015).
47. A. Maireche, *J. Nano- Electron. Phys.* **7** No 4, 04021 (2015).
48. A. Maireche, *J. Nano- Electron. Phys.* **8** No 1, 01020 (2016).
49. A. Maireche, *J. Nano- Electron. Phys.* **8** No 1, 01021 (2016).
50. A. Maireche, *J. Nano- Electron. Phys.* **8** No 2, 02027 (2016).
51. A. Maireche, *J. Nano- Electron. Phys.* **8** No 2 02046 (2016).
52. A. Maireche, *Med. J. Model. Simul.* **4**, 060 (2015).
53. A. Maireche, *NanoWorld J.* **1** No 4, 122 (2015).
54. A. Maireche, *Life Sci. J.* **11** No 6, 353 (2014).
55. A. Maireche, *Lat. Am. J. Phys. Educ.* **9** No 1, 1301 (2015).
56. A.E.F. Djemei, H. Smail, *Commun. Theor. Phys.* **41**, 837

(2004).
57. S. Cai, T. Jing, *Int. J. Theor. Phys.* **49** No 8, 1699 (2010).
58. J. Lee, *J. Korean Phys. Soc.* **47** No 4, 571 (2005).
59. A. Jahan, *Brazil. J. Phys.* **37** No 4, 144 (2007).
60. A.F. Dossa, G.Y.H. Avossevou, *J. Mod. Phys.* **4**, 1400 (2013).
61. Y. Yuan, et al., *Chin. Phys. C* **34** No 5, 543 (2010).
62. J. Mamat, S. Dulat, H. Mamatabdulla, *Int. J. Theor. Phys.* **55**, 2913 (2016).
63. B. Mirza, et al., *Commun. Theor. Phys.* **55**, 405 (2011).
64. Y. Xiao, Z. Long, S. Cai, *Int. J. Theor. Phys.* **50**, 3105 (2011).
65. A.E.F. Djemaï, H. Smail, *Commun. Theor. Phys.* **41**, 837 (2004).
66. A. Al-Jamel, *J. Theor. Appl. Phys.* **5** No 1, 21 (2011).
67. I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series and Products* (7th. ed.: Elsevier: 2007).