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Short Communication

Dynamic Conduction in 2-Dimensional Conductor: Magneto-Conductivity Tensor under Rapid Oscillatory Electric Field

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The conduction mechanism of metals under rapidly oscillating electric field and static perpendicular magnetic field has been investigated within the regime $\omega \gg \frac{1}{\tau}$. The conventional Lorentz force equation has been used to calculate the conduction current density within the metal. It was found that the conductivity of the metal is anisotropic in nature. We also found that the diagonal elements of the conductivity tensor are equal while the off-diagonal elements are equal in magnitude but opposite in sign. Further it is also found that the diagonal components are imaginary and inversely varies with ω while the off-diagonal components are inversely proportional to ω^2.

Keywords: Metal, Conduction, Magneto-conductivity, Anisotropy.

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1. THEORETICAL BACKGROUND

In the free electron theory (FET) of metal it is assumed that the electrons in metals can move almost freely throughout the metal.

A conductor is characterized by a large number of free electrons moving randomly throughout the conductor almost freely as assumed by the free electron model. When an emf is applied across the conductor then an electric field is set up within the conductor and the direction of the electric field is determined by the polarity of the source of emf. Under the action of this electric field the electrons exhibit a drift motion leading to a current flowing through the conductor.

If $\vec{k}(t)$ be the wave vector of these free electrons at an instant t, then the momentum of the electrons is given by [1],

$$m\vec{v}(t) = \hbar\vec{k}(t) \tag{1}$$

When such an electron system is placed in simultaneous external electric field $\vec{E}$ and magnetic field $\vec{B}$, then electric and magnetic forces act on the electrons which is given by the Lorentz force and the equation of motion of these electrons can be expressed as [2]:

$$\vec{F} = m\frac{d\vec{v}}{dt} = \hbar\frac{d\vec{k}}{dt} = -e\left[\vec{E} + \vec{v}\times\vec{B}\right] \tag{2}$$

During this motion the electrons make collisions with themselves, with lattice defects, phonons and impurities present in the metal [2-6]. Considering these collisions the equation of motion can be expressed as:

$$\hbar\frac{d\left(\Delta\vec{k}\right)}{dt} + \hbar\frac{\Delta\vec{k}}{\tau} = \vec{F} = -e\left[\vec{E} + \vec{v}\times\vec{B}\right] \tag{3}$$

Now, $m\vec{v} = \hbar\Delta\vec{k}$, this yields,

$$\hbar\frac{d}{dt}\left(\frac{m\vec{v}}{\hbar}\right) + \frac{\hbar}{\tau}\left(\frac{m\vec{v}}{\hbar}\right) = -e\left[\vec{E} + \vec{v}\times\vec{B}\right]$$

or

$$\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = -e\left[\vec{E} + \vec{v}\times\vec{B}\right]$$

Considering the motion of the electrons to be restricted in two dimensions (2-D) X-Y plane, and the magnetic field be along z-direction, i.e., $\vec{B} = \hat{k}B_z$, then,

$$\vec{v}\times\vec{B} = \left(\hat{\imath}v_x + \hat{\jmath}v_y\right)\times\left(\hat{k}B_z\right) = -\hat{\jmath}v_xB_z + \hat{\imath}v_yB_z$$

Hence the equation of motion becomes,

$$\frac{d\vec{v}}{dt} + \frac{m}{\tau}\vec{v} = -e\left[\vec{E} + \vec{v}\times\vec{B}\right] = -e\left[\left(\hat{\imath}E_x + \hat{\jmath}E_y\right) + \left(-\hat{\jmath}v_xB_z + \hat{\imath}v_yB_z\right)\right]$$

This equation can be separated into component form as:

$$m\left(\frac{dv_x}{dt} + \frac{v_x}{\tau}\right) = -e\left[E_x + v_yB_z\right] \tag{4}$$

$$m\left(\frac{dv_y}{dt} + \frac{v_y}{\tau}\right) = -e\left[E_y - v_xB_z\right] \tag{5}$$

Considering the dynamic conduction of electrons under the condition $\omega \gg \frac{1}{\tau}$

The Eq. (4) and (5) become

$$m\left(\frac{dv_x}{dt}\right) = -e\left[E_x + v_yB_z\right] \tag{6}$$

$$m\left(\frac{dv_y}{dt}\right) = -e\left[E_y - v_xB_z\right] \tag{7}$$

Now, under dynamic conduction, $v_x = v_{x0}e^{-j\omega t}$ and $v_y = v_{y0}e^{-j\omega t}$

Hence the solution of Eq. (6) and (7) becomes:

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$$i\omega v_x = \frac{e}{m} E_x + \omega_0 v_y \quad (8)$$

$$i\omega v_y = \frac{e}{m} E_y - \omega_0 v_x \quad (9)$$

Where, $\omega_0 = \frac{eB_z}{m}$, is called the cyclotron frequency.

After some algebraic calculation we arrive at

$$v_x = \frac{1}{(\omega_0^2 - \omega^2)}\left[i\frac{e\omega}{m}E_x + \frac{e\omega_0}{m}E_y\right]$$

$$v_y = \frac{1}{(\omega^2 - \omega_0^2)}\left[\frac{e\omega_0}{m}E_x - i\frac{e\omega}{m}E_y\right]$$

The conduction current density components are then given by

$$J_x = -nev_x = \frac{-ne}{(\omega_0^2 - \omega^2)}\left[i\frac{e\omega}{m}E_x + \frac{e\omega_0}{m}E_y\right]$$

$$= \left[-\frac{ine^2\omega}{m(\omega_0^2 - \omega^2)}E_x - \frac{-ne^2\omega_0}{m(\omega_0^2 - \omega^2)}E_y\right]$$

$$J_y = -nev_y = \frac{-ne}{(\omega^2 - \omega_0^2)}\left[\frac{e\omega_0}{m}E_x - i\frac{e\omega}{m}E_y\right]$$

$$= \left[\frac{-ne^2\omega_0}{m(\omega^2 - \omega_0^2)}E_x + \frac{ine^2\omega}{m(\omega^2 - \omega_0^2)}E_y\right]$$

The above three equations can be put into a matrix form:

$$\begin{pmatrix} J_x \\ J_y \end{pmatrix} = \frac{ne^2}{m(\omega_0^2 - \omega^2)}\begin{pmatrix} -i\omega & \omega_0 \\ -\omega_0 & -i\omega \end{pmatrix}\begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

This equation suggests that the current density components are not linearly related to the electric field components.

The magneto-conductivity tensor is thus given by:

$$\overleftrightarrow{\sigma} = \frac{ne^2}{m(\omega_0^2 - \omega^2)}\begin{pmatrix} -i\omega & \omega_0 \\ -\omega_0 & -i\omega \end{pmatrix}$$

Hence the components of dynamic magneto-conductivity tensor are:

$$\sigma_{xx} = -i\frac{ne^2\omega}{m(\omega_0^2 - \omega^2)}, \quad \sigma_{xy} = \frac{ne^2\omega_0}{m(\omega_0^2 - \omega^2)},$$

$$\sigma_{yx} = -\frac{ne^2\omega_0}{m(\omega_0^2 - \omega^2)}, \quad \sigma_{yy} = -i\frac{ne^2\omega}{m(\omega_0^2 - \omega^2)}$$

Thus we found that

$$\sigma_{xx} = \sigma_{yy}$$

$$\sigma_{yx} = -\sigma_{xy}$$

Under the condition $\omega \gg \omega_0$

$$\sigma_{xx} = \sigma_{yy} = i\frac{ne^2}{m\omega} = i\frac{\omega_p^2}{4\pi\omega}$$

$$\sigma_{yx} = -\sigma_{xy} = \frac{\omega_p^2\omega_0}{4\pi\omega^2}$$

Here, $\omega_p^2 = 4\pi\frac{ne^2}{m}$, is called the plasma frequency of the free electron oscillation [2].

2. CONCLUDING REMARKS

Thus it is found that the magneto-conduction of metals in 2-D under electric and magnetic field is non-linear in nature. This nonlinearity is expressed in terms of the magneto-conductivity tensor which is very important to understand the behaviour of electron in metal in 2-D under electromagnetic field. We also found that the diagonal elements of the conductivity tensor are equal while the off-diagonal elements are equal in magnitude but opposite in sign. Further it is also found that the diagonal components are imaginary and inversely varies with ω while the off-diagonal components are inversely proportional to ω^2.

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