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Quantum Hamiltonian and Spectrum of Schrödinger Equation with Companied Harmonic Oscillator Potential and its Inverse in Both Three Dimensional Non-commutative Real Space and Phase

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In present search, we have studied the effect of the both non commutativity of three dimensional space and phase on the Schrödinger equation with companied Harmonic oscillator potential and it's inverse, know by isotopic Harmonic oscillator plus inverse quadratic (h.p.i.) potential, we shown that the Hermitian NC Hamiltonian formed anisotropic operator and described many physics phenomena's, we have also derived the exact degenerated spectrum for studied potential in the first order of two infinitesimal parameters Θ and $\bar{\theta}$ associated for noncommutative space and phase, respectively.

Keywords: The isotropic harmonic oscillator plus inverse quadratic potential and non-commutative space-phase, Boopp's shift method, Schrödinger equation.

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1. INTRODUCTION

Among the different forms of physical central potentials which appear in the operator of Hamiltonian, those received great attention the recent years in commutative and noncommutative spaces-phases at two and three dimensional spaces [1-34]. In the last seventeen years, there has been an increasing interest in noncommutative geometry both in mathematics and in physics, represent a hop to obtain new and profound interpretations at Planck's scales; it goes back to H. Snyder, who first men suggested a noncommutative structure at small length scales [17]. The new concepts of space-time, know by noncommutative spaces and phases, introduced for solved many physics problems, as a major good examples, the divergence problem of quantum field theory and unified of gravitation with standard model. the noncommutativity is introduced by many ways, the simple approach, it consider the position and momentum operators obeys to the Heisenberg commutation relation, that is similarly to quantize space-time coordinates, when the commutator $\left[x_i, x_j\right] \neq 0$, the formalism of star product, Boopp's shift method and the Seiberg-Witten map are plays a fundamental roles in this new theory. The physics idea of a noncommutative space and phase satisfied by a new mathematical product which replaces the old ordinary product known by star product, if it is applying to the fundamental commutators between coordinates and impulsions gives $(c = \hbar = 1)$ [16-34]:

$$\left[x_i, x_j\right]_* = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right]_* = i\bar{\theta}_{ij} \tag{1}$$

The parameters θ^{ij} and $\bar{\theta}_{ij}$ are an antisymmetric real matrixes, it's important to notice that, the above two fundamentals commutators are satisfied as particulars' cases from the general star product between two arbitrary functions $f(x)$ and $g(x)$ in the first order of two infinitesimal parameters θ and $\bar{\theta}_{ij}$ as follow [16-34]:

$$\begin{aligned}\delta\left(f(x) * g(x)\right) = -\frac{i}{2}\theta^{\mu\nu}\left(\partial_\mu^x f(x)\right)\left(\partial_\nu^p g(x)\right) - \\ -\frac{i}{2}\bar{\theta}^{\mu\nu}\left(\partial_\mu^p f(x)\right)\left(\partial_\nu^p g(x)\right)\end{aligned} \tag{2}$$

The provirus relation valid only in the first order of the antisymmetric parameters ($\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$) matrixes, θ_{ij} and $\bar{\theta}_{ij}$ are equals $\frac{1}{2}\varepsilon^{ijk}\theta_k$ and $\frac{1}{2}\varepsilon^{ijk}\bar{\theta}_k$, respectively and both (μ and ν) are variants from one to dimensions of the space 3. In present work, a Boopp's shift method will be used, instead of solving the (NC-3D) spaces and phases Schrödinger equation by using directly star product procedure:

$$\left[\hat{x}_i, \hat{x}_j\right] = i\theta_{ij} \text{ and } \left[\hat{p}_i, \hat{p}_j\right] = i\bar{\theta}_{ij} \tag{3}$$

The star product replaced by usual product together with a Boopp's shift [23-32]:

$$\begin{aligned}&\hat{x}_i = x_i - \frac{\theta_{ij}}{2}p_j, \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2}x_j \\ &\left[x_i, x_j\right] = 0, \left[p_i, p_j\right] = 0 \text{ and } \left[x_i, p_j\right] = i\delta_{ij}\end{aligned} \tag{4}$$

In addition to the usually uncertainty relations $\Delta x^i \Delta p^j \geq \hbar$, as an immediately consequence to provirus new two commutators for noncommutative space and phase are the generalized incertitude relations to the special coordinates and special operators of impulsions $\Delta x^i \Delta x^j \geq \frac{1}{2}\left|\theta^{ij}\right|$ and $\Delta p^i \Delta p^j \geq \frac{1}{2}\left|\bar{\theta}^{ij}\right|$. The rest of present search is organized as follows. In next

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section, we briefly review the Schrödinger equation with companied Harmonic potential and it's inverse in ordinary three dimensional space. The Section 3, reserved to derive the deformed Hamiltonians of the Schrödinger equation with companied Harmonic potential and its inverse and by applying both Boopp's shift method and stationary perturbation theory, we find the exact quantum spectrum of the lowest excitations in (NC-3D) space and phase for studied potential. Finally, the important found results and the conclusions are discussed in the four and last section.

2. THE HARMONIC OSCILLATOR POTENTIAL AND ITS INVERSE IN ORDINARY THREE DIMENSIONAL SPACES

To begin, let's give a brief review of time independent Schrödinger equation for a fermionic particle like electron of rest mass m_0 and it's energy E with companied Harmonic potential and it's inverse $\left(V(r)=ar^2+\frac{b}{r^2}\right)$, which consider a good example in a central potential, the fermionic particle with spin $(1/2)$ interacted with proton and in other hand interacted with external field produced by Harmonic oscillator and its inverse [6]:

$$\left(-\frac{\Delta}{2m_0}+ar^2+\frac{b}{r^2}\right)\Psi\left(\overleftrightarrow{r}\right)=E\Psi\left(\overleftrightarrow{r}\right) \quad (5)$$

Where Δ is the Laplacian operator and ($a=\frac{1}{2}M\omega^2, b$) are both constants. In spherical coordinates, the complete wave function $\Psi\left(\overleftrightarrow{r}\right)$ separated as follows:

$$\Psi\left(\overleftrightarrow{r}\right)=R_{nl}(r)\Upsilon(\theta,\phi) \quad (6)$$

Where the radial function $R_{nl}(r)$ satisfied the following differential equation, in 3D space respectively [6]:

$$\left[\frac{d^2}{dr^2}+\frac{2}{r}\frac{d}{dr}-\frac{l(l+12)}{r^2}+2M\left(E-ar^2-\frac{b}{r^2}\right)\right]\Psi(r)=0 \quad (7)$$

Using the following abbreviations [6]:

$$\begin{cases}\nu(\nu+1)=l(l+1)+2Mb\\ \mu^2=2Ma\\ \varepsilon^2=2ME\end{cases} \quad (8)$$

Then, a radial function satisfied the following differential equation [6]:

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{\nu(\nu+1)}{r^2}-\mu^2r^2+\varepsilon^2\right)R_{nl}(r)=0 \quad (9)$$

Winches accept the solution:

$$R_{nl}(r)=C_{nl}\exp\left(-\frac{\mu}{2}.r^2\right)r^{k-1}_{.}\ {}_1F\left(-n,k+\frac{1}{2},r^2\right) \quad (10)$$

Where the function ${}_1F\left(-n,k+\frac{1}{2},r^2\right)$ is given by [6, 35]:

$${}_1F\left(-n,k+\frac{1}{2},r^2\right)=L_{(n)}^{\left(k+\frac{1}{2}\right)}\left(r^2\right) \quad (11)$$

And C_{nl} is the normalization constant, determined from the condition:

$$\int_0^{+\infty}\left[\frac{R_{nl}(r)}{\sqrt{r}}\right]^2dr=1 \quad (12)$$

Which satisfied by applying the useful integral [6, 35]:

$$\int_0^{+\infty}r\left[L_{(n)}^{(k)}\left(r^2\right)\right]^2\exp(-r)dr=\frac{\Gamma(k+n+1)}{n!} \quad (13)$$

To obtain the normalization constant, as follows [6]:

$$C_{nl}=\sqrt{2}\mu^{\frac{1}{2}\left(k+\frac{1}{2}\right)}\left(\frac{n!}{\Gamma\left(k+n+\frac{1}{2}\right)}\right)^{\frac{1}{2}}\frac{\left(n-\frac{1}{2}+k\right)!}{n!\left(k-\frac{1}{2}\right)!} \quad (14)$$

The complete orthonormalization eignenfunctions and the energy eigenvalues respectively in three dimensional spaces [6]:

$$\begin{gathered}\Psi(r,\phi)=C_{nl}\exp\left(-\tfrac{\mu}{2}r^2\right)r^{k-1}_{.}\ {}_1F\left(-n,k+\frac{1}{2},r^2\right)\Upsilon(\theta,\phi)\\ E_{nl}=\frac{\hbar\omega}{2}\left(4n+2+\left((2l+1)^2+8Ma_2\right)^{\frac{1}{2}}\right)\end{gathered} \quad (15)$$

Where the factor k equal the values: m^2+2Mb.

3. NONCOMMUTATIVE PHASE-SPACE HAMILTONIAN FOR COMPANIED HARMONIC POTENTIAL AND ITS INVERSE

Here, we brief present, the Schrödinger equation in NC quantum mechanics, we apply the important 4 steps as [24-33]:

$$\begin{cases}\text{Ordinary}\quad\text{Hamiltonian:}\hat{H}(p_i,x_i)\rightarrow\\ \text{NC}\quad\text{Hamiltonian:}\hat{H}(\hat{p}_i,\hat{x}_i)\\ \text{Ordinary}\quad\text{complex wave function}:\Psi\left(\vec{r}\right)\rightarrow\\ \text{NC}\quad\text{complex wave function:}\widehat{\Psi}\left(\overleftrightarrow{\hat{r}}\right)\\ \text{Ordinary}\quad\text{energy:}E\rightarrow\\ \text{NC}\quad\text{energy:}E_{nc-ih}\\ \text{Ordinary}\quad\text{produch:}.\rightarrow\\ \text{New star producr}\quad\text{acting on phace and space:}*\end{cases} \quad (16)$$

Then, the Schrödinger equitation in both (NC-3D) phase and space:

$$\hat{H}\left(\hat{p}_i,\hat{x}_i\right) * \hat{\Psi}\left(\vec{r}\right) = E_{nc-ih}\hat{\Psi}\left(\vec{r}\right) \quad (17)$$

Now, we apply the Boopp's shift method on the equation (18) to obtain, the reduced Schrödinger equation:

$$H\left(\hat{p}_i,\hat{x}_i\right)\psi\left(\vec{r}\right) = E_{nc-ih}\psi\left(\vec{r}\right) \quad (18)$$

Where the two $\hat{x}_i$ and $\hat{p}_i$ operators in (NC-3D) phase and space are given by:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2}p_j \quad \text{and} \quad \hat{p}_i = p_j - \frac{\bar{\theta}_{ij}}{2}x_j \quad (19)$$

Which allow us to obtaining, in (NC-3D) space and phase, the important 6-operators: $\hat{x} \equiv \hat{x}_1$, $\hat{y} \equiv \hat{x}_2$, $\hat{z} \equiv x_2$, $\hat{p}_x \equiv p_1$, $\hat{p}_y \equiv p_2$ and $\hat{p}_z \equiv p_3$ respectively as:

$$\begin{aligned}
&\hat{x} = x - \frac{\theta_{12}}{2}p_y - \frac{\theta_{13}}{2}p_z, \quad \hat{y} = y + \frac{\theta_{12}}{2}p_x + \frac{\theta_{13}}{2}p_z \\
&\hat{z} = z + \frac{\theta_{13}}{2}p_x + \frac{\theta_{23}}{2}p_y, \hat{p}_x = p_x + \frac{\bar{\theta}_{21}}{2}y + \frac{\bar{\theta}_{31}}{2}z \\
&\hat{p}_y = p_y + \frac{\bar{\theta}_{12}}{2}y + \frac{\bar{\theta}_{23}}{2}z \text{ and } \hat{p}_z = p_z + \frac{\bar{\theta}_{13}}{2}x + \frac{\bar{\theta}_{23}}{2}y
\end{aligned} \quad (20)$$

As a direct result of the above equations, the two operators $\hat{r}^2$ and $\hat{p}^2$ in (NC-3D) spaces and phases can be written as follows

$$\begin{aligned}
&\frac{1}{r'} = \frac{1}{r} + \frac{\vec{\mathbf{L}}\vec{\Theta}}{4r^3} \\
&\hat{p}^2 = p^2 + \vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}
\end{aligned} \quad (21)$$

Where $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}$ are given by, respectively:

$$\begin{aligned}
&\mathbf{L}\Theta \equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13} \\
&\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}} \equiv L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13}
\end{aligned} \quad (22)$$

Where $\Theta \equiv \frac{\boldsymbol{\theta}}{2}$, based, on the eq. (21), to obtain, after a straightforward calculation, the three important terms, which will be use to determine the (NC-3D) spaces and phases for (h.p.i.) potential:

$$\begin{aligned}
&a\hat{r}^2 = ar^2 - a\mathbf{L}\Theta \\
&\frac{b}{\hat{r}^2} = \frac{b}{r^2} + \frac{b}{r^4}\mathbf{L}\Theta \\
&\frac{\hat{p}^2}{2m_0} = \frac{p^2}{2m_0} + \frac{\mathbf{L}\bar{\boldsymbol{\theta}}}{2m_0}
\end{aligned} \quad (23)$$

The operator of companied Harmonic potential and its inverse $V_{\text{h.p.i}}\left(\hat{r}\right)$ and NC kinetic term $\frac{\hat{p}^2}{2m_0}$ in (NC-3D) spaces and phases are determined from the projection equations:

$$\begin{cases}
V_{\text{h.p.i}}\left(\hat{r}\right) = a\hat{r}^2 + \dfrac{b}{\hat{r}^2} \\
\dfrac{\hat{p}^2}{2m_0} = -\dfrac{\Delta}{2m_0} + \dfrac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2m_0}
\end{cases} \quad (24)$$

The two terms of eq. (25) allows us to obtaining, the global potential operator $H_{\text{h.p.i}}\left(\hat{r}\right)$ for companied Harmonic potential and its inverse in both (NC-3D) phase and space as:

$$H_{\text{h.p.i}}\left(\hat{r}\right) = ar^2 + \frac{b}{r^2} + \left(\theta\left(\frac{b}{r^4} - a\right)\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2m_0}\right) \quad (25)$$

It's clearly, the two first terms are given the ordinary companied Harmonic potential and its inverse in 3D spaces, while the rest terms are proportional's with two infinitesimals parameters (Θ and $\bar{\theta}$) and then gives the terms of perturbations $H_{\text{h.p.i}}$ in NC-3D real space and phase as:

$$H_{\text{h.p.i}}\left(r\right) = 2\left(\left(\frac{b}{r^4} - a\right) + \frac{\bar{\theta}}{2m_0}\right)\overleftrightarrow{S}\overleftrightarrow{L} \quad (26)$$

We have replaced $\vec{\mathbf{L}}\vec{\Theta}$ and $\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}$ by $2\Theta\overleftrightarrow{S}\overleftrightarrow{L}$ and $2\bar{\theta}\overleftrightarrow{S}\overleftrightarrow{L}$, respectively, with $\overleftrightarrow{S} = \frac{\vec{1}}{2}$, it's possible also to replace $\left(\overleftrightarrow{S}\overleftrightarrow{L}\right)$ by $\frac{1}{2}\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)$, which allow us to write, the perturbative terms' $H_{\text{ihopiq.-}p}\left(r\right)$ as follows:

$$H_{\text{h.p.i}}\left(r\right) = \left(\Theta\left(\frac{b}{r^4} - a\right) + \frac{\bar{\theta}}{2m_0}\right)\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right) \quad (27)$$

We have replaced $\left(\overleftrightarrow{S}\overleftrightarrow{L}\right)$ by $\frac{1}{2}\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)$, as it's known, this operator traduce the coupling between spin and orbital momentum. After, a straightforward calculation, one can show that, the radial function $R_{nl}\left(r\right)$ satisfied, the differential equation:

$$\left(\begin{array}{l}
\dfrac{d^2}{dr^2} + \dfrac{1}{r}\dfrac{d}{dr} - \dfrac{m^2}{r^2} - 2M \times \\
\times\left(\begin{array}{l}
E_{NC} - ar^2 + \dfrac{b}{r^2} - \\
\left(\Theta\left(\dfrac{b}{r^4} - a\right) + \dfrac{\bar{\theta}}{2m_0}\right)\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)
\end{array}\right)
\end{array}\right)R_{nl}\left(r\right) = 0 \quad (28)$$

In Quantum mechanics, the ($\overleftrightarrow{J}^2$, $\overleftrightarrow{L}^2$, $\overleftrightarrow{S}^2$ and s_z) formed complete basis, then the operator $\left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right)$ will be gives two eigenvalues $L\left(j,l,s\right) = \frac{l}{2}$ and $L'\left(j,l,s\right) = -\frac{l+1}{2}$, corresponding $j = l + \frac{1}{2}$ (spin up) and $j = l - \frac{1}{2}$ (spin down), respectively. Then, one can form a diagonal matrix of order (3×3), with non null elements $\left(H_{(\text{h.p.i})}\right)_{11}$, $\left(H_{(\text{h.p.i})}\right)_{22}$ and $\left(H_{(\text{h.p.i})}\right)_{33}=0$ for companied Harmonic potential and its inverse in NC-3D phase and space as:

$$\begin{aligned}
&\left(H_{(\text{h.p.i})}\right)_{11}=-\frac{\Delta}{2m_0}+ar^2+\frac{b}{r^2}+\frac{l}{2}\left(\Theta\left(\frac{b}{r^4}-a\right)+\frac{\bar{\theta}}{2m_0}\right)\\
&\text{ if } j=l+\frac{1}{2}\ \Rightarrow \text{spin up}\\
&\left(H_{(\text{h.p.i})}\right)_{22}=-\frac{\Delta}{2m_0}+ar^2+\frac{b}{r^2}-\frac{l+1}{2}\left(\Theta\left(\frac{b}{r^4}-a\right)+\frac{\bar{\theta}}{2m_0}\right)\\
&\text{ if } j=l-\frac{1}{2}\ \Rightarrow \text{spin down}\\
&\left(H_{(\text{h.p.i})}\right)_{33}=-\frac{\Delta}{2m_0}+ar^2+\frac{b}{r^2}
\end{aligned}\tag{29}$$

The energies $E_{\text{nu(h.p.i.)}}$, $E_{\text{nd(h.p.i.)}}$ and $E_{\text{n(h.p.i.)}}$ of a particle fermionic with spin up , spin down and non polarised are determined, respectively, for companied Harmonic potential and its inverse in (NC-3D) phase and space as:

$$\begin{aligned}
E_{\text{nu(h.p.i.)}} &= E_{nl}+E_{\text{uh.p.i.)}}\\
E_{\text{nd(h.p.i.)}} &= E_{nl}+E_{\text{d(h.p.i.)}}\\
E_{\text{n(h.p.i.)}} &= E_{nl}
\end{aligned}\tag{30}$$

$E_{\text{u(h.p.i.)}}$ and $E_{\text{d(h.p.i.)}}$ are the modifications to the energy levels, associate with spin up and spin down, respectively, at first order of :(Θ and $\bar{\theta}$) obtained by applying the perturbation theory, as follows:

$$\begin{aligned}
E_{\text{u(h.p.i.)}} &= \frac{l}{2}\left(\Theta T_{\text{s(h.p.i.)}}+\frac{\bar{\theta}}{2m_0}T_{\text{p(h.p.i.)}}\right)\\
E_{\text{d(h.p.i.)}} &= -\frac{l+1}{2}\left(\left(\Theta T_{\text{s(h.p.i.)}}+\frac{\bar{\theta}}{2m_0}T_{\text{p(h.p.i.)}}\right)\right)
\end{aligned}\tag{31}$$

In above two equations $T_{\text{s(h.p.i.)}}$ and $T_{\text{p(h.p.i.)}}$ are given by:

$$\begin{aligned}
T_{\text{s(h.p.i.)}} &= A\int_0^{+\infty}\left[\exp\left(-\frac{\mu}{2}r^2\right)r^{k-1}\ {}_1F\left(-n,k+\frac{1}{2},r^2\right)\right]^2\left(\frac{b}{r^3}-ar\right)r^2dr\\
T_{\text{p(h.p.i.)}} &= A\int_0^{+\infty}\left[\exp\left(-\frac{\mu}{2}r^2\right)r^{k-1}\ {}_1F\left(-n,k+1,r^2\right)\right]^2 r^2dr
\end{aligned}\tag{33}$$

Where $A=\left|C_{nl}\right|^2$, the first part in eq. (33) can be equal the sum of two terms $T^1_{\text{s(h.p.i.)}}$ and $T^2_{\text{s(h.p.i.)}}$ as:

$$\begin{aligned}
T^1_{\text{s(h.p.i.)}} &= Ab\int_0^{+\infty}\exp\left(-\mu r^2\right)r^{2k-3}.\left[{}_1F\left(-n,k+\frac{1}{2},r^2\right)\right]^2dr\\
T^2_{\text{s(h.p.i.)}} &= -Aa\int_0^{+\infty}\exp\left(-\mu r^2\right)r^{2k+1}.\left[{}_1F\left(-n,k+\frac{1}{2},r^2\right)\right]^2dr
\end{aligned}\tag{34}$$

It's convent to introduce a new variable $\upsilon=\mu r^2$, then eq. (35) take the equivalent form:

$$\begin{aligned}
T^1_{\text{s(h.p.i.)}} &= \frac{bA}{2\mu^{k-1}}\int_0^{+\infty}\upsilon^{k-2}\exp(-\upsilon)\left[{}_1F\left(-n,k+\frac{1}{2},\frac{\upsilon}{\mu}\right)\right]^2d\upsilon\\
T^2_{\text{s(h.p.i.)}} &= -\frac{aA}{2\mu^{k+1}}\int_0^{+\infty}\upsilon^{k}\exp(-\upsilon)\left[{}_1F\left(-n,k+\frac{1}{2},\frac{\upsilon}{\mu}\right)\right]^2d\upsilon
\end{aligned}\tag{35}$$

Applying the following special integration [35]:

$$\begin{aligned}
&\int_0^{+\infty}\upsilon^{\alpha-1}\exp(-\upsilon)\left[{}_1F(-n,\gamma,\upsilon)\right]^2d\upsilon=\\
&=\frac{n!\Gamma(\alpha)}{\gamma(\gamma+1)\ldots(\gamma+n-1)}\\
&\left(\begin{array}{c}1+\dfrac{n(\gamma-\alpha-1)(\gamma-\alpha)}{1^2\gamma}+\\ +\dfrac{n(n-1)(\gamma-\alpha-2)(\gamma-\alpha-1)(\gamma-\alpha)(\gamma-\alpha+1)}{1^22^2\gamma(\gamma+1)}\end{array}\right)
\end{aligned}\tag{36}$$

Then, we can prove that first integral in equation (34) can be deduced from:

$$\begin{aligned}
&\int_0^{+\infty}\exp(-\upsilon)\upsilon^{k-2}\left[{}_1F\left(-n,k+\frac{1}{2},\frac{\upsilon}{\mu}\right)\right]^2d\upsilon=\\
&K\left(1+\frac{n(k+1-(k-1)-1)(k+1-(k-1))}{k+1}+\ldots\right)
\end{aligned}\tag{37}$$

Where $K=\dfrac{n!\Gamma(k-1)}{\left(k+\frac{1}{2}\right)\left(k+\frac{3}{2}\right)\ldots\left(k+n-\frac{1}{2}\right)}$, the eq. (37) gives $T^1_{\text{s(h.p.i.)}}$ as:

$$T^1_{\text{s(h.p.i.)}}=\frac{Ab}{2\mu^{k-1}}\frac{K(3n+4)}{4k+2}\tag{38}$$

And the second integral in equation (34) can be also deduced from:

$$\begin{aligned}
&\int_0^{+\infty}\upsilon^{k}\exp(-\upsilon)\left[{}_1F\left(-n,\left(k+\frac{1}{2}\right),\frac{\upsilon}{\mu}\right)\right]^2d\upsilon=\\
&=J\left(1-\frac{3n}{2k+1}\right)
\end{aligned}\tag{39}$$

Where $J=\dfrac{n!\Gamma(k+1)}{\left(k+\frac{1}{2}\right)\left(k+\frac{3}{2}\right)\ldots\left(k+n-\frac{1}{2}\right)}$, which allow us to get $T^2_{\text{s(h.p.i.)}}$ as follows:

$$T^2_{\text{s(h.p.i.)}}=-\frac{AaJ}{2\mu^{k+1}}\left(1-\frac{3n}{2k+1}\right)\tag{40}$$

The obtained factor: $T_{\text{s(h.p.i.)}}$ winches represent the roll of NC-3D space

$$T_{\text{s(h.p.i.)}}=\frac{Ab}{2\mu^{k-1}}\frac{K(3n+4)}{4k+2}-\frac{AaJ}{2\mu^{k+1}}\left(\frac{2k+13-n}{2k+1}\right)\tag{41}$$

Now, if we replace ${}_1F\left(-n,k+\frac{1}{2},r^2\right)$ by $L_{(n)}^{\left(k+\frac{1}{2}\right)}\left(r^2\right)$, the second part of the equation (31), $T_{\text{p(h.p.i.)}}$, can be written as:

$$T_{\text{p((h.p.i.)}}=A\int_0^{+\infty}\exp\left(-\mu r^2\right)r^{2k}\left[L_{(n)}^{\left(k+\frac{1}{2}\right)}\left(r^2\right)\right]^2dr. \quad (42)$$

We set $r^2=x$ and then we have:

$$T_{\text{p((h.p.i.)}}=A\int_0^{+\infty}\exp\left(-\mu x\right)x^{k-\frac{1}{2}}\left[L_n^{k+\frac{1}{2}}\left(x\right)\right]^2dx \quad (43)$$

Now, we apply the integral form [6, 35]:

$$\int_0^{+\infty}x^A\exp\left(-x\right)\left[L_B^A\left(x\right)\right]^2dx=\frac{\Gamma\left(A+B+1\right)}{B!} \quad (44)$$

Which allow us to writing (NC-3D) phase contribution $T_{\text{p(h.p.i.)}}$ as:

$$T_{\text{p(h.p.i.)}}=A\frac{\Gamma\left(k+\frac{3}{2}+n\right)}{n!} \quad (45)$$

The modification to the energy levels, associate with spin up and spin down, at first order of Θ and $\bar{\theta}$ for companied Harmonic potential and its inverse in both (NC-3D) phase and space are given by:

$$\begin{gathered}E_{\text{u(h.p.i.)}}=\frac{lA}{2}\left\{\Theta\left(\frac{b}{2\mu^{k-1}}\frac{K\left(3n+4\right)}{4k+2}-\frac{aJ}{2\mu^{k+1}}\left(\frac{2k+13-n}{2k+1}\right)\right)+\frac{\bar{\theta}}{4m_0}\frac{\Gamma\left(k+\frac{3}{2}+n\right)}{n!}\right\}\\ E_{\text{d(h.p.i.)}}=-\frac{A\left(l+1\right)}{2}\left\{\Theta\left(\frac{b}{2\mu^{k-1}}\frac{K\left(3n+4\right)}{4k+2}-\frac{aJ}{2\mu^{k+1}}\left(\frac{2k+13-n}{2k+1}\right)\right)+\frac{\bar{\theta}}{4m_0}\frac{\Gamma\left(k+\frac{3}{2}+n\right)}{n!}\right\}\end{gathered} \quad (46)$$

The first two terms in above two equations are proportional's Θ while the rest parts are proportional's to $\bar{\theta}$ then clearly determine the physics contributions of (NC-3D) space and (NC-3D) phase, respectively. We conclude, from Eqs. (15), (46) and (31) that, the total energy of electron with spin up and down $E_{\text{nu(h.p.i.)}}$, $E_{\text{nd(h.p.i.)}}$ and $E_{\text{n(h.p.i.)}}$ corresponding $\left(H_{\text{(h.p.i.)}}\right)_{11}$, $\left(H_{\text{(h.p.i.)}}\right)_{22}$ and $\left(H_{\text{(h.p.i.)}}\right)_{33}$ respectively for companied Harmonic potential and its inverse in (NC-3D) phase and space as:

$$E_{\text{nu(h.p.i.)}}=\frac{\omega}{2}\left(4n+2+\left(\left(2l+1\right)^2+8Ma_2\right)^{\frac{1}{2}}\right)+\frac{lA}{2}\left\{\Theta\left(\frac{b}{2\mu^{k-1}}\frac{K\left(3n+4\right)}{4k+2}-\frac{aJ}{2\mu^{k+1}}\left(\frac{2k+13-n}{2k+1}\right)\right)+\frac{\bar{\theta}}{4m_0}\frac{\Gamma\left(k+\frac{3}{2}+n\right)}{n!}\right\} \quad (47.1)$$

And,

$$E_{\text{ndh.p.i.)}}=\frac{\omega}{2}\left(4n+2+\left(\left(2l+1\right)^2+8Ma_2\right)^{\frac{1}{2}}\right)-\frac{\left(l+1\right)A}{2}\left\{\Theta\left(\frac{b}{2\mu^{k-1}}\frac{K\left(3n+4\right)}{4k+2}-\frac{aJ}{2\mu^{k+1}}\left(\frac{2k+13-n}{2k+1}\right)\right)+\frac{\bar{\theta}}{4m_0}\frac{\Gamma\left(k+\frac{3}{2}+n\right)}{n!}\right\} \quad (47.2)$$

And,

$$E_{\text{n(h.p.i.)}}=\frac{\omega}{2}\left(4n+2+\left(\left(2l+1\right)^2+8Ma_2\right)^{\frac{1}{2}}\right) \quad (47.3)$$

Regarding to the eq. (47), the eigenvalues of energies are real and then the NC Hamiltonian is Hermitian, for companied Harmonic potential and its inverse in (NC-3D) phase and space as:

$$H_{NC-\text{(h.p.i.)}}=H_{\text{(h.p.i.)}}+H_{so-\text{(h.p.i.)}} \quad (49)$$

Where $H_{\text{(h.p.i.)}}$ and $H_{so-\text{(h.p.i.)}}$ are determined from, the following relation, respectively:

$$\begin{gathered}H_{\text{(h.p.i.)}}=\left(-\frac{\Delta}{2m_0}+ar^2+\frac{b}{r^2}\right)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\\ H_{so-\text{(h.p.i.)}}=\left(\Theta\left(\frac{b}{r^4}-a\right)+\frac{\bar{\theta}}{2m_0}\right)\begin{pmatrix}\frac{l}{2}&0&0\\0&-\frac{l+1}{2}&0\\0&0&0\end{pmatrix}\end{gathered} \quad (50)$$

$H_{\text{(h.p.i.)}}$ Represent an electron interacted exactly with companied Harmonic potential and its inverse in commutative 3D space while the matrix $H_{so-\text{(h.p.i.)}}$ denotes to the spin-orbital interaction. Furthermore, if we apply the 4-following steps:

$$\left(\left(\Theta\left(\frac{b}{r^4}-a\right)+\frac{\bar{\theta}}{2m_0}\right)\right)L_z\rightarrow\left(\alpha\left(\frac{b}{r^4}-a\right)+\frac{\bar{\varepsilon}}{2m_0}\right)\overrightarrow{B}\overrightarrow{L} \quad (51)$$

$\Theta=\alpha B$, $\bar{\theta}=\bar{\varepsilon}B$ and $\overleftrightarrow{B}=B\overleftrightarrow{k}$

Table 1

The total energy in 3D space	NC space-spin orbit	NC phase-spin orbit
$E_{nl}=\frac{\omega}{2}\left(4n+2+\left((2l+1)^2+8Ma_2\right)^{\frac{1}{2}}\right)$	$E_{ns-(\text{h.p.i.})}=\theta l T_{s(\text{h.p.i.})}$ for spin up	$E_{np-(\text{h.p.i.})}=\frac{\bar{\theta}l}{2m_0}T_{p(\text{h.p.i.})}$ for spin up
	$E_{ns-(\text{h.p.i.})}=-\theta(l+1)T_{s(\text{h.p.i.})}$ for spin down	$E_{np-sod}=-\frac{\bar{\theta}(l+1)}{2m_0}T_{p((\text{h.p.i.})}$ for spin down

Table 2

NC space-magnetic	NC phase- magnetic	The spectrum energy in (NC-3D) phase and space
$E_{np-m}=2\theta mT_{s((\text{h.p.i.})}$	$E_{np-m}=-2\bar{\theta}mT_{s(\text{h.p.i.})}$	$E_{nl}+E_{ns-sou}+E_{np-sou}+$ $+E_{ns-m}+E_{np-m}$ for spin down
		$E_{nl}+E_{ns-sod}+E_{np-sod}+$ $+E_{ns-m}+E_{np-m}$ for spin down

Here α and $\bar{\varepsilon}$ are infinitesimal real proportional constants, the magnetic moment $\overleftrightarrow{\mu}\equiv 1/2$ and $\left(-\overleftrightarrow{S}\overleftrightarrow{B}\right)$ denote to the ordinary Hamiltonian of Zeeman Effect, we obtains the modified new Hamiltonian for companied Harmonic potential and its inverse in (NC-3D) phase and space as:

$$H_{m-(\text{h.p.i.})}=\left(\alpha\left(\frac{b}{r^4}-a\right)+\frac{\bar{\varepsilon}}{2m_0}\right)\left(\overleftrightarrow{B}\overleftrightarrow{J}-\overleftrightarrow{S}\overleftrightarrow{B}\right) \qquad (52)$$

The above operator represents two fundamentals interactions: the first one between spin and external uniform magnetic field (ordinary Zeeman Effect) while the other is a new coupling between the momentum of electron and external uniform magnetic field. Finally, we resumed obtained energy results for for companied Harmonic potential and its inverse in NC-3D phase $H_{m-(\text{h.p.i.})}$ and space is tables 1 and 2.

Where $-l\le m\le +l$, denote to quantum number of operator L_z correspond the magnetic effect and for the effect spin-orbital interaction we have seen two possible values $j=l\pm 1/2$, thus every state in usually 3D of energy for (h.p.i) potential will be in (NC-3D) phase and space: $2(2l+1)$ sub-states, this is similarly to my work [27].

4. CONCLUSION

In this work, the effect of the non commutativity was studied on the (h.p.i) potential in (NC-3D) real space and phase, we shown that the NC Hamiltonian was represented by two matrixes: the first one $H_{(\text{h.p.i.})}$described an electron interacted exactly with (h.p.i) potential in ordinary 3D space, while the matrix $H_{so-(\text{h.p.i.})}$ was represents the spin-orbital interaction. We observed in NC-3D phase, the kinetic term gives an additional correction term and the NC Hamiltonian was given another physical interpretation for the interaction between electron and an external magnetic field including ordinary Zeeman Effect $H_{m-(\text{h.p.i.})}$ and the new symmetries can be extended to include many physics interactions. Thus, the applications of companied Harmonic oscillator potential and its inverse in (NC-3D) phase and space are prolonged to be very large at height energy.

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