

A Recent Study of Quantum Atomic Spectrum of the Lowest Excitations for Schrödinger Equation with Typical Rational Spherical Potential at Planck's and Nanoscales

Abdelmadjid Maireche*

Laboratory of: Physic and Chemical Materials, Physics Department,
 Sciences Faculty, University of M'sila, M'sila, Algeria

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The exact global quantum atomic spectrum of lowest excitations states (fundamental and first excited states) for Hydrogen atom with typical rational spherical potential like singular one-fraction power (s.o.f.p.) potential were reported by using both Boopp's shift method and stationary perturbation theory, at Planck's and Nano scales, in both non commutative 2-dimensional space and phase. We have also found the deformed anisotropic Hamiltonian for studied potential.

Keywords: Hydrogen atom, Star product, Noncommutative space and phase and the rational spherical potential.

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1. INTRODUCTION

The study of bound-state solutions of the three fundamental equations: relativistic Klein-Gordon equation (for scalar bosonic particles like pions with spin zero) and (Dirac and its adjoint) equations for the electron and positron with spin 1/2 for negative and positive charge and the same mass and non-relativistic Schrödinger equation (for the electron) are played crucial roles for describing physics phenomena in gauge theory, standard model, and quantum mechanics at high and low energy, in the case commutative and non commutative spaces, respectively. The exact solutions for a Colombian and for an Harmonic oscillator as well as in an arbitrary number of spherical and non spherical potentials represent a typical models in quantum mechanics and Dirac theory, in two and three dimensions spaces, like (s.o.f.p.) potential [1-31]. In 1947, H. Snyder, who introduce the notions of an increasing interest in noncommutative geometry both in mathematics and in physics, which represent a hop to obtain new and profound interpretations at Planck's and Nano scales [12]. The rich mathematical structure of the noncommutative theory gives a rise to the hop to get a better understanding of physics phenomena at smallness distances like Planck's and Nano scales. The physical idea of a noncommutative space satisfied by a new mathematical product, we replace the ordinary product by star product and the new noncommutative operators will be functions of the old operators and its derivative. From the references [9-26], we can deduce the new star product between two functions $f(x)$ and $g(x)$ modified by ($c = \hbar = 1$):

$$\delta(f(x) * g(x)) = -\frac{i}{2} \theta^{\mu\nu} (\partial_\mu^x f(x)) (\partial_\nu^p g(x)) - \frac{i}{2} \bar{\theta}^{\mu\nu} (\partial_\mu^p f(x)) (\partial_\nu^x g(x)) \quad (1)$$

The above relation valid in the first order of the antisymmetric parameters ($\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$) matrixes, here

$\delta(f(x) * g(x)) \equiv f(x) * g(x) - f(x)g(x)$, $\mu, \nu = \overline{1, N}$ and N denote to the dimensions of the space. As an immediately consequence of the above star product is the satisfaction of the important two commutators, which are playing a crucial role in noncommutative space-phase, as:

$$[x_i, x_j]_* = i\theta_{ij} \quad \text{and} \quad [\hat{p}_i, \hat{p}_j]_* = i\bar{\theta}_{ij} \quad (2)$$

The objective of this paper is to study of the effect of the noncommutativity of space and phase on Hydrogen atom with typical rational spherical potential like (s.o.f.p.) potential, in both noncommutative two dimensional real spaces and phases (NC-2D spaces and phases). The physical importance of the (s.o.f.p.) potential appears in many quantum physics and chemical phenomena [2]. A Boopp's shift method will be used in present article, instead of solving the (NC-2D spaces and phases) Schrödinger equation by using star product procedure:

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \quad \text{and} \quad [\hat{p}_i, \hat{p}_j] = i\bar{\theta}_{ij} \quad (3)$$

The star product replaced by usual product together with a Boopp's shift [14, 15, 20-26]:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j, \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j$$

$$[x_i, x_j] = 0$$

$$[p_i, p_j] = 0 \quad (4)$$

It's clearly that, the noncommutativity applied in present work concerned both the spatial operators and the impulsions operators. The rest of present article is organized as follows: In newt section, we briefly review the Hydrogen atom in ordinary 2 D spaces with (s.o.f.p.) potential. The Section 3, reserved to derive the deformed Hamiltonians of the Hydrogen atom with (s.o.f.p.) potential and by applying the perturbation theory we find the quantum spectrum of the lowest excita-

* abmaireche@gmail.com

tions in (NC-2D spaces and phases) for (s.o.f.p.) potential. In section 4, we examine the obtained results. Finally, the important found results and the conclusions are discussed in last section.

2. THE (S.O.F.P.) POTENTIAL IN ORDINARY 2D REAL SPACE

As it's mentioned in the previously section, the (s.o.f.p.), represent a good example of a central typical rational spherical potential $V(r)$, the important of this potential appears in many quantum physics and chemical phenomena, which proportional on the inverse of two terms $r^{1/2}$ and $r^{3/2}$ as [2]:

$$V(r) = \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} \tag{5}$$

Where a and b are two both real constants characterized the nature of studied subject. It is known that the nonrelativistic Schrödinger equation describing a fermionic particle moving in (s.o.f.p.) central potential, in polar coordinates $\vec{r}(r, \phi)$, can be defined by the following equation ($c = \hbar = 1$) [2]:

$$\left(-\frac{1}{2m_0} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) + \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} \right) \Psi(\vec{r}) = E \Psi(\vec{r}) \tag{6}$$

Where m_0 and E are the ordinary mass of a fermionic particle and the energy, respectively. The complex wave function $\Psi(\vec{r})$ is written as follows:

$$\Psi(\vec{r}) = \frac{R(r)}{\sqrt{r}} \exp(\pm im\phi) \tag{7}$$

Where m denote to the eigenvalue of the operator L_z . The radial function $R(r)$ satisfied the following equation [2]:

$$\frac{d^2 R(r)}{dr^2} + \left(E - \frac{a}{r^{1/2}} - \frac{b}{r^{3/2}} - \frac{m^2 - \frac{1}{4}}{r^2} \right) R(r) = 0. \tag{8}$$

The complete complex wave function and the quantum spectrum of the lowest excitations corresponding to (stationary state: $\Psi_m^{(0)}(\vec{r})$ and $E_m^{(0)}$, first excited states: $\Psi_m^{(1)}(\vec{r})$ and $E_m^{(1)}$) and P^{th} order excited states: $\Psi_m^{(p)}(\vec{r})$ and $E_m^{(p)}$ are, respectively [2]:

$$\frac{\Psi_m^{(0)}(\vec{r})}{\exp(\pm im\phi)} = a_0 r^m \exp \left(-\sqrt{-E_m^{(0)}} r - \frac{a}{\sqrt{-E_m^{(0)}}} r^{1/2} \right)$$

$$\text{and } E_m^{(0)} = - \left[\frac{a^2}{8 \left(\frac{1}{2} + m \right)} \right]^{\frac{3}{2}} \tag{9.1}$$

$$\frac{\Psi_m^{(1)}(\vec{r})}{\exp(\pm im\phi)} = (a_0 + a_1 r^{1/2}) r^m \times \exp \left(-\sqrt{-E_m^{(1)}} r - \frac{a}{\sqrt{-E_m^{(1)}}} r^{1/2} \right) \tag{9.2}$$

$$\text{and } E_m^{(1)} = - \left[\frac{a^2}{4 \left(\frac{5}{2} + m \right)} \right]^{\frac{2}{3}}$$

and:

$$\frac{\Psi_m^{(p)}(\vec{r})}{\exp(\pm im\phi)} = (a_0 + a_1 r^{1/2} + \dots + a_p r^{p/2}) \times r^{\delta-1/2} \exp \left(-\sqrt{-E_m^{(p)}} r - \frac{a}{\sqrt{-E_m^{(p)}}} r^{1/2} \right) \tag{9.3}$$

$$\text{and } E_m^{(p)} = - \left[\frac{a^2}{4(1+p+2m)} \right]^{\frac{2}{3}}$$

3. THE (S.O.F.P.) POTENTIAL IN NC 2D SPACES

3.1 The Perturbative (S.O.F.P.) Potential

To obtain, the Schrödinger equation on NC quantum mechanics, we replace: ordinary Hamiltonian $\hat{H}(p_i, x_i)$, ordinary complex wave function $\Psi(\vec{r})$, E_{sofp} and the old product by: NC Hamiltonian $\hat{H}(\hat{p}_i, \hat{x}_i)$, NC complex wave function $\hat{\Psi}(\vec{r})$, NC energy $E_{nc-sofp}$ and star product $*$, respectively [20-26]:

$$\hat{H}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}(\vec{r}) = E_{nc-sofp} \hat{\Psi}(\vec{r}). \tag{10}$$

Know, we apply the Boopp's shift method on the equation (10) to obtain, the reduced Schrödinger equation:

$$H(\hat{p}_i, \hat{x}_i) \psi(\vec{r}) = E_{nc-sofp} \psi(\vec{r}) \tag{11}$$

Where the two operators in (NC-2D) \hat{x}_i and \hat{p}_i are given by:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \text{ and } \hat{p}_i = p_i - \frac{\bar{\theta}_{ij}}{2} x_j \tag{12}$$

Which allow us to obtaining, in (NC-2D) space and phase, the four new operators $\hat{x} \equiv \hat{x}_1$, $\hat{y} \equiv \hat{x}_2$, \hat{p}_x and \hat{p}_y respectively as:

$$\hat{x} = x - \frac{\theta}{2} p_y, \quad \hat{y} = y + \frac{\theta}{2} p_x$$

$$\hat{p}_x = p_x + \frac{\bar{\theta}}{2} p_y, \quad \hat{p}_y = p_y - \frac{\bar{\theta}}{2} p_x \tag{13}$$

Here $\theta \equiv \theta^{12}$ and $\bar{\theta} \equiv \bar{\theta}^{12}$, as a direct result of the above equations, the two operators \hat{r}^2 and \hat{p}^2 in (NC-2D spaces and phases) can be written as follows [21-24]:

$$\begin{aligned}\hat{r}^2 &= r^2 - \theta L_z \\ \hat{p}^2 &= p^2 + \bar{\theta} L_z\end{aligned}\quad (14)$$

Here ($L_z \equiv xp_y - yp_x$), furthermore, one can show that, the only none null, the two commutators $[\hat{x}, \hat{y}]$ and $[\hat{p}_x, \hat{p}_y]$ are written as follows [21-25]:

$$\begin{aligned}[\hat{x}, \hat{y}] &= i\theta \\ [\hat{p}_x, \hat{p}_y] &= -i\bar{\theta}\end{aligned}\quad (15)$$

On based, on the eq. (14), to obtain, after a straightforward calculation, the three important terms, which use to determine the (NC-2D spaces and phases) (s.o.f.p.) potential:

$$\begin{aligned}\frac{a}{\hat{r}^{1/2}} &= \frac{a}{r^{1/2}} + \frac{\theta a}{4r^{5/2}} & L_z \frac{b}{\hat{r}^{3/2}} &= \frac{b}{r^{3/2}} + \frac{\theta b}{4r^{7/2}} L_z \\ \frac{\hat{p}^2}{2m_0} &= \frac{p^2}{2m_0} + \frac{\bar{\theta}}{2m_0} L_z\end{aligned}\quad (16)$$

The operator of (s.o.f.p.) potential $V_{sofp}(\hat{r})$ and NC kinetic term $\frac{\hat{p}^2}{2m_0}$ in (NC-2D spaces and phases) are determined from the projection equation:

$$\begin{aligned}V_{sofp}(\hat{r}) &= \frac{a}{\hat{r}^{1/2}} + \frac{b}{\hat{r}^{3/2}} \\ \frac{\hat{p}^2}{2m_0} &= -\frac{1}{2m_0} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) + \frac{\bar{\theta}}{2m_0} L_z\end{aligned}\quad (17)$$

The eq. (17) allows us to obtaining, the potential operator $H_{sofp}(\hat{r})$ as:

$$H_{sofp}(\hat{r}) = \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} + \left(\frac{a\theta}{4r^{5/2}} + \frac{3b\theta}{4r^{7/2}} + \frac{\bar{\theta}}{2m_0} \right) L_z \quad (18)$$

It's clearly, the two first terms are given the ordinary (s.o.f.p.) potential, we note to the rest terms by $H_{sofp-p}(r)$:

$$H_{sofp-p}(r) = \left(\frac{a\theta}{4r^{5/2}} + \frac{3b\theta}{4r^{7/2}} + \frac{\bar{\theta}}{2m_0} \right) L_z \quad (19)$$

After a straightforward calculation, one can prove that the noncommutative modified radial function satisfied the following equation:

$$\begin{aligned}\frac{d^2 R(r)}{dr^2} &+ \left(E - \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} - \right. \\ &\left. - \left(\frac{a\theta}{4r^{5/2}} + \frac{3b\theta}{4r^{7/2}} + \frac{\bar{\theta}}{2m_0} \right) L_z - \frac{m^2 - \frac{1}{4}}{r^2} \right) R(r) = 0\end{aligned}\quad (20)$$

It's clearly that the modified potential $H_{sofp-p}(r)$ is proportional to two parameters θ and $\bar{\theta}$.

3.2 The Exact N.C. Modification of the Energy Levels for Stationary State

The exact NC modification of the energy levels, in the first order of θ and $\bar{\theta}$, for the stationary state E_{nc0} , can be determined by applying the perturbation theory and using Eqs. (9.1) and (19):

$$\begin{aligned}\frac{E_{nc0}}{2\pi\alpha_0^2 m} &= \theta \int_0^{+\infty} \exp(-2Ar - Br^{1/2}) \left(\frac{a}{4} r^{2m-3/2} \right. \\ &\left. + \frac{3b}{4} r^{2m-5/2} \right) dr + \\ &+ \frac{\bar{\theta}}{2m_0} \int_0^{+\infty} r^{2m} \exp(-2Ar - Br^{1/2}) dr\end{aligned}\quad (21)$$

Where ($A \equiv \sqrt{-E_m^{(0)}}$ and $B \equiv \frac{2a}{\sqrt{-E_m^{(0)}}}$), the above relation, reduced to the form:

$$E_{nc0} = 2\pi\alpha_0^2 m \left(\theta (L_{sofp}^1 + L_{sofp}^2) + \frac{\bar{\theta}}{2m_0} L_{sofp}^3 \right) \quad (22)$$

Where, the three terms L_{sofp}^1 , L_{sofp}^2 and L_{sofp}^3 are determined from the two relations, respectively:

$$\begin{aligned}L_{sofp}^1 &= \frac{a}{4} \int_0^{+\infty} \exp(-2Ar - Br^{1/2}) r^{2m-3/2} dr \\ L_{sofp}^2 &= \frac{3b}{4} \int_0^{+\infty} \exp(-2Ar - Br^{1/2}) r^{2m-5/2} dr \\ L_{sofp}^3 &= \int_0^{+\infty} \exp(-2Ar - Br^{1/2}) r^{2m} dr\end{aligned}\quad (23)$$

If we change the variable r by new variable X^2 , then the above two equations are reducing to the form:

$$\begin{aligned}L_{sofp}^1 &= \frac{a}{2} \int_0^{+\infty} \exp(-2AX^2 - BX) X^{(4m-1)-1} dX \\ L_{sofp}^2 &= \frac{3b}{2} \int_0^{+\infty} \exp(-2AX^2 - BX) X^{(4m-3)-1} dX \\ L_{sofp}^3 &= 2 \int_0^{+\infty} \exp(-2AX^2 - BX) X^{(m+2)-1} dX\end{aligned}\quad (24)$$

Now, using the special integral [32]:

$$\begin{aligned}\int_0^{+\infty} x^{v-1} \exp(-\beta x^2 - \gamma x) dx &= \\ &= (2\beta)^{-v/2} \Gamma(v) \exp\left(\frac{\gamma^2}{8\beta}\right) D_{-v}\left(\frac{\gamma}{\sqrt{2\beta}}\right)\end{aligned}\quad (25)$$

Where D_{-v} , is the parabolic cylinder function, as a condition ($\text{Re}l(\beta) > 0$ and $\text{Re}l(v) > 0$), $\Gamma(v)$ is gamma function, which allow us to obtaining the factors: L_{sofp}^1

, L_{sofp}^2 and L_{sofp}^3 , respectively as

$$\begin{aligned} & \frac{L_{sofp}^1}{\frac{a}{2} \exp\left(\frac{B^2}{16A}\right) \left(4\sqrt{-E_m^{(0)}}\right)^{\frac{4m-1}{2}}} = \\ & = \Gamma(4m-1) \exp\left(\frac{B^2}{16A}\right) D_{-(4m-1)}\left(\frac{a}{E_m^{(0)}}\right) \\ & \frac{L_{sofp}^2}{\frac{3b}{2} \exp\left(\frac{B^2}{16A}\right) \left(4\sqrt{-E_m^{(0)}}\right)^{\frac{4m-3}{2}}} = \\ & = \Gamma(4m-3) \exp\left(\frac{B^2}{16A}\right) D_{-2(4m-3)}\left(\frac{a}{\sqrt{2}b^{3/4}}\right) \\ & \frac{L_{sofp}^3}{\exp\left(\frac{B^2}{16A}\right) \left(4\sqrt{-E_m^{(0)}}\right)^{\frac{m+2}{2}}} = \\ & = \Gamma(m+1) D_{-(m+2)}\left(\frac{a}{\sqrt{2}b^{3/4}}\right) \end{aligned} \tag{26}$$

Then, the modified energy eigenvalues E_{nc0} can be written as:

$$E_{nc0} = 2\pi a_0^2 m F(m, a, b, \theta, \bar{\theta}) \tag{27}$$

Where the factor $F(m, a, b)$ is given by:

$$F(m, a, b, \theta, \bar{\theta}) = \left(\theta (L_{sofp}^1 + L_{sofp}^2) + \frac{\bar{\theta}}{2m_0} L_{sofp}^3 \right) \tag{28}$$

3.3 The Exact N.C. Modification of the Energy Levels for First Excited State

Furthermore, the exact NC modification of the energy levels for first excited state E_{nc1} , in the first order of θ , can be determined by using perturbation theory and Eqs. (9.2) and (19):

$$\begin{aligned} \frac{E_{nc1}}{2\pi m} &= \theta \int_0^{+\infty} \left[(a_0 + a_1 r^{1/2}) r^m \exp\left(-\sqrt{-E_m^{(1)}} r - \frac{a}{\sqrt{-E_m^{(1)}}} r^{1/2}\right) \right]^2 \times \\ & \times \left(\frac{a}{4r^{5/2}} + \frac{3b}{4r^{7/2}} \right) dr + \frac{\bar{\theta}}{2m_0} \int_0^{+\infty} (a_0 + a_1 r^{1/2})^2 r^{2m} \times \\ & \times \exp\left(-2\sqrt{-E_m^{(1)}} r - \frac{2a}{\sqrt{-E_m^{(1)}}} r^{1/2}\right) dr \end{aligned} \tag{29}$$

The above integral can be written as follows:

$$E_{nc1} = 2\pi m \left(\theta \sum_{\mu=1}^6 S_{ofp}^{\mu} + \frac{\bar{\theta}}{2m_0} \sum_{\mu=7}^9 S_{ofp}^{\mu} \right) \tag{30}$$

Where $(A' \equiv \sqrt{-E_m^{(1)}} \text{ and } B' \equiv \frac{2a}{\sqrt{-E_m^{(1)}}})$, the sixes

terms of S_{ofp}^{μ} ($\mu = \overline{1,9}$) are given by:

$$\begin{aligned} S_{ofp}^1 &= \frac{aa_0^2}{4} \int_0^{+\infty} r^{2m-5/2} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^2 &= \frac{aa_1^2}{4} \int_0^{+\infty} r^{2m-3/2} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^3 &= \frac{aa_0a_1^2}{2} \int_0^{+\infty} r r^{2m-4/2} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^4 &= \frac{3ba_0^2}{4} \int_0^{+\infty} r^{2m-7/2} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^5 &= \frac{3ba_1^2}{4} \int_0^{+\infty} r^{2m-5/2} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^6 &= \frac{3a_0a_1b}{2} \frac{3a_0a_1b}{2} \int_0^{+\infty} r^{2m-6/2} \exp(-2A'r - B'r^{1/2}) dr \end{aligned} \tag{31.1}$$

and

$$\begin{aligned} S_{ofp}^7 &= a_0^2 \int_0^{+\infty} r^{2m} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^8 &= a_1^2 \int_0^{+\infty} r^{2m+1} \exp(-2A'r - B'r^{1/2}) dr \\ S_{ofp}^9 &= 2a_0a_1 \int_0^{+\infty} r^{2m+\frac{1}{2}} \exp(-2A'r - B'r^{1/2}) dr \end{aligned} \tag{31.2}$$

similarly to previous integrals, we introduce also, new variable $r = X^2$. After a straightforward calculation we can obtain:

$$\begin{aligned} S_{ofp}^1 &= \frac{aa_0^2}{2} \int_0^{+\infty} X^{(4m-3)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^2 &= \frac{aa_1^2}{2} \int_0^{+\infty} X^{(4m-1)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^3 &= aa_0a_1 \int_0^{+\infty} X^{(4m-2)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^4 &= \frac{3ba_0^2}{2} \int_0^{+\infty} X^{(4m-5)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^5 &= \frac{3ba_1^2}{2} \int_0^{+\infty} X^{(4m-3)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^6 &= 3a_0a_1b \int_0^{+\infty} X^{(4m-4)-1} \exp(-2A'X^2 - B'X) dX \end{aligned} \tag{32.1}$$

and

$$\begin{aligned} S_{ofp}^7 &= 2a_0^2 \int_0^{+\infty} X^{(4m+2)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^8 &= 2a_1^2 \int_0^{+\infty} X^{(4m+4)-1} \exp(-2A'X^2 - B'X) dX \\ S_{ofp}^9 &= 4a_0a_1 \int_0^{+\infty} X^{(4m+3)-1} \exp(-2A'X^2 - B'X) dX \end{aligned} \tag{32.2}$$

By applying the special integral represented by eq. (25), we obtain the exact values of S_{ofp}^{μ} , as follows:

$$\begin{aligned} \frac{S_{ofp}^1}{\frac{\alpha\alpha_0^2}{2}} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m-3}{2}} \Gamma(4m-3) \times \\ &\times \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m-3)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \\ \frac{S_{ofp}^2}{\frac{\alpha\alpha_0^2}{2}} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m-1}{2}} \Gamma(4m-1) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m-1)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \\ \frac{S_{ofp}^3}{\alpha\alpha_0\alpha_1} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m-2}{2}} \Gamma(4m-2) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m-2)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \\ \frac{S_{ofp}^4}{\frac{3b\alpha_0^2}{2}} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m-5}{2}} \Gamma(4m-5) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m-5)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \quad (33.1) \\ \frac{S_{ofp}^5}{\frac{3b\alpha_1^2}{2}} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m-3}{2}} \Gamma(4m-3) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m-3)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \\ \frac{S_{ofp}^6}{3\alpha_0\alpha_1b} &= \left(4\sqrt{-E_m^{(1)}}\right)^{(2m-2)} \Gamma(4m-4) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m-4)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \end{aligned}$$

and

$$\begin{aligned} \frac{S_{ofp}^7}{\frac{2\alpha_0^2}{2}} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m+2}{2}} \Gamma(4m+2) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m+2)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \\ \frac{S_{ofp}^8}{\frac{2\alpha_1^2}{2}} &= \left(4\sqrt{-E_m^{(1)}}\right)^{-(m+2)} \Gamma(4m+4) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m+4)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \quad (33.2) \\ \frac{S_{ofp}^9}{4\alpha_0\alpha_1} &= \left(4\sqrt{-E_m^{(1)}}\right)^{\frac{4m+3}{2}} \Gamma(4m+3) \exp\left(\frac{\alpha^2}{4(-E_m^{(1)})^2}\right) D_{-(4m+3)}\left(-\frac{\alpha}{E_m^{(1)}}\right) \end{aligned}$$

Which allow us to obtaining the energy corrections E_{nc1} to first excited state for (s.o.f.p.) potential in (NC-2D spaces and phases).

3.4 The Atomic Quantum Spectrum of Lowest Excitations States for the Magnetic Effect

Now, we summarize the obtained results of the quantum spectrum of the lowest excitations corresponding to (stationary state and first excited states), (E_{ncf0} and E_{ncf1}), corresponding to the first order of θ and $\bar{\theta}$, respectively:

$$E_{ncf0} = E_m^{(0)} + E_{nc0} \quad E_{ncf1} = E_m^{(1)} + E_{nc1} \quad (34)$$

Evidently, eq. (34) is the sum of ordinary energy and the obtained corrections. Thus, on based on equations (9.1), (9.2) and (33), we have obtained the explicitly results for the quantum spectrum of the lowest excitations in (NC-2D spaces and phases):

$$\begin{aligned} E_{ncf0} &= -\left(\frac{\alpha^2}{4+2m}\right)^{2/3} + 2\pi\alpha_0^2 m F(m, \alpha, b, \theta, \bar{\theta}) \\ E_{ncf1} &= -\left(\frac{\alpha^2}{4+2m}\right)^{2/3} + 2\pi m \theta S(\alpha, b, E_m^{(1)}, \theta, \bar{\theta}) \end{aligned} \quad (35)$$

$$\text{Here } S(\alpha, b, E_m^{(1)}, \theta, \bar{\theta}) \equiv \left(\theta \sum_{\mu=1}^6 S_{ofp}^{\mu} + \frac{\bar{\theta}}{2m_0} \sum_{\mu=7}^9 S_{ofp}^{\mu}\right).$$

Furthermore, we can construct the deformed (NC-2D spaces and phases) Hamiltonian H_{ncsofp} as a sum of two fundamental operators H_{sofp} and H_{sofp-m} , the first one is only the ordinary Hamiltonian operator for (s.o.f.p.) potential in 2D space, while the second operator, which determined on based to eq. (19) as follows:

$$\begin{aligned} H_{sofp} &= -\frac{1}{2m_0} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r}\right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}\right) + \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} \\ H_{sofp-m} &= \alpha \frac{6b}{4r^{7/2}} \overleftrightarrow{S}\overleftrightarrow{B} + \left(\frac{\alpha\alpha}{4r^{5/2}} + \frac{\bar{\varepsilon}}{2m_0}\right) \overleftrightarrow{J}\overleftrightarrow{B} \end{aligned} \quad (36)$$

We replaced $\left(\theta f(r) + \frac{\bar{\theta}}{2m_0}\right) L_z$ by $\left(\alpha f(r) + \frac{\bar{\varepsilon}}{2m_0}\right) \overleftrightarrow{B}L$, ($\theta = \alpha B$, $\bar{\theta} = \bar{\varepsilon} B$ and $\overleftrightarrow{B} = B\overleftrightarrow{k}$), here α and $\bar{\varepsilon}$ are infinitesimal real proportional constants, $f(r) = \frac{a}{4r^{5/2}} + \frac{3b}{4r^{7/2}}$, the magnetic moment $\overleftrightarrow{\mu} \equiv 1/2$, \overleftrightarrow{J} the global angular momentum and $(-\overleftrightarrow{S}\overleftrightarrow{B})$ denote to the ordinary Hamiltonian of Zeeman Effect H_Z [9]:

$$H_Z = -\overleftrightarrow{S}\overleftrightarrow{B} \quad (37)$$

The operator H_{sofp-m} denote to the modified Zeeman effect, in (NC-2D) real space. It's important to notice that ($-l \leq m \leq +l$), and then we have $(2l+1)$ possible values for m , thus every state will be $(2l+1)$ sub-states under the magnetic effect.

3.5 The Atomic Quantum Spectrum of Lowest Excitations States for Spin-orbital Effect

On another hand, it's possible to rewriting eq. (19), as:

$$H_{sofp-p}(r) = \left\{\theta \left(\frac{a}{2r^{5/2}} + \frac{3b}{2r^{7/2}}\right) + \frac{\bar{\theta}}{m_0}\right\} \overleftrightarrow{S}\overleftrightarrow{L} \quad (38)$$

We oriented the direction of spin parallel of the (Oz) axes, let us write the spin-orbital interaction $\overleftrightarrow{S}\overleftrightarrow{L}$ as follows [9]:

$$\overleftrightarrow{S}\overleftrightarrow{L} = \frac{1}{2} \left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right) \quad (39)$$

This allows us to obtaining the modified potential operator for (s.o.f.p.) potential:

$$H_{sofp-p}(r) = \frac{1}{2} \left\{\theta \left(\frac{a}{2r^{5/2}} + \frac{3b}{2r^{7/2}}\right) + \frac{\bar{\theta}}{m_0}\right\} \left(\overleftrightarrow{J}^2 - \overleftrightarrow{L}^2 - \overleftrightarrow{S}^2\right) \quad (40)$$

A similarly calculation of previously section gives the modified recent quantum spectrum of the lowest excitations $E_{NCsofp-0}(j, l, s)$ and $E_{NCsofp-1}(j, l, s)$, corre-

spending spin-orbital interaction, respectively:

$$E_{Nsofp-0} = - \left[\frac{a^2}{8 \left(\frac{1}{2} + m \right)} \right]^{\frac{3}{2}} + 2\pi a_0^2 \begin{cases} L F(a, b, \theta, \bar{\theta}) \\ \text{for -spin- up,} \\ L' F(a, b, \theta, \bar{\theta}) \\ \text{for -spin- down.} \end{cases} \quad (41.1)$$

and

$$E_{NCsofp-1} = - \left[\frac{a^2}{4 \left(\frac{5}{2} + m \right)} \right]^{\frac{2}{3}} + 2\pi \begin{cases} L S(a, b, E_m^{(1)}, \theta, \bar{\theta}) \\ \text{for -spin- up,} \\ L' S(a, b, E_m^{(1)}, \theta, \bar{\theta}) \\ \text{for -spin- down.} \end{cases} \quad (41.2)$$

Where the two important factors $L \left(j = l + \frac{1}{2}, s \right)$ and $L' \left(j = l + \frac{1}{2}, s \right)$ are determined from the following relations:

$$L \left(j = l + \frac{1}{2}, s \right) = \frac{\ell}{2} \text{ and } L' \left(j = l + \frac{1}{2}, s \right) = -\frac{\ell+1}{2} \quad (42)$$

Then, we can construct a diagonal matrix of order (2×2) , with elements $(H_{nc-sofp})_{11}, (H_{nc-sofp})_{22}, (H_{nc-sofp})_{12}$ and $(H_{nc-sofp})_{21}$, as it has been constructed in our four references, respectively [21-24]:

$$(H_{nc-sofp})_{11} = -\frac{1}{2m_0} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) + \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} + \alpha \frac{6b}{4r^{7/2}} \overleftrightarrow{SB} + \left(\frac{\alpha a}{4r^{5/2}} + \frac{\bar{\varepsilon}}{2m_0} \right) \overleftrightarrow{JB} + \left\{ \theta \left(\frac{a}{2r^{5/2}} + \frac{3b}{2r^{7/2}} \right) + \frac{\bar{\theta}}{m_0} \right\} \frac{l}{2} \quad (43.1)$$

and

$$(H_{nc-sofp})_{22} = -\frac{1}{2m_0} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) + \frac{a}{r^{1/2}} + \frac{b}{r^{3/2}} + \alpha \frac{6b}{4r^{7/2}} \overleftrightarrow{SB} + \left(\frac{\alpha a}{4r^{5/2}} + \frac{\bar{\varepsilon}}{2m_0} \right) \overleftrightarrow{JB} - \left\{ \theta \left(\frac{a}{2r^{5/2}} + \frac{3b}{2r^{7/2}} \right) + \frac{\bar{\theta}}{m_0} \right\} \times \frac{l+1}{2} (H_{nc-sofp})_{12} = (H_{nc-sofp})_{21} = 0. \quad (43.2)$$

It's clearly, from the obtained results for states of energy (40) are degenerated and different with ordinary energy (9), it is depended with three quantum numbers (j, l, s) , furthermore it describes two fermionic particles, the first with spin up and spin down as it has been observed in Dirac theory at high energy. It's worth to notice that $j = l \pm \frac{1}{2}$ and then we have 2 possible values for j , thus every state will be 2 sub-states under the effect of spin-orbital interaction.

4. EXAMINATIONS OF OBTAINED RESULTS

As a typical application, let us consider an electron

with spin down transition from first excited state to the stationary one; after a straightforward calculation, we can obtain the expression of produced energy $(\Delta E_{NC-sofp} = E_{ncf1} - E_{ncf0})$ and the modified ionization energy $E_{ion-sofp}$ as:

$$\Delta E_{NC-sofp} = -a^{4/3} \left(\frac{1}{(14)^{2/3}} - \frac{1}{(4)^{2/3}} \right) + 2\pi \theta S(a, b, E_m^{(1)}, \theta, \bar{\theta}), \quad (44)$$

$$E_{ion-sofp} = \left[\frac{a^2}{8 \left(\frac{1}{2} + m \right)} \right]^{\frac{3}{2}} - 2\pi a_0^2 m \theta F(m, a, b, \theta, \bar{\theta}).$$

The first terms are the ordinary produced energy and ionization energy, while the add parts are the contributions of the physics proprieties of (NC-2D spaces and phases). The obtained results were proportional to the infinitesimal parameter θ . Finally, when the parameters $(\theta \text{ and } \bar{\theta} \rightarrow 0)$, in the obtained results for (NC-2D) spaces: ((10), (15), (19), (35), (36), (38), (40), (41), (43) and (44)) we obtain all ordinary 2D space results for (s.o.f.p) potential, which prove the correct obtained recent results. Regarding, two obtained specters for the magnetic and spin-orbital interaction, one can deduce that every state degenerated to $2(2l+1)$ sub states, as it has been obtained in our reference [22]. Know, the global quantum spectrum of lowest excitations states (fundamental $E_{NC0}(m, j, l, s)$ and first excited states $E_{NC1}(m, j, l, s)$) for Hydrogen atom with (s.o.f.p) potential:

$$E_{NC0}(m, j, l, s) = - \left[\frac{a^2}{8 \left(\frac{1}{2} + m \right)} \right]^{\frac{3}{2}} + \begin{cases} 2\pi a_0^2 \left(m + \frac{\ell}{2} \right) F(a, b, \theta, \bar{\theta}) \\ \text{for - spin -up,} \\ 2\pi \left(m - \frac{\ell+1}{2} \right) F(a, b, \theta, \bar{\theta}) \\ \text{for - spin -down.} \end{cases} \quad (45.1)$$

and

$$E_{NC1}(m, j, l, s) = - \left[\frac{a^2}{4 \left(\frac{5}{2} + m \right)} \right]^{\frac{2}{3}} + \begin{cases} 2\pi \left(m + \frac{\ell}{2} \right) S(a, m, E_m^{(1)}) \\ \text{for - spin -up,} \\ 2\pi \left(m - \frac{\ell+1}{2} \right) S(a, m, E_m^{(1)}) \\ \text{for -spin -down.} \end{cases} \quad (45.2)$$

Finally, it's important to notice that, the using of complex coordinates, which indicted in reference [25], permitted to obtain two terms of perturbations; this is deferent totally for our present work and [22-24], based on real coordinates, which allow us to obtaining only one term of perturbation.

5. CONCLUSIONS

We have used the Boopp's shift method and perturbation theory to study the effect of the non commutativity on the central typical rational spherical potential like (s.o.f.p.) potential in (NC-2D spaces and phases). The exact global atomic quantum spectrum of lowest

excitations states were reported, we observed the every ordinary state of energy was changed radically and degenerated to $2(2l+1)$ sub-states, depended with 4 quantum numbers: m and $(j, l$ and $s)$, and correspond two fermionic particles, the first with spin up while the second with spin down as it was observed in Dirac theory at high energy.

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