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Electron Scattering for Uniaxially Deformed *n*-Ge Single Crystals

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The electron scattering in the ($L_1 - \Delta_1$) model of the conduction band of *n*-Ge single crystals formed by uniaxial pressure along the crystallographic direction [100] has been investigated. Obtained experimental results and performed theoretical calculations show that nonequivalent intervalley electron scattering between L_1 and Δ_1 minima becomes significant for the range of uniaxial pressures from 1.4 to 2.3 GPa, and the relative contribution of this scattering depends on the magnitude of the uniaxial pressure. The presence of the maximum for the dependences of the Hall coefficient on the uniaxial pressure for $P \sim 2.1$ GPa, when the energy gap between L_1 and Δ_1 "is closed", is explained by the greatest efficiency of the given scattering mechanism for such pressures.

Keywords: Uniaxial deformation, Equivalent and nonequivalent intervalley scattering, *n*-Ge single crystals, L_1 and Δ_1 minima, Hall coefficient.

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1. INTRODUCTION

Single crystal germanium has always been and remains one of the promising materials for semiconductor electronics. In particular, it is used as a raw material for the production of integrated circuits, radiation-monitoring instruments, dark matter detectors, in designs of the ground, air, and sea-based thermal imaging devices operating in the wavelength range of 2.5 … 14 μm [1-3]. High electron mobility in germanium allows to create nanotransistors with high-conductivity channels used in NMOSFET and CMOS technologies [4-7].

Electrical and optical properties of semiconductors and semiconductor nanostructures depend significantly on the external and internal deformation fields. Band structure of semiconductors is changed with deformation. In particular, the authors of [8] have studied the influence of the internal mechanical stresses arising in Si/Ge heterostructure on the transformation of the band structure of silicon and germanium. It is shown that the value of such internal stresses depends on the ratio between the thicknesses of silicon and germanium layers that should be taken into account when designing electronic devices based on the strained heterostructures. Moreover, deformation can influence the change in the relative contribution of different scattering mechanisms. It was shown in [9] based on the measurements of the temperature dependences of the resistivity for undeformed and uniaxially deformed *n*-Si single crystals that the *f*-type intervalley scattering is completely "switched off" because of strong uniaxial pressure of $P = 1.3$ GPa along the crystallographic direction [100]. Studies of the intervalley electron scattering in deformed *n*-Ge single crystals were carried out in the works [10, 11]. At that, the intervalley electron scattering between the equivalent minima of the L_1 or Δ_1 type was only considered. Intervalley electron scattering between nonequivalent L_1 and Δ_1 minima is also possible in germanium [12]. At weak electric fields and low temperatures, thermal energy of an electron will be much less than the value of the energy gap between these minima, which for undeformed *n*-Ge is equal to 0.18 eV. At that, all electrons will be located in lower (by energy scale) L_1 minima, and this scattering mechanism will not be observed. However, under the conditions of strong electric fields or substantial deformations, when position of the given minima is changed by the energy scale, the relative contribution of the above mentioned scattering mechanism cannot be neglected.

Therefore, electron scattering in the ($L_1 - \Delta_1$) model of the conduction band of germanium, when both L_1 and Δ_1 minima participate in the conduction, was studied in this work. The given model can be implemented under uniaxial deformation of *n*-Ge single crystals along the crystallographic direction [100], since, as it was shown in [13], energy gap between L_1 and Δ_1 in *n*-Ge decreases for these conditions.

2. THEORETICAL CALCULATIONS OF THE RESISTIVITY AND HALL CONSTANT FOR *n*-Ge SINGLE CRYSTALS UNIAXIALLY DEFORMED ALONG THE CRYSTALLOGRAPHIC DIRECTION [100]

Specific electrical conductivity of *n*-Ge single crystals uniaxially deformed along the crystallographic direction [100] can be written as

$$\sigma = q(n_{L_1}\mu_{L_1} + n_{\Delta_1}\mu_{\Delta_1}), \quad (1)$$

where n_{L_1}, n_{Δ_1}, $\mu_{L_1}, \mu_{\Delta_1}$ are the electron concentrations and mobility for L_1 and Δ_1 minima, respectively; q is the electron charge.

For non-degenerate electron gas

$$n_{L_1} = 2\left(\frac{2\pi m_{L_1} kT}{\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_F - E_{L_1}}{kT}}, \quad n_{\Delta_1} = 2\left(\frac{2\pi m_{\Delta_1} kT}{\hbar^2}\right)^{\frac{3}{2}} e^{\frac{E_F - E_{\Delta_1}}{kT}}. \quad (2)$$

Then

$$\frac{n_{L_1}}{n_{\Delta_1}} = \left(\frac{m_{L_1}}{m_{\Delta_1}}\right)^{\frac{3}{2}} e^{\frac{\Delta E(P)}{kT}} = A, \quad (3)$$

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where $\Delta E(P) = E_{\Delta_1} - E_{L_1}$, E_{L_1}, E_{Δ_1} are the energies of L_1 and Δ_1 minima in the deformed crystal; m_{L_1} and m_{Δ_1} are the effective masses of the density of states in the specified minima.

For undeformed n-Ge single crystals, energy gap between L_1 and Δ_1 is equal to 0.18 eV and decreases linearly with respect to the uniaxial pressure P [13]. Then, for the case of the deformed sample one can write

$$\Delta E(P) = 0.18 - \beta P , \quad (4)$$

where $\beta = 8.97 \cdot 10^{-11} \mathrm{eV} \cdot \mathrm{Pa}^{-1}$ is the pressure coefficient of the change in the energy gap between L_1 and Δ_1 minima at the uniaxial pressure P//[100] [13].

For the case, when energy levels of impurities are completely ionized and intrinsic conductivity is not yet manifested

$$n_{L_1} + n_{\Delta_1} = n = N_D . \quad (5)$$

Taking into account expressions (3) and (5), resistivity for the uniaxially deformed n-Ge single crystals

$$\rho = \frac{1}{\sigma} = \frac{A+1}{qN_D\left(A\mu_{L_1} + \mu_{\Delta_1}\right)} . \quad (6)$$

Constant-energy surfaces for both L_1 and Δ_1 minima are the ellipsoids of revolution. Then, charge carrier mobility in an arbitrary direction can be determined from the relation [14]

$$\mu = \mu_{\perp} \sin^2\theta + \mu_{\parallel} \cos^2\theta , \quad (7)$$

where θ is the angle between the considered direction and the principal axis of the ellipsoid; $\mu_{\perp}$ and $\mu_{\parallel}$ are the mobilities of charge carriers across and along the ellipsoid axis.

According to (1), for the L_1 minimum

$$\mu^{L_1} = \frac{1}{3}\mu_{\parallel}^{L_1} + \frac{2}{3}\mu_{\perp}^{L_1} \quad (8)$$

and for the Δ_1 minimum, when P//J//[100],

$$\mu^{\Delta_1} = \mu_{\parallel}^{\Delta_1} . \quad (9)$$

Components of the mobility tensor $\mu_{\perp}$ and $\mu_{\parallel}$ can be expressed through the components of the relaxation time tensor and effective mass for the given minima

$$\mu_{\parallel}^{L_1,\Delta_1} = \frac{q}{m_{\parallel}^{L_1,\Delta_1}} < \tau_{\parallel}^{L_1,\Delta_1} >, \qquad \mu_{\perp}^{L_1,\Delta_1} = \frac{q}{m_{\perp}^{L_1,\Delta_1}} < \tau_{\perp}^{L_1,\Delta_1} > . \quad (10)$$

Based on the theory of anisotropic scattering, we write the expressions for $\tau_{\parallel}^{L_1,\Delta_1}$ and $\tau_{\perp}^{L_1,\Delta_1}$ under the conditions of electron scattering by impurity ions and acoustic phonons (intravalley scattering) [15]

$$\tau_{\parallel}^{L_1,\Delta_1} = \frac{a_{\parallel}^{L_1,\Delta_1}}{\sqrt{k_B}T^{\frac{3}{2}}} \cdot \frac{x^{\frac{3}{2}}}{x^2 + b_0^{L_1,\Delta_1}}, \qquad \tau_{\perp}^{L_1,\Delta_1} = \frac{a_{\perp}^{L_1,\Delta_1}}{\sqrt{k_B}T^{\frac{3}{2}}} \cdot \frac{x^{\frac{3}{2}}}{x^2 + b_1^{L_1,\Delta_1}} \quad (11)$$

(The necessary designations in expressions (11) are presented in the Appendix).

Along with the electron scattering by acoustic phonons and impurity ions in n-Ge, it is also necessary to take into account for the L_1 model of the conduction band the electron scattering by optical phonons, whose frequencies correspond to the temperature of $T_{C1} = 430$ K (intravalley scattering) and the intervalley scattering by acoustic phonons with the characteristic temperature of $T_{C2} = 320$ K [11]. The equivalent intervalley scattering by acoustic and optical phonons with the characteristic temperatures of $T_{C1} = 100$ K and $T_{C1} = 430$ K between valleys located on the same axis (g is the scattering) [10] takes place for the two-valley Δ_1 model of the conduction bands of n-Ge single crystals formed by a uniaxial pressure P//[100].

The role of the nonequivalent intervalley scattering increases due to the decrease in the energy gap between L_1 and Δ_1 minima at a uniaxial pressure. Nonequivalent $L_1 \leftrightarrow \Delta_1$ intervalley electron scattering is conditioned by their interaction with acoustic phonons with the characteristic temperature of $T_{C2} = 320$ K [12]. Electron scattering by optical and intervalley phonons can be described by the scalar relaxation time τ_{ij} [12]:

$$\frac{1}{\tau_{ij}} = a_{ij}\phi_i , \quad (12)$$

where

$$a_{ij} = \frac{\Xi_{ij}^2\left(m_d^j\right)^{\frac{3}{2}}}{\sqrt{2}\pi\rho\hbar^2\left(kT_{Cj}\right)^{\frac{1}{2}}}\left(T_{Cj}/T\right)^{\frac{1}{2}},$$

$$\phi_i(x) = \frac{1}{e^{T_{Cj}/T} - 1}\left[\left(x + \Delta E^*(P) + T_{Cj}/T\right)^{\frac{1}{2}} + e^{T_{Cj}/T}\theta(x;T_{Cj}/T)\left(x + \Delta E^*(P) - T_{Cj}/T\right)^{\frac{1}{2}}\right],$$

m_d^j is the combined mass of the density of states for the j-th minimum; Ξ_{ij} is the constant of the intervalley or optical deformation potential; ρ is the crystal density; T_{Cj} is the temperature of the j-th intervalley or optical phonon; $x = \varepsilon/kT$ is the dimensionless electron energy; $\theta(x;T_{Cj}/T)$ is the step function; $\Delta E^*(P) = \Delta E(P)/kT$.

For the equivalent intervalley scattering [12]

$$m_d^j = \left(m_{\parallel j} m_{\perp j}^2\right)^{\frac{1}{3}}\left(Z_j - 1\right) \quad (13)$$

and for the nonequivalent one –

$$m_d^j = \left(m_{\parallel j} m_{\perp j}^2\right)^{\frac{1}{3}} Z_j , \quad (14)$$

where $m_{\parallel j}$ and $m_{\perp j}$ are the longitudinal and transverse components of the effective mass tensor for the electrons inside ellipsoid of the j-th type; Z_j is the number of equivalent ellipsoids of the conduction band of the j-th type.

For the intravalley electron scattering by optical phonons

$$m_d^j = \left(m_{\parallel j} m_{\perp j}^2\right)^{\frac{1}{3}} Z_j^{\frac{1}{3}} . \qquad (15)$$

Then, for the most general case of electron scattering by acoustic phonons, impurity ions, optical and intervalley phonons, expressions for the components of the relaxation time tensor for L_1 and Δ_1 minima can be represented as follows:

$$\frac{1}{\tau_{\parallel}^{L_1}} = \frac{1}{\tau_{\parallel}} + \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_{12}} + \frac{1}{\tau_{21}}, \quad \frac{1}{\tau_{\perp}^{L_1}} = \frac{1}{\tau_{\perp}} + \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_{12}} + \frac{1}{\tau_{21}}, \qquad (16)$$

$$\frac{1}{\tau_{\parallel}^{\Delta_1}} = \frac{1}{\tau_{\parallel}} + \frac{1}{\tau_3} + \frac{1}{\tau_4} + \frac{1}{\tau_{12}} + \frac{1}{\tau_{21}}, \qquad (17)$$

where $\tau_{\parallel}^{L_1,\Delta_1}$ and $\tau_{\perp}^{L_1,\Delta_1}$ are the longitudinal and transverse components of the relaxation time tensor during scattering by acoustic phonons (intravalley scattering) and impurity ions for L_1 and Δ_1 minima; τ_1 and τ_2 are the relaxation times for the intervalley scattering by acoustic phonons with the characteristic temperature of $T_{C1} = 320$ K and scattering by optical phonons with the characteristic temperature of $T_{C2} = 430$ K (intravalley scattering) for L_1 minima; τ_3 and τ_4 are the relaxation times for the intervalley electron scattering (*g* is the scattering) by acoustic and optical phonons with the characteristic temperatures of $T_{C1} = 100$ K and $T_{C2} = 430$ K for Δ_1 minima; τ_{12} and τ_{21} are the relaxation times for the nonequivalent $L_1 \to \Delta_1$ and $\Delta_1 \to L_1$ intervalley electron scattering by acoustic phonons with the characteristic temperature of $T_C = 320$ K.

For non-degenerate electron gas

$$\left\langle \tau_{\parallel}^{L_1,\Delta_1} \right\rangle = \frac{4}{3\sqrt{\pi}} \int_0^{\infty} dx x^{\frac{3}{2}} e^{-x} \tau_{\parallel}^{L_1,\Delta_1}, \quad \left\langle \tau_{\perp}^{L_1,\Delta_1} \right\rangle = \frac{4}{3\sqrt{\pi}} \int_0^{\infty} dx x^{\frac{3}{2}} e^{-x} \tau_{\perp}^{L_1,\Delta_1}. \qquad (18)$$

In the study of the electrical properties and scattering mechanisms of charge carriers in semiconductors, along with such electrical parameters as charge carrier mobility and concentration, resistivity, it is also necessary to know the value of the Hall coefficient. The Hall coefficient in the presence of charge carriers with different mobility and concentration can be written as [16]

$$R = \frac{r_1 q_1 n_1 \mu_1^2 + r_2 q_2 n_2 \mu_2^2}{\left(r_1 q_1 n_1 \mu_1 + r_2 q_2 n_2 \mu_2\right)^2}, \qquad (19)$$

where r_1, q_1, n_1, μ_1 are the Hall-factor, charge, concentration and mobility for charge carriers of the first sort, and r_2, q_2, n_2, μ_2, respectively, – of the second one.

For the electrons of L_1 and Δ_1 minima in n-Ge $r_1 = r_{L1}$, $r_2 = r_{\Delta 1}$, $n_1 = n_{L1}$, $n_2 = n_{\Delta 1}$, $\mu_1 = \mu_{L1}$, $\mu_2 = \mu_{\Delta 1}$, $q_1 = q_2 = q$.

Then, taking into consideration expressions (3), (5), and (19), the Hall coefficient for the uniaxially deformed n-Ge will take the following form:

$$R = \frac{(A+1)\left(A r_{L_1} \mu_{L_1}^2 + r_{\Delta_1} \mu_{\Delta_1}^2\right)}{qn\left(A\mu_{L_1} + \mu_{\Delta_1}\right)^2}. \qquad (20)$$

The Hall factor for different types of minima, according to [16], is determined as

$$r_{L_1} = \frac{\left\langle \tau_{L_1}^2 \right\rangle}{\left\langle \tau_{L_1} \right\rangle^2}, \quad r_{\Delta_1} = \frac{\left\langle \tau_{\Delta_1}^2 \right\rangle}{\left\langle \tau_{\Delta_1} \right\rangle^2}. \qquad (21)$$

Taking into account expressions (7)-(18) and (21), it is possible based on the expression (20) to obtain the Hall coefficient for the uniaxially deformed n-Ge during the electron scattering by impurity ions, acoustic and optical phonons, intervalley phonons, which correspond to the equivalent and nonequivalent intervalley scattering.

3. RESULTS OF THE EXPERIMENTAL STUDY AND NUMERICAL CALCULATIONS

To compare the performed theoretical calculations and experimental results we have carried out measurements of the temperature dependences of the resistivity for the uniaxially deformed along the crystallographic direction [100] at different pressures n-Ge, doped with Sb impurity, of the concentration of $N_D = 5 \cdot 10^{14}$ cm^{-3} (Fig. 1). For the calculation of the resistivity and Hall coefficient for n-Ge under the existing experimental conditions, it is necessary to have the optical and intervalley deformation potential constants, components of the acoustic deformation potential tensor for the intravalley scattering, effective mass of the density of states and components of the effective mass tensor for L_1 and Δ_1 minima. A significant number of these parameters was obtained in the works [11, 13, 17]. In Table 1 we present the parameters of L_1 and Δ_1 minima necessary for the calculations.

In Fig. 1 we illustrate the results of the theoretical calculations of the resistivity for uniaxially deformed n-Ge single crystals with (solid curves 1-6) and without (dashed curves 1'-6') taking into account the nonequivalent intervalley electron scattering between L_1 and Δ_1 minima.

As seen from Fig. 1, for the undeformed and strongly deformed under the pressure of $P = 2.8$ GPa n-Ge single crystals it is possible to neglect the nonequivalent intervalley electron scattering between L_1 and Δ_1 minima. At that, it is sufficient to confine ourselves to the present electron scattering mechanisms in the L_1 and Δ_1 models of the conduction band of germanium. For the range of uniaxial pressures from 1.4 to 2.3 GPa, it is necessary to additionally take into account the nonequivalent intervalley scattering. In Fig. 2 we present the dependences of the Hall constant on the uniaxial pressure along the crystallographic direction [100] for n-G, doped with Sb impurity, of the concentration of $N_D = 5 \cdot 10^{14}$ cm^{-3} at different temperatures.

Presence of the maximum for all curves at uniaxial pressures of $P \sim 2.1$ GPa indicates the maximum efficiency of nonequivalent intervalley scattering under these pressures. This, in turn, explains the biggest difference between the curves 3 and 3' in Fig. 1.

Table 1 – Parameters of the L_1 and Δ_1 minima of the conduction band of germanium

Parameters		Conduction band of germanium	
Name	Designation	L_1 minima	Δ_1 minima
Components of the effective mass tensor	$m_\parallel$	$1.58m_0$ [18]	$1.65m_0$ [17]
	$m_\perp$	$0.082m_0$ [18]	$0.32m_0$ [17]
Effective mass of the density of states	$m_{L1,\Delta1}$	$0.55m_0$ [18]	$0.88m_0$ [17]
Components of the acoustic deformation potential tensor for the intervalley scattering	Ξ_u (eV)	16.4 [18]	11.82 [13]
	Ξ_d (eV)	– 6.4 [18]	– 1.29 [13]
Optical deformation potential constant, $T_C = 430$ K	Ξ_{430} (eV/cm)	$4\cdot10^8$ [11]	
Intervalley deformation potential constant, $T_C = 320$ K	Ξ_{320} (eV/cm)	$1.4\cdot10^8$ [11]	
Intervalley deformation potential constant for the non-equivalent $L_1 \leftrightarrow \Delta_1$ scattering, $T_C = 320$ K	Ξ_{320} (eV/cm)	$5.5\cdot10^8$ [12]	
Intervalley deformation potential constants for the g–scattering, $T_{C1} = 100$ K, $T_{C2} = 430$ K	Ξ_{100} (eV/cm)		$7.89\cdot10^7$ [12]
	Ξ_{430} (eV/cm)		$1.57\cdot10^8$ [12]

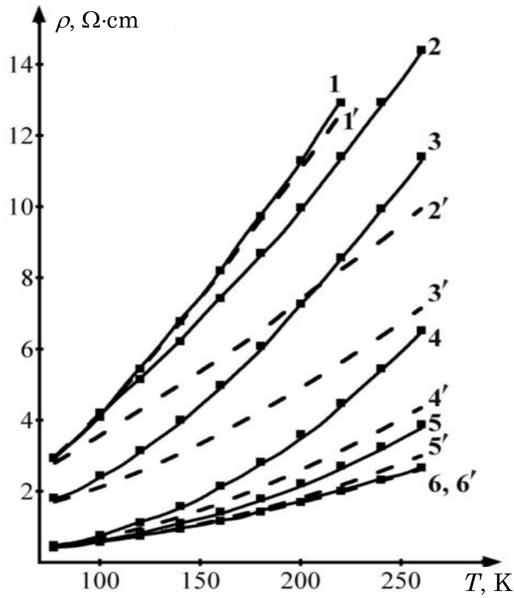

Fig. 1 – The temperature dependences of the resistivity ρ for the uniaxially deformed along the crystallographic direction [100] at different pressures n-Ge, doped with Sb impurity, of the concentration of $N_D = 5\cdot1014$ cm^{-3}: 1 – 2.8 GPa, 2 – 2.3 GPa, 3 – 2.1 GPa, 4 – 1.8 GPa, 5 – 1.4 GPa, 6 – undeformed sample. Solid lines are the theoretical calculations taking into account the nonequivalent intervalley scattering, and dashed lines are those without taking into account the nonequivalent intervalley scattering; ■ – experimental results

For these values of the uniaxial pressures, according to the data of [13], energy gap between L_1 and Δ_1 minima is "closed". With increasing or decreasing uniaxial pressure, relative value of the Hall constant decreases that is explained by the decrease in the contribution of the nonequivalent intervalley electron scattering between L_1 and Δ_1 minima.

4. CONCLUSIONS

The performed theoretical and experimental studies show that for the uniaxially deformed along the crystallographic direction [100] n-Ge single crystals it is necessary to take into account the nonequivalent intervalley electron scattering between L_1 and Δ_1 minima, whose efficiency depends on the value of the uniaxial pressure.

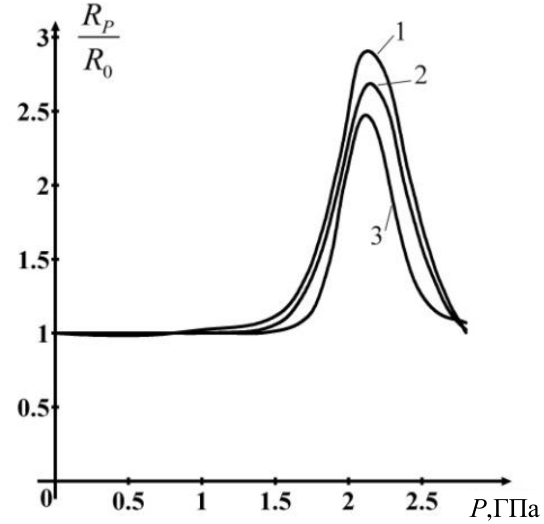

Fig. 2 – Dependence of the Hall constant on the uniaxial pressure along the crystallographic direction [100] for n-Ge, doped with Sb impurity, of the concentration of $N_D = 5\cdot10^{14}$ cm^{-3} at different temperatures: 1 – 180 K, 2 – 150 K, 3 – 110 K

Probability of this scattering mechanism is maximum at uniaxial pressures of $P \sim 2.1$ GPa, when L_1 and Δ_1 minima are equal by the energy scale.

The obtained results should be taken into consideration when designing elements of modern nanoelectronics based on strained heterostructures with a germanium component.

APPENDIX A

$$a_\parallel = \frac{\pi C_{11}\hbar^4}{k\Xi_d^2\sqrt{2m_\parallel m_\perp^2}}\cdot\frac{1}{\Phi_{0a}},\; a_\perp = \frac{\pi C_{11}\hbar^4}{k\Xi_d^2\sqrt{2m_\parallel m_\perp^2}}\cdot\frac{1}{\Phi_{1a}}, \quad (A1)$$

$$b_0 = \frac{a_\parallel\cdot\Phi_{0i}}{\sqrt{k}T^{\frac{3}{2}}\tau_{0i}(kT)},\; b_1 = \frac{a_\perp\cdot\Phi_{1i}}{\sqrt{k}T^{\frac{3}{2}}\tau_{0i}(kT)}, \quad (A2)$$

$$\tau_{0i}(kT) = \frac{\sqrt{2}m_\perp\varepsilon^2(kT)^{\frac{3}{2}}}{\pi Ne^4\sqrt{m_\parallel}}, \quad (A3)$$

where

$$\Phi_{1a} = 1 + \frac{1+\beta^2}{\beta^2}\left(2 + \frac{3}{\beta^2} - \frac{3(1+\beta^2)}{\beta^3}\alpha\right)\frac{\Xi_u}{\Xi_d} +$$
$$\frac{(1+\beta^2)}{\beta^4}\frac{\Xi_u^2}{\Xi_d^2}\left((1+\beta^2)\left(1 + \frac{15}{4\beta^2} - \frac{3}{4\beta^3}(5+3\beta^2)\alpha\right) + \right. \quad (A4)$$
$$\left. + \frac{C_{11}}{4C_{44}}\left(-13 - \frac{15}{\beta^2} + \frac{3(1+\beta^2)}{\beta^3}(5+\beta^2)\alpha\right)\right),$$

$$\Phi_{0a} = 1 + \frac{2(1+\beta^2)}{\beta^2}\left(1 - \frac{3}{\beta^2} + \frac{3}{\beta^3}\alpha\right)\frac{\Xi_u}{\Xi_d} +$$
$$+ \frac{(1+\beta^2)}{\beta^4}\frac{\Xi_u^2}{\Xi_d^2}\left((1+\beta^2)\left(1 - \frac{6}{\beta^2} - \frac{3}{2\beta^2(1+\beta^2)} + \frac{15\alpha}{2\beta^3}\right) + \right. \quad (A5)$$
$$\left. + \frac{C_{11}}{C_{44}}\left(2 + \frac{15}{2\beta^2} - \frac{3}{2\beta^3}(5+3\beta^2)\alpha\right)\right),$$

$$\Phi_{0i} = \frac{3}{2\beta^3}\left(\left(\frac{\beta}{1+\beta^2} - \alpha\right)\ln\gamma^2 - \alpha\ln(1+\beta^2) + \right.$$
$$\left. + 2L(\alpha) + \frac{\beta\gamma^2}{2}\left(\frac{\beta^2-1}{\beta^2+1} + \frac{\alpha(\beta^2+1)}{\beta}\right)\right),$$

$$\Phi_{1i} = \frac{3}{4\beta^3}\left(\left((1-\beta^2)\alpha - \beta\right)\ln\gamma^2 + 2(\beta^2-1)L(\alpha) - 2\beta^2\alpha - \right.$$
$$\left. - (\beta^2-1)\alpha\ln(1+\beta^2) + \frac{\gamma^2}{2}\left(\beta(1+3\beta^2) + \alpha(3\beta^4+2\beta^2-1)\right)\right) \quad (A6)$$

Here $\alpha = \mathrm{arctg}\,\beta$, $\beta^2 = \frac{m_\parallel - m_\perp}{m_\perp}$, $\gamma = \sqrt{\frac{\pi\hbar^2 e^2 n}{2m_\parallel \varepsilon\varepsilon_0 kT}}$ and $L(\alpha) = -\int_0^\alpha \ln\cos\phi\, d\phi$ is the Lobachevsky function, n is the electron concentration.

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