

Deformed Quantum Energy Spectra with Mixed Harmonic Potential for Nonrelativistic Schrödinger Equation

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(Received 20 January 2015; revised manuscript received 04 June 2015; published online 10 June 2015)

The paper describes the deformed Hamiltonian for Schrödinger equation with mixed harmonic potential known by sextic potential and the corresponding spectrum of energies which depended with 3-new quantum numbers ($j = l \pm 1/2$, l) and $s = 1/2$ in the non-commutativity infinitesimal parameter θ .

Keywords: Hydrogen atom, Star product, Noncommutative space and the mixed harmonic potential.

PACS numbers: 11.10.Nx, 32.30. - r, 03.65. - w

1. INTRODUCTION

The major of physics applications of quantum mechanics is based on the Schrödinger equation, it was most successful in describing physics phenomena's in 2 and 3 dimensional spaces and in a particular in the central potentials at week energy to study the atoms nuclei, molecules and their spectral behaviours, this theory will correctly described phenomena only when the velocities are small compared to the light [1-19]. The second revolution in the last century it was the standard model, in where the three fundamental forces in nature: electromagnetic, week and strong, are successful unfitted in the framework of gauge field theories. But the last forth forces, gravitation, its out of this model of unification. The hop to get a new gauge theory, in which the four forces at include in this theory, is possible, when the symmetries will be huge, which satisfied by the notion of the noncommutativity of space-time, which is extended to the canonical commutation relations between position coordinates and their momentums themselves. This new concept considers a new revolution in the modern physics, and plays a crucial role in quantum mechanics particularly. The physics idea of a noncommutative space satisfied by anew mathematical product, which replaces the old ordinary product by a new one, known by star product, noted by (*) [20-37]:

$$[x_i, x_j]_* = i\theta_{ij} \quad (1)$$

Throughout this paper the natural unites are employed. The star product between two arbitrary functions $f(x)$ and $g(x)$ in the first order of θ defined as follow:

$$f(x) * g(x) = f(x) \cdot g(x) - \frac{i}{2} \theta^{ij} (\partial_i f(x)) (\partial_j g(x)) \quad (2)$$

It's possible to apply the notion of the star product to obtain the new NC commutator represented by equation (1). Much effort has been put in to find the approximation bound state solution in recent years to the study of noncommutative canonical type quantum mechanics, quantum field theory and string theory. We apply those notions to a mixed harmonic potential which the importance of this study appears in the field

of fiber and also applicable to molecular physics [9]. A Boopp's shift method will be used in our paper, Instead of solving the non commutative Schrödinger equation by using star product procedure:

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \quad (3)$$

We replace the star product with usual product together with a Boopp's shift:

$$x_i = \hat{x}_i - \frac{\theta^{ij}}{2} p_j \quad \text{and} \quad \hat{p}_i = p_i \quad (4)$$

The parameters θ^{ij} are an antisymmetric real matrix of dimension square length in the noncommutative canonical-type space. The main aim of this article is to give a new physics contribution in the context of the non-commutativity of mixed harmonic potential in NC 2D space. This paper is organized as follows. In section 2, we present the notions of Boopp's shift method. In section 3, we present the Hydrogen atom in ordinary 2D space with mixed harmonic potential. In section 4 we derive the deformed Hydrogen atom Hamiltonian with mixed harmonic potential. We apply the perturbation theory to obtain the non-commutative modification of the energy levels. Finally, in section 5, we draw our conclusions.

2. THE BRIEFLY REVIEW OF BOOPP'S SHIFT METHOD

As it is mentioned in the introduction, a Boopp's shift method will be used in our paper, instead of solving the non commutative Schrödinger equation by using star product procedure; we replace the star product with usual product together with a Boopp's shift [20, 36, 37, 40]:

$$\begin{aligned} \hat{x}_i &= x_i - \frac{\theta^{ij}}{2} p_j & \hat{p}_i &= p_i & i &= 1, 2 \\ \theta^{12} &= \theta & \text{and} & & \theta^{21} &= -\theta \end{aligned} \quad (5)$$

Then, we can show that, the commutator $[\hat{x}, \hat{y}]$, in NC 2D real space $R(2)$, will be written as follows:

$$[\hat{x}, \hat{y}] = i\theta \quad (6)$$

Furthermore, the eq. (5) follows to write the two oper-

ators \hat{x} and \hat{y} as a function to old positions, as follows:

$$\hat{x} = x - \frac{\theta}{2} p_y \quad \text{and} \quad \hat{y} = y + \frac{\theta}{2} p_x \quad (7)$$

Then, one can deduce the square \hat{x}^2 and \hat{y}^2 respectively:

$$\hat{x}^2 = x^2 - \theta x p_y \quad \text{and} \quad \hat{y}^2 = y^2 + \theta y p_x \quad (8)$$

Which allow obtaining the operator \hat{r}^2 , in NC 2D real space $R_{def}(2)$, as follows [37, 40]:

$$\hat{r}^2 = r^2 - \theta L_z \quad (9)$$

Based, on the eq. (9), one can obtain, after a straightforward calculation, the 3-important terms which will be use to obtain the NC deformed Hamiltonian, in the next 4th section:

$$\begin{cases} \hat{r}^2 = r^2 - \theta L_z \\ \hat{r}^4 = r^4 - 2\theta r^2 L_z \\ \hat{r}^6 = r^6 - 3\theta r^4 L_z \end{cases} \quad (10)$$

3. THE MIXED HARMONIC POTENTIAL IN ORDINARY TWO DIMENSIONAL SPACES

We represent the Schrödinger equation with the sextic potential ($V(r) = ar^2 + br^4 + cr^6$), which accompanied with 3-terms, the first describe the usual vibration of electron, while the rest 2-terms are represents doubly vibration of electron [9]. The best formulation is to work on the polar coordinate $\vec{r}(r, \phi)$, in which we have the general form of Schrödinger equation:

$$\left(-\frac{1}{2m_0} \Delta + ar^2 + br^4 + cr^6 \right) \Psi(\vec{r}) = E \Psi(\vec{r}) \quad (11)$$

Where Δ represent the Laplacienne operator in polar coordinate and $\Psi(\vec{r})$ obtain by the method of separation of coordinates as follows:

$$\Psi(\vec{r}) = \frac{R_m(r)}{\sqrt{r}} \Phi(\phi) \quad (12)$$

Where $R_m(r)$ and $\Phi(\phi)$ are the radial functions and the angler function satisfied the two equations, respectively:

$$\frac{d^2 R_m(r)}{dr^2} + \left(E - V(r) - \frac{m^2 - \frac{1}{4}}{r^2} \right) R_m(r) = 0 \quad (13)$$

$$\frac{d^2 \Phi(\phi)}{d\phi^2} + m^2 \Phi(\phi) = 0$$

Where m and E denote the two values of momentum and energy, respectively. The standard solution of $\Phi(\phi)$ is given by [9]:

$$\Phi(\phi) = \exp(\pm im\phi) \quad \text{where } m = 0; 1, 2, \dots \quad (14)$$

And $R_m(r)$ determine from the equation:

$$R_m(r) = \exp(p_m(r)) \sum_{n=0}^{\infty} a_n r^{2n+\delta} \quad (15)$$

Where:

$$\begin{cases} p_m(r) = \frac{1}{2} \alpha r^2 + \frac{1}{2} \beta r^4 \\ \beta = -\sqrt{c} \quad \alpha = \frac{b}{2\beta} \quad \text{and} \quad \delta = m + \frac{1}{2} \end{cases} \quad (16)$$

The radial functions of the (stationary state $R_m^{(0)}$ and first excited states $R_m^{(1)}$) and corresponding energies (E_0, E_1) are determined from the relation, respectively [9]:

$$\begin{cases} \frac{R_m^{(0)}}{\exp\left(-\frac{b}{4\sqrt{c}}r^2 - \frac{\sqrt{c}}{4}r^4\right)} = a_0 r^\delta & E_0 = \frac{3b}{2\sqrt{c}} \\ \frac{R_m^{(1)}}{\exp\left(-\frac{b}{4\sqrt{c}}r^2 - \frac{\sqrt{c}}{4}r^4\right)} = (a_0 + a_1 r^2) r^\delta & E_1 = \frac{b(m+2)}{2\sqrt{c}} \pm \frac{\sqrt{b^2(m+2)-4c(1+m)}(2+2\sqrt{c(m+2)})}{\sqrt{c}} \end{cases} \quad (17)$$

Then, the complete normalized wave functions ($\Psi^{(0)}(\vec{r}), \Psi^{(1)}(\vec{r})$) and corresponding energies (E_0, E_1), respectively [9]:

$$\begin{cases} \Psi^{(0)}(\vec{r}) = a_0 r^{\delta-\frac{1}{2}} \exp\left(-\frac{b}{4\sqrt{c}}r^2 - \frac{\sqrt{c}}{4}r^4\right) & E_0 = \frac{3b}{2\sqrt{c}} \\ \Psi^{(1)}(\vec{r}) = (a_0 + a_1 r^2) r^{\delta-\frac{1}{2}} \exp\left(-\frac{b}{4\sqrt{c}}r^2 - \frac{\sqrt{c}}{4}r^4\right) & E_1 = \frac{b(m+2)}{2\sqrt{c}} \pm \frac{\sqrt{b^2(m+2)-4c(1+m)}(2+2\sqrt{c(m+2)})}{\sqrt{c}} \end{cases} \quad (18)$$

And the generalized normalized wave function [9]:

$$\begin{aligned} \Psi^{(p)}(\vec{r}) &= (a_0 + a_1 r^2 + \dots a_p r^{2p}) r^{\delta-\frac{1}{2}} \times \\ &\times \exp\left(-\frac{b}{4\sqrt{c}}r^2 - \frac{\sqrt{c}}{4}r^4\right) \end{aligned} \quad (19)$$

4. THE MIXED HARMONIC (SEXTIC) POTENTIAL IN NC TWO DIMENSIONAL SPACES $R_{def}(2)$

By applying the Boopp's shift method, the deformed Hamiltonian operator, associated with sextic potential $H_{NC-sext}$, in the NC 2D space, determined from the relation:

$$H_{NC-sext} = -\frac{\Delta}{2m_0} + V_{NC-S}(\hat{r}) \quad (20)$$

Where m_0 is the rest masses, \hat{r} is the operator po-

sition in NC 2D space. The operator of sextic potential in NC 2D space $V_{NC-S}(\hat{r})$, take the form:

$$V_{NC-S}(\hat{r}) = a\hat{r}^2 + b\hat{r}^4 + c\hat{r}^6 \quad (21)$$

Using the two equations (20) and (21) to write the Schrödinger equation in NC 2D space as follows:

$$\left(-\frac{\Delta}{2m_0} + V_{NC-S}(\hat{r}) \right) \Psi(\vec{r}) = E_{NC} \Psi(\vec{r}) \quad (22)$$

Where E_{NC} is the eigenvalues of energy in NC 2D space associated with mixed harmonic potential. The separation variable method to equation (22) gives:

$$\Psi(\vec{r}) = \frac{R_m(r)}{\sqrt{r}} \Phi(\phi) \quad (23)$$

The radial function $R_m(r)$ and the angler function $\Phi(\phi)$ in NC 2D space satisfied, respectively:

$$\frac{d^2 R_m(r)}{dr^2} + \left(E - V(\hat{r}) - \frac{m^2 - \frac{1}{4}}{r^2} \right) R_m(r) = 0 \quad (24)$$

$$\frac{d^2 \Phi(\phi)}{d\phi^2} + m^2 \Phi(\phi) = 0$$

As a direct result to the eq. (9), the different terms of sextic potential $V(\hat{r})$ will be:

$$\begin{cases} a\hat{r}^2 = ar^2 - a\theta L_z \\ b\hat{r}^4 = br^4 - 2b\theta r^2 L_z \\ c\hat{r}^6 = cr^6 - 3c\theta r^4 L_z \end{cases} \quad (25)$$

Which allow writing the sextic potential in NC 2D space as follows:

$$V_{NC-S}(\hat{r}) = V(r) + V_{sex-pert}(r) \quad (26)$$

Where $V(r)$ is ordinary potential and the modified term $V_{pert}(r)$ is determined by the following equation:

$$V_{sex-pert}(r) = -\theta \left(a + 2br^2 + 3cr^4 \right) L_z \quad (27)$$

We observe, that the term $V_{sex-pert}(r)$ is proportional to the smallness parameter θ , then we considers as a perturbation term. A straightforward calculation, lead to get the radial function in NC 2D space corresponding $V_{sex-pert}(r)$:

$$\frac{d^2 R_m(r)}{dr^2} + \left(E - V(r) - V_{sex-pert}(r) - \frac{m^2 - \frac{1}{4}}{r^2} \right) R_m(r) = 0 \quad (28)$$

We apply the perturbation theory to obtain the modification to the energy:

$$E_p = \int \Psi^{(p)*}(\vec{r}) V_{sex-pert}(r) \Psi^{(p)}(\vec{r}) ds \quad (29)$$

Where $ds = r dr d\phi$, the non-commutative modification of the energy levels, in the first order of θ corresponding the stationary state E_{NC0} , determined by using the equations (18), (27) and (29) to obtain:

$$E_{NC0} = -2\pi m \alpha_0^2 \theta \sum_{\alpha=1}^3 T^\alpha \quad (30)$$

Where m is eigenvalues of the operator of angular momentum L_z , and the three terms T^1 , T^2 and T^3 are given by:

$$\begin{cases} T^1 = a \int_0^{+\infty} r^{2\delta} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \\ T^2 = 2b \int_0^{+\infty} r^{2\delta+2} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \\ T^3 = 3c \int_0^{+\infty} r^{2\delta+4} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \end{cases} \quad (31)$$

We change the variable by introduces $r^2 = X$, and then eq. (31) will be to the form:

$$\begin{cases} T^1 = a \int_0^{+\infty} X^{(\delta+1)-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \\ T^2 = b \int_0^{+\infty} X^{(\delta+2)-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \\ T^3 = \frac{3}{2} c \int_0^{+\infty} X^{(\delta+3)-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X\right) dX \end{cases} \quad (32)$$

Now using the special integral [38]:

$$\begin{aligned} \int_0^{+\infty} x^{v-1} \exp(-\beta x^2 - \gamma x) dx = \\ = (2\beta)^{-\frac{v}{2}} \Gamma(v) \exp\left(\frac{\gamma^2}{8\beta}\right) D_{-v}\left(\frac{\gamma}{\sqrt{2\beta}}\right) \end{aligned} \quad (33)$$

To obtain the following results:

$$\begin{cases} T^1 = a \left(\sqrt{c}\right)^{-\frac{\delta+1}{2}} \Gamma(\delta+1) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+1)}\left(\frac{b}{2c^{\frac{1}{8}}}\right) \\ T^2 = b \left(\sqrt{c}\right)^{-\frac{\delta+2}{2}} \Gamma(\delta+2) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+2)}\left(\frac{b}{2c^{\frac{1}{8}}}\right) \\ T^3 = \frac{3}{2} c \left(\sqrt{c}\right)^{-\frac{\delta+3}{2}} \Gamma(\delta+3) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+3)}\left(\frac{b}{2c^{\frac{1}{8}}}\right) \end{cases} \quad (34)$$

Which allow writing the non-commutative modification of the energy levels E_{10} as follows

$$E_{10} = -m\theta S(a, b, c) \quad (35)$$

Where, the factor $S(a, b, c)$ given by:

$$S(a, b, c) = a_0^2 \begin{pmatrix} a(\sqrt{c})^{-\frac{\delta+\frac{1}{2}}{2}} \Gamma(\delta - \frac{1}{2}) D_{-(\delta-\frac{1}{2})} \left(\frac{b}{2c^{\frac{1}{8}}} \right) + \\ + b(\sqrt{c})^{-\frac{\delta+\frac{1}{2}}{2}} \Gamma(\delta + \frac{1}{2}) D_{-(\delta+\frac{1}{2})} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \\ + \frac{3}{2} c(\sqrt{c})^{-\frac{\delta+\frac{3}{2}}{2}} \Gamma(\delta + \frac{3}{2}) D_{-(\delta+\frac{3}{2})} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \end{pmatrix} \times \exp\left(\frac{b^2}{16c}\right) \quad (36)$$

The non-commutative modification of the energy levels E_{NC1} associated with the first order of θ , corresponding the first excited states, determined by using equations (18), (27) and (29) to obtain:

$$E_{NC1} = 2\pi m \theta \sum_{\alpha=1}^5 A^\alpha \quad (37)$$

Where A^α are determined from:

$$\begin{cases} A^1 = A_1 \int_0^{+\infty} r^{2\delta+1} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \\ A^2 = \frac{A_2}{2} \int_0^{+\infty} r^{2\delta+3} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \\ A^3 = A_3 \int_0^{+\infty} r^{2\delta+5} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \\ A^4 = A_4 \int_0^{+\infty} r^{2\delta+7} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \\ A^5 = A_5 \int_0^{+\infty} r^{2\delta+9} \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) dr \end{cases} \quad (38)$$

And:

$$\begin{cases} A_1 = a_0^2 a, & A_2 = (2ba_0^2 + 2a_1 a_0 a) \\ A_3 = 3ca_0^2 + 4ba_1 a_0 + a_1^2 a, & A_4 = 6ca_1 a_0 + 2ba_1^2 \\ & A_5 = 3ca_1^2 \end{cases} \quad (39)$$

If we change the variable, similarly to above correction, we can rewrite the eq. (39) as follow:

$$\begin{cases} A^1 = \frac{A_1}{2} \int_0^{+\infty} X^{(\delta+\frac{3}{2})-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \\ A^2 = \frac{A_2}{2} \int_0^{+\infty} X^{(\delta+2)-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \\ A^3 = \frac{A_3}{2} \int_0^{+\infty} X^{(\delta+3)-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \\ A^4 = \frac{A_4}{2} \int_0^{+\infty} X^{(\delta+4)-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \\ A^5 = \frac{A_5}{2} \int_0^{+\infty} X^{(\delta+\frac{11}{2})-1} \exp\left(-\frac{b}{2\sqrt{c}} X - \frac{\sqrt{c}}{2} X^2\right) dX \end{cases} \quad (40)$$

We apply the above one get:

$$\begin{cases} A^1 = \frac{A_1}{2} (\sqrt{c})^{-\frac{(\delta+\frac{3}{2})}{2}} \Gamma(\delta + \frac{3}{2}) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+\frac{3}{2})} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \\ A^2 = \frac{A_2}{2} (\sqrt{c})^{-\frac{(\delta+2)}{2}} \Gamma(\delta + 2) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+2)} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \\ A^3 = \frac{A_3}{2} (\sqrt{c})^{-\frac{(\delta+3)}{2}} \Gamma(\delta + 3) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+3)} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \\ A^4 = \frac{A_4}{2} (\sqrt{c})^{-\frac{(\delta+4)}{2}} \Gamma(\delta + 4) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+4)} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \\ A^5 = \frac{A_5}{2} (\sqrt{c})^{-\frac{(\delta+\frac{11}{2})}{2}} \Gamma(\delta + \frac{11}{2}) \exp\left(\frac{b^2}{16c^{\frac{3}{2}}}\right) D_{-(\delta+\frac{11}{2})} \left(\frac{b}{2c^{\frac{1}{8}}} \right) \end{cases} \quad (41)$$

Which allow to gives the non-commutative modification of the energy levels E_{NC1} .

Now, the non-commutative modification of the energy levels E_{NCp} , in the first order of θ , corresponding the p^{th} order excited states, using eqs. (19), (27) and (29) to get:

$$E_{NCp} = -2\pi m \theta A \quad (42)$$

Where:

$$A = \int_0^{+\infty} \left[(a_0 + a_1 r^2 + \dots a_p r^{2p}) \right]^2 r^{2\delta+1} \times \exp\left(-\frac{b}{2\sqrt{c}} r^2 - \frac{\sqrt{c}}{2} r^4\right) \left((a + 2br^2 + 3cr^4) \right) dr \quad (43)$$

We summarize the obtained results of energies levels ($E_{NC0-m}, E_{NC1-m}, E_{NCp-m}$) corresponding first order of θ to the stationary state, the first existed states and the p^{th} excited states respectively:

$$\begin{cases} E_{NC0-m} = E_0 + E_{0m} \\ E_{NC1-m} = E_1 + E_{1m} \\ E_{NCp-m} = E_p + E_{pm} \end{cases} \quad (44)$$

Then, we have, the explicitly obtained results:

$$\begin{cases} E_{NC0-m} = \frac{3b}{2\sqrt{c}} - m\theta S(a, b, c) \\ E_{NC1-m} = \frac{b(m+2)}{2\sqrt{c}} \pm \frac{\sqrt{b^2(m+2) - 4c(1+m)(2+2\sqrt{c}(m+2))}}{\sqrt{c}} + \\ + 2\pi\theta \left(\sum_{\alpha=1}^5 A^\alpha \right) m \\ E_{NCp-m} = E_p - 2\pi m \theta A \end{cases} \quad (45)$$

Now, in order to construct the NC Hamiltonian, this will be realized by two principal's parts, the first deformed NC Hamiltonian H_{NC-S} , will be construct on based to eqs. (11), (20) and (27), we can write:

$$H_{NC-S} = \left[-\frac{1}{2m_0} \Delta + ar^2 + br^4 + cr^6 \right] + \left[-\theta (a + 2br^2 + 3cr^4) L_z \right] \quad (46)$$

For a better understanding of the physics content to the operator H_{NC-S} , we dives it to 2- operators, H_{Sext0}

and H_{sext-m} as follows:

$$H_{hlc1} = H_{Sext0} + H_{sext-m} \quad (47)$$

Where the first operator is given by:

$$H_{sext0} = -\frac{1}{2m_0} \Delta + ar^2 + br^4 + cr^6 \quad (48)$$

While the second operator H_{sext-m} is given by:

$$H_{sext-m} = -\theta(a + 2br^2 + 3cr^4)L_z \quad (49)$$

Furthermore, if we choices, the parameter θ and the vector of a magnetic field, which oriented with (Oz.) axes, as follows [38]:

$$\theta = \alpha B \quad \text{and} \quad \vec{B} = B\vec{k} \quad (50)$$

Where α is a proportional constant, and the magnetic moment $\vec{\mu} = \vec{S}$, after a straightforward calculation, we have the following important results [38]:

$$\theta L_z = \alpha \vec{J}\vec{B} - \alpha \vec{S}\vec{B} \quad (51)$$

Where $(\vec{J} = \vec{L} + \vec{s})$, using two eqs. (49) and (51), we can write the operator H_{sext-m} as follows:

$$H_{sext-m} = \alpha(a + 2br^2 + 3cr^4)(\vec{J}\vec{B} - H_z) \quad (52)$$

Where H_z , determined from [39]:

$$H_z = -\vec{S}\vec{B} \quad (53)$$

θ_1 (1/2)

Physically, the operator (48), as it is mentioned in the section 3, represented a particle with spin interacted with sextic potential in ordinary 2D space, while the operator (52) represented two interactions between a particle with spin (1/2) and a external magnetic field, the first one represent the ordinary Zeeman effect and the new interaction represent a coupling between the total monument \vec{J} and external magnetic field \vec{B} . Now, regarding to the relation (27), this can be rewritten to the form:

$$V_{sext-pert}(r) = -\theta(a + 2br^2 + 3cr^4)\vec{S}\vec{L} \quad (54)$$

Which allow writing, the perturbative term $V_{sext-p}(r)$ as follows:

$$V_{sext-p}(r) = -\frac{\theta}{2}(a + 2br^2 + 3cr^4)(\vec{J}^2 - \vec{L}^2 - \vec{S}^2) \quad (55)$$

Furthermore, the operator $(\vec{S}\vec{L})$ traduced physically, the coupling between spin and orbital momentum, then, the modified energy levels (E_{NC0-SO}, E_{NC1-SO}) produced, will be:

$$E_{NC0-SO} = \frac{3b}{2\sqrt{c}} + \begin{cases} \theta L_{1s} S(a, b, c) \Rightarrow \text{spin up} \\ \theta L_{2s} S(a, b, c) \Rightarrow \text{spin down} \end{cases}$$

$$E_{NC1-SO} = \frac{b(m+2)}{2\sqrt{c}} \pm \frac{\sqrt{b^2(m+2) - 4c(1+m)(2 + 2\sqrt{c}(m+2))}}{\sqrt{c}} +$$

$$+ \begin{cases} 2\pi\theta L_{1s} \sum_{\alpha=1}^5 A^\alpha \Rightarrow \text{spin up} \\ 2\pi L_{2s} \theta \sum_{\alpha=1}^5 A^\alpha \Rightarrow \text{spin down} \end{cases} \quad (56)$$

Where L_1 and L_2 are given by:

$$L_{1s} = (l + \frac{1}{2})(l + \frac{3}{2}) - l(l+1) - \frac{3}{4}$$

$$L_{2s} = (l - \frac{1}{2})(l + \frac{1}{2}) - l(l+1) - \frac{3}{4} \quad (57)$$

Then, the second part of NC Hamiltonian corresponding noted by H_{sext2} and given from the relation:

$$H_{sext2} = H_{sext0} + H_{sext-so} \quad (58)$$

Where the operator of spin-orbital interaction H_{hlc-so} takes the form:

$$H_{sext-so} = -\theta(a + 2br^2 + 3cr^4)(\vec{J}^2 - \vec{L}^2 - \vec{S}^2) \quad (59)$$

In another hand, it's evidently to consider the infinitesimal parameter θ , the sum of 2- infinitesimal parameters and θ_2 , then the complete NC Hamiltonian equal the sum of three fundamentals parts, the first one, it has been seen from eq. (42), the second part, it has been seen from eq. (43), while the last part, it has seen from the eq. (53). Thus, the final expression of deformed NC Hamiltonian $H_{NC-sext}$, takes the following relation:

$$H_{NC-sext} = H_{sext0} + H_{sext-m} + H_{sext-so} \quad (60)$$

It's worth to mention, that the above obtained NC Hamiltonian, satisfied by applying the physical superposition principal. Thus, the obtained NC Hamiltonian was extended to describing, in addition the usual interaction, in 2D space, 2-new physics phonemes, the modified Zeeman effect and the spin orbital interaction, which are introduced in the new theory automatically. Regarding the eq. (60), we can deduce the total NC quantum spectrum of Hydrogen atom with sextic potential, $E_{NC}(j, l, s, m)$, evidently equal the sum 3 values of energies $((E_0, E_1), (E_{NC0-SO}, E_{NC1-SO})$ and (E_{NC0-m}, E_{NC1-m}) , corresponding $(H_{sext0}, H_{sext-m}$ and $H_{sext-so})$, respectively, as follows:

$$E_{NC0} = \frac{3b}{2\sqrt{c}} + \begin{cases} S(a, b, c)(-\theta_1 m + \theta_2 L_1) \Rightarrow \text{spin up} \\ S(a, b, c)(-\theta_1 m + \theta_2 L_2) \Rightarrow \text{spin down} \end{cases} \quad (61)$$

And

$$E_{NCI} = \frac{b(m+2)}{2\sqrt{c}} \pm \sqrt{\frac{b^2(m+2) - 4c(1+m)(2+2\sqrt{c}(m+2))}{\sqrt{c}}} + \begin{cases} 2\pi \left(\sum_{\alpha=1}^5 A^\alpha \right) (m\theta_1 + \theta_2 L_1) \Rightarrow \text{spin up} \\ 2\pi \left(\sum_{\alpha=1}^5 A^\alpha \right) (m\theta_1 + L_2 \theta_2) \Rightarrow \text{spin down} \end{cases} \quad (62)$$

5. CONCLUSION

The deformation spectrum is studied for the sextic potential in NC 2D spaces in the case of low energy by applying the Boopp's shift method to first order in the non-commutativity parameter θ , the modified of the energies levels at the fundamental state and first excited states are established. The new spectrums of energy are changed radically; it depended with 3 new

quantum numbers ($j = l \pm 1/2, l$) and $s = 1/2$ in addition to the quantum number m which was characterized the original commutative spectrum. We have seen that the Non commutative Hamiltonian was constructed from 3-operators, the first it's naturally to describing usual H_{sext0} , the second sub-Hamiltonian H_{sext-m} describing the new modified Zeeman effect while the last sub-Hamiltonian $H_{sext-so}$ correspond the spin-orbital interaction.

ACKNOWLEDGMENTS

This work was supported with search laboratory of: Physique et Chimie des Matériaux, in university of M'sila, Algeria.

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