Spin Waves in a Ferromagnetic Film with a Periodic System of Antidots

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In the paper, spin waves in a thin film (composed of a uniaxial ferromagnet) with a two-dimensional periodic system of antidots are studied. The film ferromagnet is considered to have the "easy axis" type. To describe such waves, the magnetostatic approximation with account for the magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects is used. For such waves, an equation for the magnetic potential is derived; for the case of remote antidots, the dispersion relation and the transverse wavenumber spectrum are found. For the case of a film thin compared to the exchange length and for the case of a film bounded by a high-conductivity metal, the longitudinal wavenumber spectrum and the frequency spectrum of such spin waves are also obtained.

Keywords: Spin waves, Nanomagnetism, Thin magnetic film, Antidot, Dipole-exchange theory.

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1. INTRODUCTION

Spin waves in magnetically ordered materials [1, 2] have been actively studied in recent years both theoretically and experimentally. Spin waves are the subject of research for new areas of physics, namely, magnonics [1] and spintronics [3], and are promising for numerous practical applications, in particular, for creation of new data storage, transmission and processing devices [4, 5].

Spin waves in nanostructures are particularly important and popular topic for investigation in recent years. The authors of numerous papers have studied spin waves in thin ferromagnetic films [6], micron-sized magnetic quantum dots [7-9], nanowires [10-12] and more complex nanosystems (see, for example, [13]). Since magnetic properties of nanostructures significantly depend on their shape and sizes, spin waves are studied in the nanosystems of different configurations separately.

Magnetic dots and their systems and also magnetic antidots and their systems hold a special place among magnetic nanostructures. Systems of ferromagnetic dots [14, 15] and two-dimensional lattices of magnetic granules [16] have been intensively studied during the last years. However, systems of ferromagnetic antidots [17, 18] remain comparatively little studied, and spin waves in such systems are almost not studied theoretically. At that, systems of magnetic antidots are promising from the point of view of the application in engineering (in information storage devices [19], magnon waveguides [4], as the base of magnetic meta-materials [20], as two-dimensional magnonic crystals [21], etc.) that makes investigation of their magnetic properties, in particular, spin waves in such system, relevant.

In the work, we consider spin waves in a ferromagnetic film with a two-dimensional periodic system of antidots. The magnetostatic approximation, which takes into account the magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects, is used for the description of these waves. A differential equation for the magnetic potential of such waves is derived; and using this equation the dispersion relation and the transverse wavenumber spectrum for the case of the system of remote antidots is found. The longitudinal wavenumber spectrum and the spin wave frequency spectrum are also obtained for two particular cases.

2. STATEMENT OF THE PROBLEM

We will consider a ferromagnetic film of thickness l composed of a uniaxial ferromagnet which has the "easy axis" type. Let a periodic system of identical circular antidots with distances between centers of neighboring antidots a and radiuses R be present in this film (see Fig. 1). We will assume that a ferromagnet, of which the film consists, is characterized by the following parameters: exchange energy constant α, uniaxial anisotropy constant β (is considered constant), gyromagnetic ratio γ (is considered constant). We will also suggest that easy magnetization axis (and, thus, equilibrium magnetization $\vec{M}_0$ which is considered constant in the whole volume of the film) is directed orthogonally to the film; we will choose the Oz-axis in this direction. We will assume that the external magnetic field $\vec{H}_0^{(e)}$, in which the film is placed, is uniform and directed along the Oz-axis.

We will consider a spin wave propagating in the film described above. We take into account in the Landau-Lifshitz equation both the magnetic dipole-dipole and the exchange interactions (we suggest that the film is thin and distances between antodots are both small, so that both the magnetic dipole-dipole and the exchange interactions can be significant, and rather enough to neglect the exchange interaction). Moreover, since we consider a uniaxial ferromagnet, we should also reserve the term accounting anisotropy. We neglect dissipation and, correspondingly, damping of spin waves in the film thus omitting the relaxation term in the Landau-Lifshitz equation.

We will apply the linearized theory of spin waves considering that magnetization $\vec{m}$ and magnetic field $\vec{h}$ of a spin wave are small perturbations of the total magnetization $\vec{M}$ and the internal magnetic field $\vec{H}^{(i)}$ of the film, respectively. Thus, inequality $|\vec{m}| << |\vec{M}_0|$ holds for perturbations of the magnetization $\vec{m}$; $|\vec{h}| << |\vec{H}_0^{(i)}|$ – for perturbations of the magnetic field $\vec{h}$, where $\vec{M}_0$ is the saturation magnetization; $\vec{H}_0^{(i)}$ is the equilibrium value of the internal magnetic field (so $\vec{M}(\vec{r},t) = \vec{M}_0 + \vec{m}(\vec{r},t)$, $\vec{H}^{(i)}(\vec{r},t) = \vec{H}_0^{(i)} + \vec{h}(\vec{r},t)$).

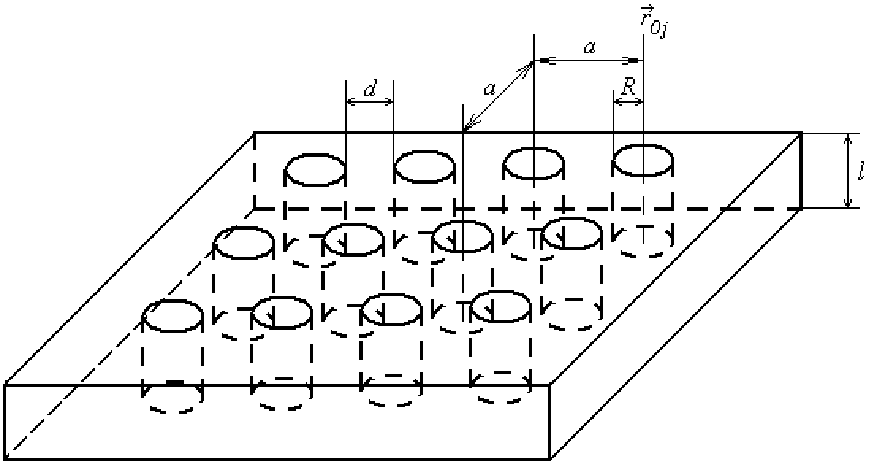

Fig. 1 – The considered system of antidots

The aim of the present work is to obtain the differential equation for the magnetic potential of the above spin waves in the magnetostatic approximation, as well as the dispersion relation and the transverse wavenumber spectrum for these waves.

3. EQUATION FOR THE MAGNETIC POTENTIAL

Let us write the linearized Landau-Lifshitz equation for a spin wave in the above ferromagnetic film with a system of antidots. Taking into account that magnetization and wave field change periodically in time, so that $\vec m(\vec r,t)=\vec m_0(\vec r)\exp(i\omega t)$, $\vec h(\vec r,t)=\vec h_0(\vec r)\exp(i\omega t)$, from [2] we obtain the equation for $\vec m_0$, $\vec h_0$ as follows:

$$i\omega\vec m_0=\gamma\left(M_0\vec e_z\times\left(\vec h_0+\alpha\Delta\vec m_0-\left(\beta+\frac{H_0^{(i)}}{M_0}\right)\vec m_0\right)\right); \quad (1)$$

for our system we can assume $\vec H_0^{(i)}=\vec H_0^{(e)}$. To make the system of equations complete, we will use the magnetostatic approximation (see, for example, [2]) considering the spin wave field $\vec h$ potential, i.e. $\vec h=-\nabla\Phi$, where Φ is the magnetic potential. In this approximation, from the Maxwell equation $\operatorname{div}\vec h=-4\pi\cdot\operatorname{div}\vec m$ we obtain the second equation of the system:

$$\Delta\Phi_0=4\pi\operatorname{div}\vec m_0\,. \quad (2)$$

Here Φ_0 is the potential of the field $\vec h_0$ perturbation amplitude, so that $\vec h_0=-\nabla\Phi_0$, $\Phi(\vec r,t)=\Phi_0(\vec r)\exp(i\omega t)$.

We will obtain the equation for the magnetic potential Φ_0 amplitude excluding the amplitude of the magnetic moment $\vec m_0$ density perturbation from the system of equations (1) and (2). Let us multiply vectorially the first equation of the system on the left by the unit vector $\vec e_z$ and take divergence from both sides of the equation. Taking into account that $m_{0z}=0$, and from equation (2) $\operatorname{div}\vec m_0=\frac{\Delta\Phi_0}{4\pi}$, we obtain

$$\begin{aligned}&-\frac{i\omega}{\gamma M_0}\operatorname{div}\left(\vec e_z\times\vec m_0\right)=\\&=-\Delta\Phi_0+\frac{\partial^2\Phi_0}{\partial z^2}+\frac{1}{4\pi}\left(\alpha\Delta-\left(\beta+\frac{H_0^{(i)}}{M_0}\right)\right)\Delta\Phi_0.\end{aligned} \quad (3)$$

Applying operator $\alpha\Delta-\left(\beta+\frac{H_0^{(i)}}{M_0}\right)$ to both sides of the equation, after some transformations we have

$$\begin{aligned}&\left(\frac{\omega^2}{\gamma^2M_0^2}-\left(\frac{H_0^{(i)}}{M_0}+\beta-\alpha\Delta\right)\left(\left(\frac{H_0^{(i)}}{M_0}+\beta\right)+4\pi-\alpha\Delta\right)\right)\times\\&\times\Delta\Phi_0+4\pi\left(\frac{H_0^{(i)}}{M_0}+\beta-\alpha\Delta\right)\frac{\partial^2\Phi_0}{\partial z^2}=0.\end{aligned} \quad (4)$$

As seen, equation we have obtained is similar to the known equation for a cylindrical nanowire (see, for example, [10]).

We note that equation (4) does not contain geometric parameters of the system. This equation will be true for any system composed of a uniaxial ferromagnet, as long as equilibrium magnetization is directed along the easy magnetization axis (and, therefore, external magnetic field is absent or directed also along this direction), along which the Oz-axis is also directed. To use the symmetry of the considered system, it is also necessary that this axis coincides with the normal to the film plane.

4. DISPERSION RELATION

Let us find the dispersion relation for spin waves under consideration using equation (4).

We note that pattern of spin waves in the system substantially depends on the minimum distance between neighboring antidots d. If this distance is less or of the same order of magnitude than the characteristic exchange interaction length l_{ex}, description of spin waves in the system is complicated. However, at $d >> l_{ex}$ spin wave can freely propagate in between antidots. Let us consider this case.

Let us assume that symmetry of the solution of equation (4) coincides with the symmetry of the system and introduce the radius-vector $\vec r_\perp$ in the plane orthogonal to

$\vec{M}_0$ which will be considered the xOy plane. Let us choose the Ox and Oy axis in this plane in such a way that their directions coincide with the translation directions of the system. For the coordinate system chosen in such a way, orthogonal radius-vectors of the centers of antidots are written in the form of $\vec{r}_{0j} = a\left(N_{1j}\vec{e}_x + N_{2j}\vec{e}_y\right)$, where N_{1j}, N_{2j} are the integers; j is the number of an antidot. In this case, we can seek the magnetic potential in the form of superposition of cylindrical wave fronts with the same transverse wavenumber $k_\perp$:

$$\Phi = \exp\left(i\left(k_{||}z - \omega t\right)\right)\times \times\sum_{j,n}\left(A_{jn}J_n\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|\right) + B_{jn}N_n\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|\right)\right)\exp\left(in\theta_j\right), \quad (5)$$

here θ_j is the polar angle measured from the center of j-th antidot.

According to our assumption, symmetry of the resulting pattern of spin waves coincides with the symmetry of the system. Thus, for each n all A_{jn} and all B_{jn} are equal (we will denote them A_j and B_j, respectively), and number n should be 4-fold. Taking this into account, we will re-write (5) in the following form:

$$\Phi = \exp\left(i\left(k_{||}z - \omega t\right)\right)\times \times\sum_{j,n}\left(A_nJ_{4n}\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|\right) + B_nN_{4n}\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|\right)\right)\exp\left(4in\theta_j\right). \quad (6)$$

We will substitute the solution (6) into the equation (4). From the properties of linearity of the Laplace operator we will obtain for each wave (and, thus, for the superposition of waves)

$$\Delta\Phi = -\left(k_\perp^2 + k_{||}^2\right)\Phi . \quad (7)$$

Thus, after substitution of the solution (5) into the equation (4) we obtain the dispersion equation similar to the dispersion equation for a cylindrical nanotube and a cylindrical nanowire:

$$\alpha^2\left(k_{||}^2 + k_\perp^2\right)^3 + 2\alpha\left(\tilde\beta + 2\pi\right)\left(k_{||}^2 + k_\perp^2\right)^2 + +\left(\tilde\beta\left(\tilde\beta + 4\pi\right) - \frac{\omega^2}{\gamma^2M_0^2} - 4\pi\alpha k_{||}^2\right)\left(k_{||}^2 + k_\perp^2\right) - 4\pi\tilde\beta k_{||}^2 = 0, \quad (8)$$

where $\tilde\beta = \frac{H_0^{(e)}}{M_0} + \beta$. Hence we can obtain the dispersion equation in the form of

$$\omega = \gamma M_0\sqrt{\alpha^2k^4 + 2\alpha\left(2\pi + \tilde\beta\right)k^2 + \tilde\beta\left(4\pi + \tilde\beta\right) - 4\pi k_{||}^2\left(\alpha + \frac{\tilde\beta}{k^2}\right)}, \quad (9)$$

where $k^2 = k_{||}^2 + k_\perp^2$ is the total wavenumber.

5. WAVENUMBER SPECTRUM

Let us note that the wavenumber of a spin wave has two components, the longitudinal and the transverse ones, and, thus, for the description of a spin wave, equation (9) should be supplemented with the spectrum of, at least, one of these components. Let us find the transverse wavenumber $k_\perp$ spectrum for the case of remote antidots described in the previous section.

In order to obtain the spectrum of $k_\perp$ it is necessary, in general, to impose the boundary conditions on the boundaries of antidots (if boundary conditions do not violate the symmetry of the problem, they should be identical on each antidot and transform into themselves when rotated by angle $\pi/2$ relative to the antidote axis). However, since, according to the assumption made in the previous section, symmetry of the solution corresponds to the symmetry of the problem, it is possible to find the transverse wavenumber from considerations of symmetry without imposition of the boundary conditions.

As seen, the translation symmetry with the translation period a is present in the system. We will use the Bloch theorem analogue. Cylindrical functions entering the solution (6) are not periodic ones, however, at large distances from the central antidot they asymptotically tend to the following expressions:

$$J_n\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|\right) \to \sqrt{\frac{2}{\pi k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|}}\cos\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right| - \frac{\pi n}{2} - \frac{\pi}{4}\right), \quad (10)$$

$$N_n\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|\right) \to \sqrt{\frac{2}{\pi k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right|}}\sin\left(k_\perp\left|\vec{r}_\perp - \vec{r}_{0j}\right| - \frac{\pi n}{2} - \frac{\pi}{4}\right). \quad (11)$$

From considerations of symmetry on the translation period a phase of these functions should be changed by the value multiply of 2π. Hence we obtain that transverse wavenumber of a spin wave can be represented in the form

$$k_\perp = \frac{2\pi N}{a}, \quad (12)$$

where $N \in \aleph\cup\{0\}$ is the number of the transverse mode. Substituting this form of the transverse wavenumber into equation (9), we find the dispersion relation as follows:

$$\omega_N\left(k_{||}\right) = \gamma M_0\sqrt{\alpha^2\left(k_{||}^2 + \left(\frac{2\pi N}{a}\right)^2\right)^2 + 2\alpha\left(2\pi + \tilde\beta\right)\left(k_{||}^2 + \left(\frac{2\pi N}{a}\right)^2\right) + \tilde\beta\left(4\pi + \tilde\beta\right) - 4\pi k_{||}^2\left(\alpha + \frac{\tilde\beta}{k_{||}^2 + \left(\frac{2\pi N}{a}\right)^2}\right)}. \quad (13)$$

After specifying the film thickness and boundary conditions on the film boundaries, it is also possible to obtain the longitudinal wavenumber spectrum and, therefore, the spin wave frequency spectrum. Thus, for a film thinner than the exchange length l_{ex}, we can neglect the spin oscillations in the Oz direction taking $k_{||} = 0$ and write the spin wave frequency spectrum in the following form:

$$\omega_N = \gamma M_0\sqrt{\alpha^2\left(\frac{2\pi N}{a}\right)^4 + 2\alpha\left(2\pi + \tilde\beta\right)\left(\frac{2\pi N}{a}\right)^2 + \tilde\beta\left(4\pi + \tilde\beta\right)}. \quad (14)$$

If the film thickness exceeds l_{ex}, but a non-magnetic metal with a relatively high conductivity is located on its boundaries, so that when writing the boundary conditions we can assume it ideal, boundary conditions for the magnetic potential are reduced to the conditions of nulling of its normal derivative $\partial\Phi/\partial z$ on the film boundaries, wherefrom we obtain

$$k_{||p} = \frac{2\pi p}{l}, \quad (15)$$

$$\omega_{Np} = \gamma M_0 \sqrt{\frac{16\pi^4\alpha^2}{D_{Np}^4} + \frac{8\pi^2\alpha\left(2\pi+\tilde{\beta}\right)}{D_{Np}^2} + \tilde{\beta}\left(4\pi+\tilde{\beta}\right) - 16\pi^3\left(\frac{p}{l}\right)^2\left(\alpha+\frac{\tilde{\beta}}{4\pi^2 D_{Np}^2}\right)}; \quad (16)$$

here $D_{Np} = 1/\sqrt{\left(\frac{p}{l}\right)^2 + \left(\frac{N}{a}\right)^2}$, p is the integer number – the number of the longitudinal mode.

6. CONCLUSIONS

Thus, the dipole-exchange linear spin waves in a thin ferromagnetic film with a two-dimensional periodic system of antidots have been studied in the work. The differential equation for the magnetic potential of such waves has been obtained in the magnetostatic approximation. This equation has been solved for the case when antidots are rather remote, so that the minimum distance between them much exceeds the exchange length. The dispersion relation for such spin waves has been derived. The transverse wavenumber spectrum has been also obtained for the case of remote antidots.

The spin wave frequency spectrum has been found for two particular cases (a film thin compared with the exchange length and a film restricted by a metal with high conductivity).

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