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Quantum Properties of Bloch Point as Nanosized Soliton in Ferromagnetics

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It is established that magnetic soliton – Bloch point – has quantum properties which are manifested in the effects of tunneling and above-barrier reflection in a subhelium temperature range. The conditions of the given phenomena are determined.

Keywords: Quantum tunneling, Bloch point, Domain boundary, Vertical Bloch line, Ferromagnetic materials, Potential barrier, Soliton, Quantum depinning, Above-barrier reflection.

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1. INTRODUCTION

Mesoscopic systems realized in uniaxial ferromagnets with a strong uniaxial magnetic anisotropy are the subject of scrutiny (see, for example, [1, 2] and references therein). Among these systems, one can single out the domain boundaries (DB) and elements of their internal structure, namely, vertical Bloch lines (VBL) and Bloch points (BP) [3]. VBL and BP are stable local inhomogeneities of DB with the typical size of ~ 10^2 nm and considered as the element base in magnetic storage devices [4]. Moreover, structures like VBL and BP take place in ferromagnetic nanowires and nanoribbons [5-8], i.e. in materials promising for application in spintronics.

We should note that from the mathematical point of view DB, VBL and BP are nonlinear wave formations – solitons with a certain topology. As a result, state of the given structures is characterized by the so-called topological charge (chirality) [3], which determines the turn of the magnetization vector $\vec{M}$ in the system center. It is obvious that because of the nature, this parameter is degenerate. However, in the low-temperature range, i.e. $T < 1$ K, splitting along the directions of the vector $\vec{M}$ is possible using the under-barrier quantum tunneling. Quantum fluctuations of this type in DB of different ferro- and antiferromagnetic materials were considered in the works [9-12]. Tunneling of a topological charge of VBL in ultrathin magnetic film was studied in [13].

We note that quantum depinning of DB and VBL takes place in a subhelium temperature range [14, 15]. At the same time, the aspects conditioned by the point soliton (BP) nucleation processes [16-18] definitely indicate the presence of quantum properties of the given element of the DB internal structure. Investigation of this question for BP in DB of ferromagnets with the material quality factor (ratio of the magnetic anisotropy energy to the magnetostatic one) $Q >> 1$ is the aim of our work. We will study the quantum effects conditioned by the interaction of BP with a defect: quantum tunneling and above-barrier reflection. We will also establish the realization conditions of these phenomena.

2. QUANTUM TUNNELING OF THE BLOCH POINT

We will consider the domain boundary, in which VBL and Bloch point dividing a Bloch line into two regions with different signs of a topological charge are the elements of the internal structure. We will introduce the Cartesian coordinate system with the origin in the BP center and direct the OZ-axis along the anisotropy axis, the OY-axis – along the normal to the DB plane. Based on the Slonchevsky equations [3], one can show that in the area of the domain boundary $\Delta < r \le \Lambda$, where Δ is the DB width, $r = \sqrt{x^2+z^2}$, $\Lambda = \Delta\sqrt{Q}$ is the typical size of VBL, distortion of the magnetic structure of the last BP takes place, and the following “vortex solution” corresponds to this distortion [19]:

$$\operatorname{tg}\varphi = z/x\,, \qquad (1)$$

where $\varphi = \operatorname{arctg}(M_y/M_x)$, $M_{x,y}$ are the components of $\vec{M}$. At the same time, distribution of magnetization along the OY-axis has a Bloch view: $\sin\theta = ch^{-1}(y/\Delta)$, where θ is the polar angle in the chosen coordinate system.

We note that the above indicated area is supposed to be a characteristic scale of BP, since exactly this area of DB conditions the main contribution into $m_{BP} = \Delta\,\gamma^2$ (γ is the gyromagnetic ratio) – the effective BP mass [19].

Taking into account (1) and assuming that the BP motion along the DB is self-similar ($\varphi = \varphi(z - z_0, x)$, z_0 is the coordinate of the BP center), after some transformations, the interaction energy between the Bloch point W_H and the external magnetic field $\vec{H}_y = -H\vec{e}_y$ can be written as follows

$$W_H = -M_S\pi\Delta z_0 H \int\limits_{\Delta < r \le \Lambda} dxdz\left(\cos\phi\frac{\partial\phi}{\partial z}\right)\Bigg|_{z_0=0} \approx \qquad (2)$$
$$\approx -M_S\pi^2\Delta\Lambda z_0 H,$$

M_S is the saturation magnetization of the material.

For the description of the BP dynamic behavior in the defect field H_d we will use the Lagrangian formalism. Then, based on (2), “potential energy” $W(z_0)$ in the Lagrange function $\mathcal{L} = m_{BP}\dot{z}_0^2/2 - W(z_0)$ will be represented in the form

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$$W(z_0) = -M_S^2\pi^2\Lambda\Delta\int\limits_0^{z_0} dz'\left(H - H_d(z')\right). \quad (3)$$

Expanding $H_d(z_0)$ into series near the defect position, the defect field can be written as

$$H_d(z_0) = H_c\left(1 - (z_0 - d)^2 / 2D^2\right), \quad (4)$$

where H_c is the coercive force of the defect; d is the coordinate of the defect center; $D^{-2} = \frac{1}{H_c}\frac{\partial^2 H_d}{\partial z_0^2}\bigg|_{z_0 = d}$; D is the barrier width.

It is natural to assume that the characteristic change of the defect field is determined by the size factor of interacting with it magnetic irregularity. In our case, $\partial^2 H_d / \partial z_0^2 \sim H_c / \Lambda^2$ and, correspondingly, $D \sim \Lambda$. We note that the above stated proposition about the defect field also correlates with the results of the work [20] which imply the dependence of H_c on the characteristic size of DB, VBL or BP.

Then, substituting (4) into (3) and taking into account that in the point $z_0 = 0$ potential W has a local metastable minimum (see Fig. 1), we obtain the following expression:

$$W(z_0) = \frac{\pi^2 Q^{-1/2} M_S H_c}{2}\left(-\frac{z_0^3}{3} + dz_0^2\right), \quad (5)$$

where $d = \Lambda\sqrt{2\varepsilon}$, $\varepsilon = 1 - H/H_c << 1$ (we consider the values of the magnetic fields H near H_c that substantially decreases the potential barrier height).

Here, potential $W(z_0)$ satisfies the normalization condition $W(z_{0,1}, z_{0,2}) = 0$, where $z_{0,1} = 0$ and $z_{0,2} = 3\Lambda\sqrt{2\varepsilon}$ are the barrier coordinates. We should also note that expression (5) corresponds to the model potential used in the works [14, 15] in the study of the quantum depinning of DB and VBL.

Following the VBL approximation formalism, the BP tunneling amplitude P will be defined by the formula

$$P \sim \exp(-B),$$

where $B = \frac{2}{\hbar}\int\limits_{z_{0,1}}^{z_{0,2}} |\dot{z} m_{BP}| dz$, $\hbar$ is the Plank constant.

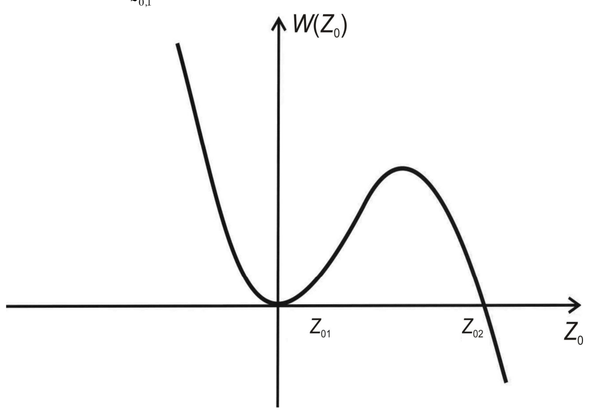

Fig. 1 – Interaction potential $W(z_0)$ of the Bloch point with the external magnetic field H and defect field H_d

After variation of the Lagrange function $\mathcal{L}$ and integration of the obtained differential equation with the boundary condition in the point $z_0 = 0$, $\dot{z}_0 \to 0$, $t \to -\infty$ that corresponds to the BP pinning on the defect in the field $\vec{H}_x$ absence, we determine the BP pulse and, correspondingly, the tunneling exponent

$$B = \frac{2}{\hbar}\int\limits_{z_{0,1}}^{z_{0,2}} \sqrt{2m_{BP}W(z)}dz. \quad (6)$$

Taking into consideration (5), expression (6) can be rewritten as follows

$$B = \frac{8\Delta^3 Q h_c^{1/2}\varepsilon^{5/4}}{\hbar\omega_M}(4\pi M_S)^2, \quad (7)$$

where $h_c = H_c/8M_S$, $\omega_M = 4\pi\gamma M_S$.

Temperature T_{cr}, at which quantum BP motion mode is actual, follows from the relation $T_{cr} = W_{max}/k_B B$, where W_{max} is the maximum value of the potential barrier; k_B is the Boltzmann constant. Then, according to (5) and (7), we obtain

$$T_{cr} = \frac{\sqrt{2}\varepsilon^{1/4} h_c^{1/2}\hbar\omega_M}{12k_B}. \quad (8)$$

Substituting into (7) and (8) parameters corresponding to the uniaxial magnetic films, namely, $Q \sim$ 5-10, $\Delta \sim 10^{-6}$ cm, $4\pi M_S \sim (10^2\text{-}10^3)$ Gs, $H_c \sim (10\text{-}10^2)$ Oe [20], $\gamma \sim 10^7$ Oe^{-1}s^{-1}, for $\varepsilon \sim 10^{-4}\text{-}10^{-2}$ we obtain $B \approx$ 1-30 and $T_{cr} \sim (10^{-3}\text{-}10^{-2})$ K.

The obtained estimate $B \le 30$ agrees with the corresponding values of the tunneling exponent for magnetic nanostructures [21] which indicate the possibility of realization of the given quantum effect. Here, as seen from the definition of the BP effective mass, in contrast to the tunneling through the DB defect and VBL, the BP tunneling is implemented by the "transfer" through the potential barrier of the whole BP effective mass at once. The given feature is the consequence of the size factor of quasi-particles. Thus, the characteristic size of BP is $\sim \Lambda^3$. At the same time, the characteristic scales of DB and VBL are $\Delta L_y h$ (L_y is the film length along the OY-axis; h is the film thickness) and $\Lambda\Delta h$, respectively. It is clear that in this case overcoming of the potential barrier of DB or VBL requires successive quantum displacements of small regions of the area (for DB) or length (VBL) of the given nano-objects.

After integration of the equation of BP motion obtained by the variation of the Lagrange function, we find the instanton trajectories z_{BP} and frequency ω_0 of a Bloch point which characterize its motion in the space with the "imagine" time ($\tau = it$): from the point $z_1 = 0$ at $\tau = -\infty$ to the point $z_2 = 3\Lambda\sqrt{2\varepsilon}$ at $\tau = 0$; and back to the point z_1 at $\tau = +\infty$

$$z_{BP} = 3\Lambda\sqrt{2\varepsilon} / ch^2(\omega_{in}\tau),$$
$$\omega_0 = \omega_M h_c^{1/2}(2\varepsilon)^{1/4} / 2. \quad (9)$$

Having determined the BP instanton frequency, we will consider a question about the correctness of the use of the VBL formalism. As known, according to [22], the usability condition of the VBL method is the fulfillment of the following inequality:

$$m\hbar\left|F\right|/p^3 << 1\,, \tag{10}$$

where p is the pulse; m is the quasi-particle mass; F is the force acting on the quasi-particle.

It is obvious that in our case $F = m_{BP}\omega_0^2\xi$, $p = m_{BP}\omega_0\xi$, $\xi \sim \Lambda\sqrt{2\varepsilon}$. Taking into account (9), formula (10) can be rewritten in the form

$$\hbar\gamma^2\omega_M^{-1}h_c^{-1/2}\left(2\varepsilon\right)^{-5/4}Q^{-1}/\Delta^3 << 1\,. \tag{11}$$

Substituting into (11) the above specified parameters (see estimation of expressions (7), (7)), it is easy to make sure in fulfillment of the given relation that, in turn, implies the legitimacy of the application of the quasi-classical approximation to the considered problem.

Now we will evaluate the influence of the dissipation on the BP tunneling. To this end, we compare the force F acting on the quasi-particle with the braking force F_r, which in our case is equal to ~ $\alpha\omega_M\omega_0\Lambda\sqrt{2\varepsilon}m_{BP}$, where $\alpha \sim 10^{-3}$-10^{-2} is the damping parameter of the magnetization. Then, taking into account the explicit view of F, we find the following expression:

$$F_r/F = 2\alpha/h_c^{1/2}\left(2\varepsilon\right)^{1/4}.$$

Analysis of the last expression shows that $F_r/F << 1$ at $10^{-2} \le h_c \le 10^{-1}$ and $\varepsilon \sim 10^{-4}$-10^{-2}. The obtained result indicates that in the consideration of the BP quantum tunneling we can neglect the influence of the braking force in the given materials.

We note that nature of the force F_r was studied in the work [23] and is conditioned by accounting of the terms of the exchange nature in the Landau-Lifshitz equation for the ferromagnet magnetization [24].

3. ABOVE-BARRIER REFLECTION OF THE BLOCH POINT

We have established in the previous Section that quantum depinning of BP, in contrast to DB and VBL, is realized by the under-barrier transition at once of the whole effective mass of the quasi-particle. The given result implies that the presence of a metastable minimum in the interaction potential of BP and defect is not necessary. The latter indicates that there is a principal possibility of realization of the quantum effect of BP above-barrier reflection. In this case, velocity of BP fall on the barrier can be conditioned by the pulse of the magnetic field applied to BP (velocity with which). Obviously, the effect is most noticeable when the BP energy does not much exceed the potential barrier height U_0.

Based on formula (2), we write equation of the BP dynamics in the pulsed magnetic field $H_y(t) = H_0\chi(1 - t/T)$ in the following form:

$$m_{BP}\,\partial\upsilon/\partial t + F_r = \pi^2\Lambda\Delta M_S H_y\left(t\right), \tag{12}$$

$v = \partial z_0/\partial t$ is the BP velocity; $\chi(1 - t/T)$ is the Heaviside function; H_0 is the amplitude; T is the pulse duration.

Integrating equation (12) at $T \le t << \alpha^{-1}\omega_M^{-1}$, we find $v(t) = \pi^2 M_S\Lambda\Delta H_0 T/m_{BP}$ – the BP velocity upon termination of the pulse. Correspondingly, the BP energy E_{BP} in the considered time interval has the view

$$E_{BP} = m_{BP}\upsilon^2/2 = \pi^2\omega_M^2T^2\Lambda^2\Delta H_0^2/32\,. \tag{13}$$

We note that investigation carried out for the time $t << \alpha^{-1}\omega_M^{-1}$ (or taking into account the value of the magnetization damping $\omega_M t << (10^2$-$10^3)$ allows us to neglect the influence on the process of the braking force F_r, which in the given case is ~ $\alpha\omega_M m_{BP}v$.

Assuming that defect is located in the point $z_0 = 0$, by analogy with [25] we write its potential U_d in the form

$$U_d\left(z_0\right) = U_0 ch^{-2}\left(z_0/\Lambda\right), \tag{14}$$

where according to (2) $U_0 = \pi^2\Lambda^2\Delta M_S H_c$.

Such a view of the potential is not only general but also agrees with the approach expressed by the formulas (3)-(5). Indeed, having supplemented (14) with the term $W_H = -M_S^2\pi^2\Lambda\Delta z_0 H$, after series expansion of the final expression in the point $\tilde{z}_0 = \Lambda Ar\sinh\left(1/\sqrt{2}\right)$, inflection point of the function $\mathrm{ch}^{-2}(z_0/\Lambda)$ and potential normalization (in the coordinate system with the center at $\tilde{z}_0$), we obtain (5). It is clear that just at $z_0 = \tilde{z}_0$ the defect field is maximum. Therefore, if quasi-particle overcomes the barrier in the given point, then the tunneling process in whole takes place for it. Consideration of the quantum depinning of DB and VBL in the works [14, 15] was based, in essence, on this fact. Asymptotic of the potential near its maximum value is actual in our case.

In the framework of the VBL approximation, using the formalism proposed in [26], we determine the coefficient of the BP above-barrier reflection by the formula

$$R = e^{-\beta}\,, \tag{15}$$

where $\beta = -\dfrac{2}{\hbar}\mathrm{Im}\int\limits_{z_{0,1}^*}^{z_{0,2}^*}dz\sqrt{2m_{BP}\left(E_{BP} - U_d\left(z\right)\right)}$, $z_{0,1}^*$, $z_{0,2}^*$ are the roots of the equation $E_{BP} - U_d(Z_0) = 0$.

Expanding into series expression (14) and taking into account (15), we obtain

$$\beta = \pi\sqrt{2m_{BP}}E_{BP}\Delta\varepsilon'/\hbar\sqrt{U_0}\,, \tag{16}$$

here parameter $\varepsilon' = (E_{BP} - U_0)/E_{BP} << 1$ (we remind that the case of close values of E_{BP} and U_0 is considered).

Using (13), formula (16) can be rewritten in the form

$$\beta = \pi\left(2M_S H_c\right)^{1/2}\varepsilon'\gamma^{-1}\Delta^3 Q^{1/2}/\hbar\,. \tag{17}$$

Substituting into expressions (15) and (17) the corresponding numerical parameters, at $\varepsilon' \ge 5\cdot10^{-5}$ we obtain $R \le 0.1$ that agrees with the criterion of applicability of formula (15) (see in [22, 26]).

We should note that it follows from expressions (15) and (17) that $R \to 0$ at $U_0 \to 0$, i.e. we obtain physically consistent conclusion about disappearance of the above-barrier reflection effect of the quasi-particle in the potential barrier absence.

Based on $\tau \sim \Delta\left(m_{BP}/U_0\right)^{1/2} = 4\left(M_S/H_c\right)^{1/2} Q^{-1/2}\omega_M^{-1}$ and the above stated parameters, we determine the characteristic interaction time τ between the BP and defect as $0.6 \le \omega_M\tau \le 2.3$. It is easy to see that τ satisfies relation $\omega_M\tau < \omega_M t \sim 10\text{-}10^2$, which in conjunction with the estimate for R indicates the principal possibility of realization of the considered quantum effect.

Now we will investigate the question about the correctness of applicability of the VBL formalism. Since in our case $E_{BP} \approx U_0$, then conditions of the quasi-classicality of BP and potential barrier coincide, and according to the work [22] they are reduced to the fulfillment of the following inequality:

$$\delta z_0\sqrt{m_{BP}U_0}/\hbar >> 1 \,, \tag{18}$$

where $\delta z_0 = \Delta\sqrt{2\left(E_{BP} - U_0\right)/U_0} \approx \Delta\sqrt{2\varepsilon'}$.

Taking into account the explicit form of U_0, condition (18) can be re-written as follows

$$\pi\gamma^{-1}\Delta^3\left(M_S H_c\right)^{1/2} Q^{1/2}\left(\varepsilon'\right)^{1/2} >> \hbar \,.$$

Evaluation of the last expression shows its fulfillment at $\varepsilon' \ge 10^{-4}$ that, in fact, is the estimate from below for this parameter.

Critical temperature T'_{cr} corresponding to the studied effect can be defined from simple qualitative considerations. Indeed, process of the BP above-barrier reflection is equivalent to the quasi-particle fluctuations with the frequency of $\nu = 2\Delta^{-1}\sqrt{U_0}/\sqrt{m_{BP}}$ along the imagine iz-axis between the points $z^*_{0,1,2} = \pm i\Delta\sqrt{\varepsilon}$. Since BP motion occurs in the absence of external fields, we have one unexcited quantum level ($n = 0$). In this case, BP energy E_{BP} is not changed and is equal to $\hbar\nu/2\pi$. Then, equating E_{BP} to the relation $k_B T'_{cr}$ (k_B is the Boltzmann constant), we obtain

$$T'_{cr} = \frac{\hbar U_0^{1/2}}{\pi k_B\sqrt{2m_{BP}}\Delta} = \frac{\hbar\gamma}{\sqrt{2}k_B}\left(M_S H_c\right)^{1/2}. \tag{19}$$

Evaluation of (19) shows that $T'_{cr} \sim \left(10^{-3} - 10^{-2}\right)$ K. These values of T'_{cr} are in the same range with the critical temperatures of quantum depinning of DB and elements of its internal structure, namely, VBL and BP. The given fact indicates the importance of accounting of the quantum properties of BP in the study of the low-temperature dynamics of DB with the complex internal structure in ferromagnetic materials. Here, since tunneling of BP and its above-barrier reflection occurs in different magnetic fields, then there is a practical possibility of the separate study of these effects.

4. CONCLUSIONS

It is established that Bloch point (magnetic soliton) in the uniaxial ferromagnets in subhelium temperature range has quantum properties. A principal possibility of the quantum depinning and above-barrier reflection of the Bloch point from the defect potential is shown.

Experimental realization of the considered effects can be the basis for the development of new precision methods of diagnostics of the internal structure of domain boundaries in ferromagnetic materials.

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