Effect of Spin-orbit Interaction on the Magneto-optical Spectra of InSb Quantum Dots

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The work presents the theoretical calculations of the dependences of the energy levels and the electron absorption spectra in a parabolic quantum dot on the applied magnetic field. Effect of the spin-orbit interaction in the calculations was taken into account. Spin-orbit interaction gives rise to a leap dependence of the absorption frequency of quantum dots on the applied magnetic field.

Keywords: Spin-orbit interaction, Quantum dot, Absorption spectra.

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1. INTRODUCTION

Low-dimensional systems opened a new field of physical investigations which have been conducted during the last 20 years. Using modern semiconductor technologies it was able to design and create quantum dots (QD) and systems of QD with the specified parameters. QD have much in common with atoms, but they are artificial objects developed and produced in laboratories. One of the ways for creating the QD consists in the limitation of two-dimensional electron gas in semiconductor heterostructure using an external electric field [1-6]. Such quantum structures restrict the motion of a small amount of electrons in all three spatial dimensions.

Magneto-optical properties of QD are considered in a number of works [7-8]. The influence of spin-orbit (SO) interaction connected with the perpendicular to the symmetry axis dependence of the confining potential of a quantum dot is not taken into account in these works.

The study of the electronic and optical properties of QD is important on the grounds that the given structures are promising for the development of optoelectronic devices and application in optical quantum-information technology [9, 10].

SO interaction in low-dimensional structures, such as QD, has been intensively studied lately. The present investigation is the consequence of the fact that these structures can be used in the devices connected with the spin transport [11-13]. The main goal of these works was to show that using SO interaction one can control the spin states in QD. The spin dynamics of electrons in QD was considered in the works [14-19]. Development of the data recording and read-out mechanisms of the spin information is the result of the above mentioned works. The majority of modern experimental works are focused on the magneto-transport phenomena [20-21]. The theoretical calculations of the influence of SO interaction on the QD absorption spectra, which can be studied experimentally, are presented in these works.

2. THEORY

For the description of a quantum dot we will use the following model. 2D electron gas is limited by a parabolic potential. A cylinder will be used as a quantum dot shape. Electron motion along the Z-axis is limited by an infinitely deep potential well (Fig. 1c).



Fig. 1 - View and profile of the quantum dot potential

For the description of the potential profile in the *XY*-plane we will use a parabolic potential (Fig. 1b)

$$V_c(r) = \frac{1}{2} m(E) \omega_0^2 r^2 , \qquad (1)$$

m(E) is the effective electron mass which depends on the electron energy; $\hbar \omega_0$ is the characteristic electron binding energy in a quantum dot. The typical size of a quantum dot is determined from the correlation [14]

$$r_0 = \sqrt{\frac{2E_f}{m(E_f)\omega_0^2}},$$
 (2)

where E_f is the Fermi level.

A uniform constant magnetic field, to which the vector potential

$$\mathbf{A} = \left(-\frac{B_y}{2}; \frac{B_x}{2}; 0\right), \ \mathbf{B} = (0; 0; B)$$

corresponds, is applied along the Z-axis.

One-electron Hamiltonian in the cylindrical coordinate system $\{r, \phi\}$ can be written in the form of [14]

$$\hat{H} = -\frac{\hbar^2}{2m(E)} \left[\frac{\partial}{r\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} \right] - \frac{i}{2} m \omega_c(E, B) \frac{\partial}{\partial \varphi} + \frac{1}{8} m(E) \omega_c^2(E, B) r^2 + V_c(r) + V_{so}^R(r, \varphi) + \frac{1}{2} \sigma_z \mu_B g(E) B, \quad (3)$$

where the first three terms describe the kinetic energy of an electron moving in the $\{x, y\}$ -plane; the fourth term describes the parabolic potential; the fifth term – the SO interaction, and the last one – the interaction of the magnetic field and the electron spin. Calculations of the energy spectra and view of the wave functions for the

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Hamiltonian (3) are carried out in [14].

SO interaction for a parabolic quantum dot was described by Rushba and has the form of [22-24]

$$\hat{V}_{so}^{R}(r,\phi) = \hat{\sigma}_{z} \alpha \, \frac{dV_{c}(r)}{dr} \left(k_{\varphi} + \frac{e}{2\hbar} Br \right), \tag{4}$$

where $k_{\varphi} = -i(1/\rho)\partial/\partial\varphi$, α is the parameter of SO interaction introduced by Rushba [22].

Schrödinger equation with Hamiltonian (3) has the analytical solution [14]. The eigen energies and the wave function have the following view:

$$E_{n,l,\sigma} = \hbar\Omega_{\sigma}(E_{n,l,\sigma}, B)(2n + |l| + 1) + l\frac{\hbar\omega_{c}(E_{n,l,\sigma}, B)}{2} + \sigma \left[\frac{\mu_{B}}{2}g(E_{n,l,\sigma})B + l\alpha m(E_{n,l,\sigma})\omega_{0}^{2}\right],$$
(5)
$$\psi_{n,l,\sigma}(r,\phi) = \frac{1}{\sqrt{2\pi}}\exp(il\phi)\sqrt{\frac{2}{l_{B}^{2}}} \left[\frac{n!}{(n+|l|)!}\right]^{1/2}\exp\left(-\frac{r^{2}}{2l_{B}^{2}}\right) \cdot \left(\frac{r^{2}}{l_{B}^{2}}\right)^{|l|/2} \cdot L_{n}^{|l|}\left(\frac{r^{2}}{l_{B}^{2}}\right),$$

where *n* is the principal quantum number; *l* is the quantum number responsible for the projection of the electron orbital moment on the *z*-axis; $\sigma = \pm 1$ is the quantum number responsible for the projection of the electron spin on the *z*-axis, $\mu_B = e\hbar/2m_0$ is the Bohr magneton; *e* is the electron charge; m_0 is the free electron mass; $L_n^{[4]}(x)$ is the Laguerre polynomial; $l_B = \sqrt{\hbar/m\Omega_\sigma}$ is the effective magnetic length;

$$\Omega_{\sigma}^2 = \omega_0^2 + \frac{\omega_C^2(E,B)}{4} + \sigma \alpha \frac{m(E)\omega_0^2}{\hbar} \omega_C(E,B) \ .$$

The effective electron mass is represented by the correlation [22]

$$\frac{1}{m(E)} = \frac{1}{m(0)} \frac{E_g \left(E_g + \Delta\right)}{\left(3E_g + 2\Delta\right)} \left[\frac{2}{E + E_g} + \frac{1}{E + E_g + \Delta}\right], \quad (6)$$

where E is the electron energy in the conduction band; m(0) is the effective electron mass at the conduction band bottom; E_g is the band gap; Δ is the conduction band spinorbit splitting;

$$\omega_c(E,B) = \frac{eB}{m(E)}$$

is the cyclotron electron frequency;

$$g(E) = 2 \left[1 - \frac{m_0}{m(E)} \frac{\Delta}{3(E_g + E) + 2\Delta} \right]$$
(7)

is the effective g-factor for semiconductors [25].

Electron energy levels in a quantum dot can be obtained by the self-consistent solution of equations (5)-(7). Calculations have been performed using the method of successive approximations.

For the calculation of the absorption of electromagnetic radiation by a quantum dot, we consider the case of light incidence on QD along the Z-axis. Intensity of absorption by a quantum dot within the dipole approximation is proportional to the square of matrix elements $A_{nls}^{n'ls'} = \langle \psi_{nls} | r e^{\pm i\theta} | \psi_{n'ls'} \rangle$ for the case when electron passes from the state $|\psi_{n'ls'}\rangle$ into the state $\langle \psi_{nls} |$ [7]. Sign of the exponent determines polarization of the absorbed radiation. Oscillator force is defined by the expression $f_{nls}^{n'ls'} = (2m/\hbar)\omega_{nls}^{n'ls'} |A_{nls}^{n'ls'}|^2$, where $\omega_{nls}^{n'ls'} = (E_{n'ls'} - E_{nls})/\hbar$ is the transition frequency. The rule of selection for the

allowed dipole transitions ($\Delta l = l' - l = \pm 1$, $\Delta s_z = 0$,) holds for the electron states.

All subsequent calculations will be performed in the extreme case for the temperature of T = 0 K.

3. RESULTS

The following parameters will be used for the InSb quantum dot: $m(0) = 0.014m_0$, $E_g = 0.24$ eV, $\Delta = 0.81$ eV, $\alpha = 5$ nm², $\hbar \omega_0 = 0.025$ eV [14, 26, 27].

In Fig. 2a, c we present the dependences of the electron energy levels in a quantum dot on the external magnetic field. In Fig. 2a we illustrate the dependence without SO interaction. Arrows show the allowed dipole electron transitions between energy levels. If take into account SO interaction, the energy level splitting appears in the absence of the magnetic field (Fig. 2c). The values of the photon absorption energy (Fig. 2d) are also split at zero value of the magnetic field.

At the filling of a quantum dot by two or three electrons, absorption spectra intersect taking into account SO interaction. It is well seen in Fig. 3d for three electrons.

In Fig. 3a, c we present the dependences of the energy levels of three non-interacting electrons situated in a quantum dot on the external magnetic field. The energy is calculated from the relation $E_{total} = \sum E_{n, l, \sigma}$ where n, l, σ determine the occupied electron states. Thick lines denote the energy levels of electrons which, according to the rules of selection, can participate in light absorption by QD. Taking into account SO interaction, the energy level splitting appears at zero value of the magnetic field (Fig. 3c). Dependences of the absorption spectra taking into account SO interaction and without consideration of SO interaction on the applied magnetic field are also represented in Fig. 3b, d.

A more complex pattern appears if consider a quantum dot filled with four electrons (Fig. 4). As it is seen from Fig. 4c, intersection of the lower levels occurs in the spectrum if take into account SO interaction. Thus, level with the electron states $|0,0,1\rangle$, $|0,0,-1\rangle$, $|0,-1,1\rangle$ and $|0,1,-1\rangle$ (where $|n,l,\sigma\rangle$ is the state with the corresponding quantum numbers) is filled at fields lower than B = 0.2 T. At fields higher than B = 0.2 T, level with the electron states $|0,0,1\rangle$, $|0,-1,1\rangle$ and $|0,-1,-1\rangle$ is filled. Change of the ground electron level with increasing magnetic field led to the changes in the absorption spectra (Fig. 4d, e). As seen from Fig. 4e, because of SO interaction at B = 0.2 T, absorption spectrum lines not only spasmodically change their position (absorption frequency), but also the absorption intensity.



Fig. 2 – Dependences of the electron energy levels (a, c) and absorption spectra (b, d) of the InSb quantum dot with one electron on the applied magnetic field. Dependences without (a, b) and with (c, d) taking into account the influence of SO interaction. Area of the circles in the dependences (b, d) is proportional to the absorption intensity of the quantum dot



Fig. 3 – Dependences of the total energy of three non-interacting electrons (a, c) and absorption spectra (b, d) of the InSb quantum dot with three electrons on the applied magnetic field. Dependences without (a, b) and with (c, d) taking into account the influence of SO interaction. Area of the circles in the dependences (b, d) is proportional to the absorption intensity of the quantum dot

The same pattern appears when considering a quantum dot filled with five electrons (Fig. 5). If examine the electron spectrum of a quantum dot without taking into account SO interaction (Fig. 5a), it is seen that at the value of the magnetic field B = 2.6 T lower electron levels intersect. The mentioned intersection results in an abrupt change of the absorption energy of external electromagnetic radiation by a quantum dot during the transition through the critical value of the magnetic field (Fig. 5b). One more intersection of lower levels at B = 0.8 T appears on the energy spectrum of a quantum dot when taking into account SO interaction (Fig. 5c). SO interaction also influenced the intersection earlier considered in Fig. 5a; the given intersection shifted to the region of lower fields, $B \approx 2.4$ T (Fig. 5c). Change in the ground electron level led to the change in the absorption spectrum (see Fig. 5b, d, e). As seen, changes, which are similar to those occurred in a quantum dot filled with four electrons, are observed for the absorption spectrum line.



Fig. 4 – Dependences of the total energy of four non-interacting electrons (a, c) and absorption spectra (b, d, e) of the InSb quantum dot with four electrons on the applied magnetic field. Dependences without (a, b) and with (c, d, e) taking into account the SO interaction. Thick lines in the dependence (a, c) denote the energy levels participating in absorption. Area of the circles in the dependences (b, d, e) is proportional to the absorption intensity of the quantum dot



Fig. 5 – Dependences of the total energy of five non-interacting electrons (a, c) and absorption spectra (b, d, e) of the InSb quantum dot with five electrons on the applied magnetic field. Dependences without (a, b) and with (c, d, e) taking into account the SO interaction. Thick lines in the dependence (a, c) denote the energy levels participating in absorption. Area of the circles in the dependences (b, d, e) is proportional to the absorption intensity of the quantum dot

4. CONCLUSIONS

We have theoretically studied the energy spectra and magneto-optical transitions of non-interacting electrons in a parabolic InSb quantum dot containing to five electrons. Calculations have been performed taking into account the effect of SO interaction and external magnetic field. When considering a quantum dot with one electron it is seen that SO interaction removes degeneracy at B = 0 T of the optical absorption spectrum lines. For the case of three non-interacting electrons in a quantum dot, SO interaction not only partly removes degeneracy at B = 0 T of the absorption spectra lines, but also leads to the appearance of intersection of spectral lines at $B \approx 0.2$ T and $B \approx 2.3$ T. When considering a quantum dot with four or five non-interacting electrons, ground state of electron energy levels varies with the change in the applied magnetic field. These changes lead to an abrupt change in both frequency and absorption intensity of a quantum dot. Thus, as a result of the influence of SO interaction on a quantum dot with four electrons, two lines in the absorption spectrum are reEFFECT OF SPIN-ORBIT INTERACTION ON THE MAGNETO-OPTICAL ...

placed by two other spectral lines at B = 0.2 T (both the absorption intensity and frequency are changed). The similar situation takes place for a quantum dot with five electrons. Change in the absorption lines induced by SO interaction occurs at $B \approx 0.8$ T.

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Experimental observation of these optical properties can be a very important step in the study of the control of spin dynamics of electrons in quantum structures by using SO interaction.

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