

## Labyrinth Domain Structure in the Models with Long-range Interaction

V.Yu. Kapitan<sup>1,\*</sup>, K.V. Nefedev<sup>1,2,†</sup>

<sup>1</sup> Far Eastern Federal University, School of Natural Sciences, 8, Sukhanova Str., 690950 Vladivostok, Russia

<sup>2</sup> Institute of Applied Mathematics, Far Eastern Branch Russian Academy of Science,  
7, Radio St., 690041 Vladivostok, Russia

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The numerical approaches to the study of the magnetic states, properties, and phase transitions in the XY and Ising spin systems with the long-range exchange interaction is presented. The Monte Carlo calculations have been performed for a system of spins (superspins) on a square lattice with long-range RKKY interaction. It is shown that the Monte Carlo simulation systems RKKY interaction leads to the formation of a complex of the magnetic structure. We compared the results of simulation with experimental images of labyrinth domain structure of garnet ferrite films.

**Keywords:** Ising model, XY model, Long-range interaction, Magnetic states.

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### 1. INTRODUCTION

The modern level of theoretical physics and, in particular, numerical simulation allows us to study the properties of planar nanostructures, such as 2D arrays of spins (superspin). However, the possibility of creating real 2D metals is questionable. In papers [1-3] describes the facilities of the existence in nature and the creation of real 2D metals as well as the potential to create experimental samples of 2D metals. According to the authors of the above manuscripts, the two main factors that have frustrated attempts to realize an ideal two-dimensional (2D) metal are disorder and Coulomb repulsion. It can be argued that although modern technology and material sciences have not yet reached the power to create 2D metal samples, the laws of nature do not prohibit the existence of 2D metal. Necessity of theoretical studies and computer simulation of the physical properties of magnetic spin systems due to the presence of the fundamental problems in condensed matter physics, as well as further improvement of the characteristics of the existing magnetic nanomaterials, including studies of the possibility of creating real 2D metals and practical perspective produced of magnetic materials in modern computer technology [1-4].

The thermodynamic properties of lattice spin systems using Monte Carlo methods, in the frame of models with long-range RKKY exchange interaction [5-8] were studied. Spin systems were simulated in the frame of the Ising model and the XY model.

The researches in the frame of simple models such as the XY model and the Ising model are primarily associated with the presence of unresolved problems in these approaches, such as the question of the critical exponents for the phase transition of the second kind. Nature of complex structure of low-energy magnetic states for models with RKKY exchange also poorly studied.

### 2. APPROACHES TO SIMULATION OF A PLANAR STRUCTURES

#### 2.1 The Monte Carlo Simulation

For the simulation was used the Metropolis algorithm [9]. The Monte Carlo (MC) simulation technique, with the implementation of the Metropolis algorithm, has been proved a very powerful tool for the systematic study of the magnetic behaviour both of single nanoparticles and nanoparticle assemblies [10-12]. The two major advantages [13] of this technique are

- The possibility for atomic scale treatment of the nanoparticles, so the details of their microstructure can be studied.
- The implementation of finite temperature through the Metropolis algorithm.

Although, the obtained dynamics in the Monte Carlo simulations is intrinsic and the time evolution of the system does not come from any deterministic equation for the magnetization, the results of the Monte Carlo simulations reproduce qualitatively the trend of the experimental data [14].

Actually, this good qualitative agreement between the simulation results and the experimental data enable us to have a better insight into the nanoscale phenomena, some aspects of the implementation of statistical Monte Carlo methods, including issues of their parallelization for simulation of magnetic phenomena and other problems of mathematical physics are presented in [15-21].

#### 2.2 RKKY interaction in superspins system

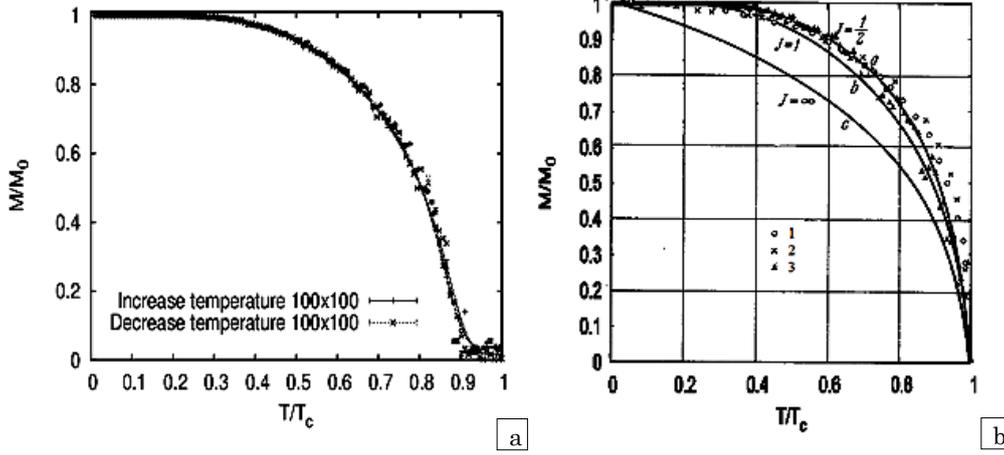
The energy of interaction for effective magnetic moments of spherical particles as follows to [23] is

$$E_{3D}(R) = -\frac{J_0}{R} \sum_{i=1}^{N-1} \sum_{j=i+1}^N F(x) m_{eff}^i m_{eff}^j, \quad (1)$$

$$F(x) = \frac{2x \cos(x) - \sin(x)}{x^4}, \quad (2)$$

\* kapitan.vyu@dvfu.ru

† nefedev.kv@dvfu.ru



**Fig. 1** – The system of spins (100 × 100) was studied by authors in two regimes: with decreasing and increasing temperature (a), 1 – Ni, 2 – Fe, 3 – MC simulation for a system of spins (50 × 50) with the direct exchange interaction between spins [22] (b)

$$m_{eff} = \frac{\pi M}{2k_f^3} (\sin(y) - 2x \cos(y)), \quad (3)$$

where  $m_{eff}^i$  is effective magnetic moment of the superspin,  $M$  is total magnetization,  $x = 2k_f R$ ,  $y = 2k_f R_c$ ,  $R$  is the distance between particles,  $R_c$  is the radius of the spherical particle,  $k_f$  is the Fermi wave vector. Under the superspin we mean area of magnetic domains in which the magnetic spins of the atoms rotate coherently. For granules and nanoparticles it is proposed, that the size of superspin is limited by their size.

As was shown in works [24, 25], the energy of the spin system, located in the plane and interacting via RKKY exchange:

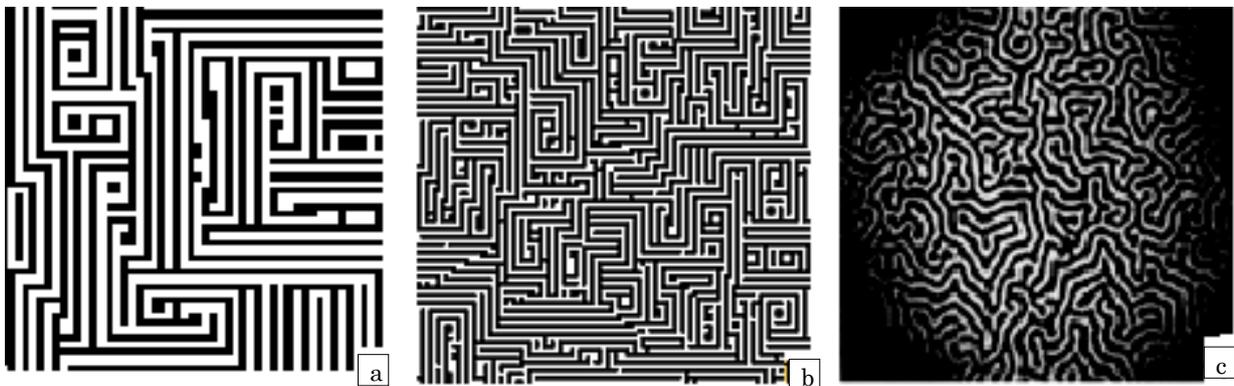
$$E_{2D}(R) = - \sum_{i=1}^{N-1} \sum_{j=i+1}^N A \frac{\sin(x)}{x^2} S_i S_j, \quad (4)$$

$$k_f^2 = 2\pi n_s, n_s \sim \frac{1}{a^2}, a \text{ is the lattice constant.}$$

### 3. THE RESULTS OF COMPUTER SIMULATION OF SYSTEMS OF SUPERSPINS

#### 3.1 Verification of the Developed Models and Algorithms

A developed software for calculating the properties



**Fig. 2** – The system of 50x50 superspins (a), the system of 100 × 100 superspins (b), the experimentally observed labyrinth domain structure in iron garnet films [30] (c)

of lattice systems with different types of interactions has been tested on a fairly simple fully connected Ising model with a direct exchange interaction between all spins in frame of the Curie-Weiss model [26, 27].

$$E_{direct} = -J \sum_{i=1}^{N-1} \sum_{j=i+1}^N S_i S_j. \quad (5)$$

The correctness of the model and the algorithm is confirmed by good convergence of the results of independent calculations of the magnetization temperature behaviour in two regimes: with decreasing temperature and with increasing temperature, and scaling of the system, see Figure 1a. In paper [22] are the experimental plots for Fe [28] and Ni [29], see Figure 1b. On this plot, there is imposed the calculated curve and got a good agreement with experiment.

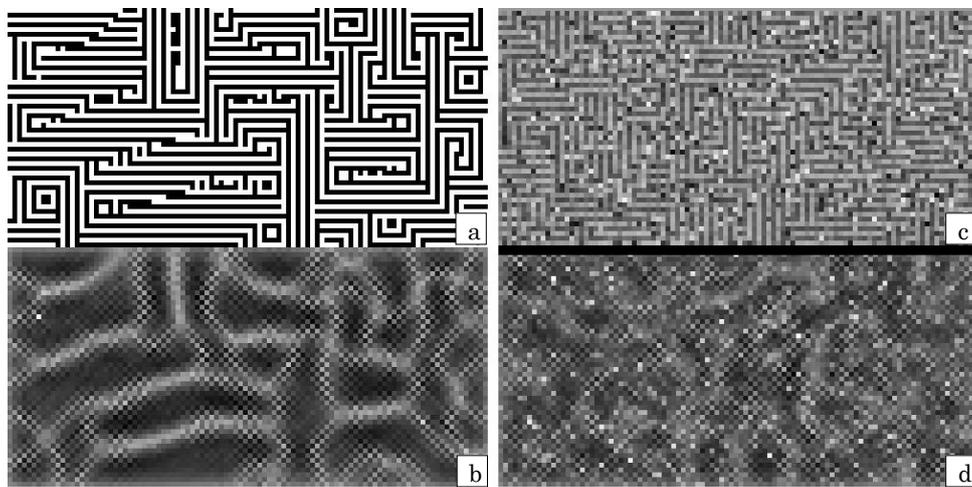
Thus, author’s software has been developed to simulate the long-range interaction in the spin lattice systems and verified the accuracy of the results. Such a clear coincidence of the results of analytical calculations in the mean field theory, the results of numerical Monte Carlo simulations, and the results of physical experiments with samples of iron and nickel raises questions about the eligibility of models with short-range interaction for magnetic systems.

### 3.2 Results of Simulation of Systems of Superspins with Indirect RKKY Interaction

In these numerical experiments, the exchange integral is equal to  $A = 10^6$ , which, on the other hand, corresponds to the use of superspin in the order of thousands of units, or blocking temperature (freezing) in thousandths of a kelvin.

For a system of  $50 \times 50$  was shown that the labyrinth domain state ( $E = -4.616 \times 10^{-4}$ ) dimensionless units have a minimum of energy compared with the ferromagnetic ( $E = 2.844 \times 10^{-4}$ ) and antiferromagnetic ( $E = -4.364 \times 10^{-4}$ ) states. Thus, comparing the energies of configurations showed that, observed in Figure 2a, b, the magnetic state of the system RKKY interacting Ising spins on a simple square lattice is energetically more favorable than the ferromagnetic or antiferromagnetic states [31].

Figure 2c shows dynamic domain structures was



**Fig. 3** – The system of  $50 \times 100$  superspins (Ising model) (a), the energy landscape for the system of  $500 \times 100$  superspins (Ising model) (b), the system of  $50 \times 100$  superspins (XY model) (c), the energy landscape for the system of  $500 \times 100$  superspins (XY model) (d)

## 4. CONCLUSION

Lattice spin system with long-range indirect exchange interaction were studied by Monte Carlo simulation. Images of domain structures are presented and a qualitative comparison with experimental data was performed. Most researchers consider that the RKKY interaction leads to frustrations and spin glass state. However, simulations show that this interaction in the system of super-spins can lead to the formation of the domain structure, even in the absence of competition between direct of the exchange interaction and the di-

pole-dipole interaction. In the experiment used an epitaxial (111) garnet ferrite films with the initial labyrinth domain structure [30]. In compounds with rare earth inclusions, the basic interaction between the spins of the rare-earth impurities is an indirect exchange interaction.

Figure 3 a, c shows the magnetic structure of the system of superspins with the RKKY interaction. In the system of spins, the finite size effect and dependence of the magnetic order of the geometric characteristics of the sample is observed. This effect is to change the energetically favorable magnetic states depending on the number of interacting particles. The energy landscape of this sample shown in Figure 3 (b,d). Dark pixels correspond to particles located at the minimum of energy and bright pixels correspond to an excited state of superspins. The interaction energy of each spin with all its neighbors are calculated according to the formula (4).

Beyond the scope of work remained investigating, the possibility of a phase transition with Ising spin systems with direct and indirect types of exchange interaction.

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