JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. **6** No 2, 02021(5pp) (2014)

ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том **6** № 2, 02021(5cc) (2014)

Spin Waves in an Arbitrary Ferromagnetic Nanosystem with a Translational Symmetry. Nanotube with a Round Cross-section. Nanotube with an Elliptic Cross-section

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(Received 25 March 2014; revised manuscript received 05 June 2014; published online 20 June 2014)

In the paper, spin waves in an arbitrary translational-symmetry composite nanosystem containing an "easy axis" ferromagnet are studied. For such a system, equation for the magnetic potential in magnetostatic approximation is obtained taking into account the magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects. The theory that allows to obtain the dispersion relation and the transverse wavenumber spectrum for a particular system of this type is proposed; dispersion relation for a system with small transverse size is obtained. The dispersion relation and the transverse wavenumber spectrum are written for nanotubes with round and elliptic cross-sections.

Keywords: Spin waves, Nanomagnetism, Ferromagnetic nanotube, Ferromagnetic nanowire, Dipole-exchange theory.

PACS numbers: 62.23.St, 75.30.Ds, 75.75. + a

1. INTRODUCTION

Spin waves in nanosystems of different configurations are an actual topic of research during the last years. Such waves are promising for engineering applications, namely, creation of new data storage and transmission devices [1, 2], new computing devices [3], etc.

It is known that pattern of spin excitations in a nanosystem substantially depends on its shape and size. Therefore, spin waves are investigated in different types of nanosystems, in particular, thin ferromagnetic films [4, 5], nanowires [6, 7], micron-sized magnetic quantum dots [8] and other nanosystems, separately both theoretically and experimentally.

Development of nanotechnologies in the last decades led to the synthesis and application of composite nanostructures. It is known that nanocomposites containing ferromagnet exhibit a number of anomalous properties [9-13]. Spin waves in nanocomposites of different configurations are intensively investigated [14, 15], but spin waves in composite nanoparticles remain comparatively poorly-studied.

Metal nanotubes, which find more and more practical applications, occupy a special place among composite nanoparticles [16, 17]. Nanowires and nanotubes of a non-circular cross-section (see, for example, [18]) are synthesized and studied during the last years, and their properties differ from the properties of round nanowires and nanotubes (see, for example, [19]). Among such nanosystems a special attention is devoted to the synthesis and investigation of nanowires and nanotubes of an elliptic cross-section [20-22]. Therefore, magnetic nanowires [23] and magnetic nanotubes [24-26], in particular, of a non-circular cross-section are of a great interest for the researches of spin waves. (We particularly note that typical synthesized magnetic nanowires and, especially, nanotubes often have a cross-section which considerably differs from a round one, see, for example, [26].) However, spin waves in such nanostructures (except spin waves in nanowires of a round cross-section, see, for example, [6, 7]) remain comparatively poorly-studied that makes their investigation actual.

In the present work we study spin waves in a translational-symmetry ferromagnetic nanosystem (nanowire of an arbitrary cross-section, in general, non-continuous; single-layer nanotubes of an arbitrary profile and solid nanowires of an arbitrary profile enter this class of nanosystems as a particular case). The theory which allows to obtain the dispersion relation and transverse wavenumber spectrum in such nanosystems is proposed and the dispersion relation for the case of a nanosystem with small transverse sizes of continuous regions of a ferromagnet (thin nanowire, thin nanotube, etc.) is written. We have written the dispersion relation and transverse wavenumber spectrum for the cases of a nanotube of a round and an elliptic cross-sections.

2. STATEMENT OF THE PROBLEM

We consider a translational-symmetry ferromagnetic nanosystem (nanotube of an arbitrary profile, continuous nanowire of an arbitrary profile, etc.).

Let us assume that the ferromagnet from which the system consists of is a single-axis crystal, whose axis is directed along the system translational direction, and has the type "easy axis", so that its equilibrium magnetization $\vec{M}_0$ is also directed along this direction. We assume that the ferromagnet is characterized by the following parameters: constant of uniaxial anisotropy β (is considered persistent); constant of exchange energy a. Gyromagnetic ratio of the ferromagnet γ is assumed to be fixed and known. Damping for spin waves is assumed to be insignificant neglecting the relaxation term in the Landau-Lifshitz equation.

Now we consider a spin wave propagating in the above described nanosystem (parallel to its axis) with small perturbations of the magnetic moment density and, correspondingly, magnetic field. Thus, we assume deviations of the magnetic moment $\vec{m}$ density and field $\vec{h}$ inside the ferromagnet from their equilibrium values $\vec{M}_0$ and $\vec{H}_0^{(i)}$, respectively, negligible in comparison with these equilibrium values: $|\vec{m}| << |\vec{M}_0|$, $|\vec{h}| << |\vec{H}_0^{(i)}|$.

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We introduce the coordinate axis Oz directed along the nanosystem axis (translational direction). Taking into account the typical sizes of nanotubes and nanowires and the corresponding restrictions to the wavenumber, in the description of a spin wave in such nanosystem we, in general, should take into consideration both the magnetic dipole-dipole and the exchange interactions in the Landau-Lifshitz equation. Since we consider a uniaxial ferromagnet, we should also leave the term describing anisotropy.

Our aim is to obtain the dispersion relation and the radial wavenumber spectrum for a such spin wave.

3. AN ARBITRARY FERROMAGNETIC SYSTEM WITH THE TRANSLATIONALSYMMETRY

3.1 Equation for magnetic potential

We will write the linearized Landau-Lifshitz equation for a spin wave described in the previous section. In order to make the system of equations complete, we use the magnetostatic approximation (see, for example, [27]) assuming field of a spin wave $\vec{h}$ potential $\vec{h} = -\nabla\Phi$, where Φ is the magnetic potential. Such system of equations will have the following form [27]:

$$\begin{cases} i\omega\vec{m}_0 = \gamma\left(M_0\vec{e}_z \times \left(-\nabla\Phi_0 + \alpha\Delta\vec{m}_0 - \left(\beta + H_0^{(i)}/M_0\right)\vec{m}_0\right)\right) \\ \Delta\Phi_0 = 4\pi\,\mathrm{div}\vec{m}_0 \end{cases} \quad (1)$$

and $\vec{H}_0^{(i)} = \vec{H}_0^{(e)} - 4\pi\hat{N}\vec{M}_0$, where $\vec{H}_0^{(e)}$ is the external field in which a nanotube is situated, $\hat{N}$ is the tensor of demagnetizing coefficients (for many configurations of the system one can assume $4\pi\hat{N}\vec{M}_0 = 0$, $\vec{H}_0^{(i)} = \vec{H}_0^{(e)}$). Here we have taken into account that magnetization and wave field are periodically changed with time, i.e. $\vec{m}(\vec{r},t) = \vec{m}_0(\vec{r})\exp(i\omega t)$, $\vec{h}(\vec{r},t) = \vec{h}_0(\vec{r})\exp(i\omega t)$; Φ_0 is the disturbance amplitude potential of the field $\vec{h}_0$, so that $\vec{h}_0 = -\nabla\Phi_0$ and $\Phi(\vec{r},t) = \Phi_0(\vec{r})e^{i\omega t}$.

We will suppose that symmetry of the nanosystem, which we consider, admits the introduction of the corresponding orthogonal cylindrical (not necessarily of a circular cylinder) coordinate system (x_1, x_2, z), so that each ferromagnet/non-magnetic medium interface is specified by the equation of the form $x_1 = a_i$, where i is the interface number (is changed from 1 to N, where N is the general amount of such surfaces), a_i are the constants. (Here we note that for an arbitrary translational-symmetry system we can introduce, for example, the coordinates of a circular cylinder (ρ, θ, z) and perform the following transformations to obtain the dispersion relation; however, when finding the wavenumber spectrum using the boundary conditions at these interfaces, we should use the coordinate system which corresponds to the nanosystem symmetry.)

At first we will obtain the equation for the magnetic potential excluding perturbations of the magnetic moment $\vec{m}_0$ density from this system of equations. We will use the above introduced coordinate system (x_1, x_2, z). Rewriting the first equation of the system (1) in the form of

$$\frac{i\omega}{\gamma M_0}\vec{m}_0 = \vec{e}_z \times \left(-\nabla\Phi_0 + \alpha\Delta\vec{m}_0 - \left(\beta + H_0^{(i)}/M_0\right)\vec{m}_0\right), \quad (2)$$

we multiply vectorially both sides from left by the unit vector $\vec{e}_z$ and take divergence from both sides of the equation. Taking into account that $m_{0z} = 0$ and from the second equation of the system (1) $\mathrm{div}\,\vec{m}_0 = \Delta\Phi_0/4\pi$ we will obtain

$$-\frac{i\omega}{\gamma M_0}\mathrm{div}\left(\vec{e}_z \times \vec{m}_0\right) = $$

$$= -\Delta\Phi_0 + \frac{\partial^2\Phi_0}{\partial z^2} + \frac{1}{4\pi}\left(\alpha\Delta - \left(\beta + H_0^{(i)}/M_0\right)\right)\Delta\Phi_0 . \quad (3)$$

For the transformation of the left side of the equation (3), we will use operator $\alpha\Delta - \left(\beta + H_0^{(i)}/M_0\right)$ to both sides. Using equation (2), vectorially multiplied by the unit vector $\vec{e}_z$ from left, we derive

$$-\frac{i\omega}{\gamma M_0}\mathrm{div}\left(\vec{e}_r \times \left(-\frac{i\omega}{\gamma M_0}\vec{e}_r \times \vec{m}_0 + \nabla\Phi_0 - \frac{\partial\Phi_0}{\partial z}\vec{e}_z\right)\right) =$$

$$= \left(\alpha\Delta - \left(\beta + H_0^{(i)}/M_0\right)\right)\left(-1 + \frac{1}{4\pi}\left(\alpha\Delta - \left(\beta + H_0^{(i)}/M_0\right)\right)\right)\Delta\Phi_0 +$$

$$+\left(\alpha\Delta - \left(\beta + \frac{H_0^{(i)}}{M_0}\right)\right)\frac{\partial^2\Phi_0}{\partial z^2}, \quad (4)$$

and since

$$\mathrm{div}\left(\vec{e}_z \times \left(\nabla\Phi_0 - \frac{\partial\Phi_0}{\partial z}\vec{e}_z\right)\right) = 0 ,$$

$$\mathrm{div}\left(\vec{e}_z \times \left(\vec{e}_z \times \vec{m}_0\right)\right) = -\frac{\Delta\Phi_0}{4\pi} ,$$

then the desired differential equation for the magnetic potential will be finally rewritten as

$$\left(\frac{\omega^2}{\gamma^2 M_0^2} - \left(H_0^{(i)}/M_0 + \beta - \alpha\Delta\right)\left(\left(H_0^{(i)}/M_0 + \beta\right) + 4\pi - \alpha\Delta\right)\right)\Delta\Phi_0 +$$

$$+4\pi\left(H_0^{(i)}/M_0 + \beta - \alpha\Delta\right)\frac{\partial^2\Phi_0}{\partial z^2} = 0 . \quad (5)$$

As one can see, the obtained equation is similar to the known equation for a cylindrical nanowire (see, for example, [7]). This dispersion equation is the same for all translational-symmetry ferromagnetic systems, and, as we see, equation (5) does not contain the geometric parameters of the system.

3.2 Dispersion relation, general case

Now we will obtain the relation between spin wave frequency and wave number using equation (5) for the magnetic potential of spin waves.

Let function $F(x_1, x_2, k_\perp)$ to be a general solution of two-dimensional Helmholtz equation in (x_1, x_2) coordinates, so that $\Delta_\perp F - k_\perp^2 F = 0$, here $k_\perp$ is the transverse wavenumber (it describes wave propagation in the direction perpendicular to the system symmetry axis). Then, the

potential of the following form is the solution of the equation (5) for a nanotube

$$\Phi_0 = F(x_1, x_2, k_\perp)\exp(ik_\parallel z), \quad (6)$$

where $k_\parallel$ is the longitudinal wavenumber. Substitution of the solution (6) into the equation (5) allows to obtain the following dispersion equation:

$$\alpha^2\left(k_\parallel^2 + k_\perp^2\right)^3 + 2\alpha\left(\tilde{\beta} + 2\pi\right)\left(k_\parallel^2 + k_\perp^2\right)^2 + \\ + \left(\tilde{\beta}\left(\tilde{\beta} + 4\pi\right) - \frac{\omega^2}{\gamma^2 M_0^2} - 4\pi\alpha k_\parallel^2\right)\left(k_\parallel^2 + k_\perp^2\right) - 4\pi\tilde{\beta}k_\parallel^2 = 0, \quad (7)$$

from which we find the dispersion relation

$$\omega = \gamma M_0\sqrt{\alpha^2 k^4 + 2\alpha\left(2\pi + \tilde{\beta}\right)k^2 + \tilde{\beta}\left(4\pi + \tilde{\beta}\right) - 4\pi k_\parallel^2\left(\alpha + \frac{\tilde{\beta}}{k^2}\right)}, \quad (8)$$

here $k^2 = k_\parallel^2 + k_\perp^2$, $\tilde{\beta} = \beta + H_0^{(i)}/M_0$ (and for the system configurations, at which $4\pi\hat{N}\vec{M}_0 = 0$, $\tilde{\beta} = \beta + H_0^{(e)}/M_0$ holds).

We should note that obtained dispersion equation (8) contains two components of the wave vector. Under the condition of a sufficiently long tube, component $k_\parallel$ can be considered such that is changed continuously; therefore, for the description of spin waves in the system one have to specify the spectrum of $k_\perp$.

If transverse sizes of continuous regions of the ferromagnetic in the nanosystem are small enough (of the order or less than the length of the exchange interaction), we can neglect the transverse oscillations assuming $k_\perp = 0$, $k = k_\parallel$, and the dispersion equation will be rewritten as follows:

$$\omega = \gamma M_0\sqrt{\alpha^2 k^4 + 2\alpha\left(2\pi + \tilde{\beta}\right)k^2 + \tilde{\beta}\left(4\pi + \tilde{\beta}\right) - 4\pi k^2\left(\alpha + \frac{\tilde{\beta}}{k^2}\right)}. \quad (9)$$

In the case, when one cannot neglect the transverse oscillations in a spin wave, in order to obtain the dispersion equation it is necessary to concretize the system profile and write the boundary conditions at its surface.

3.3 Transverse wavenumber spectrum

If system transverse sizes are not small enough in order to assume $k_\perp = 0$, then dispersion relation (8), in general, should be supplemented by the transverse wavenumber spectrum. To obtain it, we will write the boundary conditions for the magnetic potential at the ferromagnet boundary.

In the general case, we should solve equation (5) in both ferromagnet and external space, and join these solutions using boundary conditions. However, the problem is considerably simplified in the case, when ferromagnetic nanosystem is limited by metal non-magnetic surfaces, and metal conductivity is sufficiently high, so we can consider it perfect when writing the boundary conditions. In this case, boundary condition is reduced to the nulling condition of the normal derivative of the magnetic potential on the surface of the ferromagnet, see, for example, [28]

$$\nabla\Phi\vec{n}_0 = 0, \quad (10)$$

where $\vec{n}_0$ is the unit vector of the normal to the interface. This boundary condition in the introduced earlier coordinate system (x_1, x_2, z) will be written as

$$\left.\frac{\partial\Phi}{\partial x_1}\right|_{x_1 = a_i} = 0,$$

i.e., taking into account the view of magnetic potential,

$$\left.\frac{\partial F(x_1, x_2, k_\perp)}{\partial x_1}\right|_{x_1 = a_i} = 0 \quad (11)$$

for all i. Such system of boundary conditions specifies the desired wavenumber spectrum $\{k_{\perp j}\}$.

Dispersion relation (8) together with the wavenumber spectrum $\{k_{\perp j}\}$ obtained from the system of equations (11) solve the assigned task for an arbitrary ferromagnetic translational-symmetry nanosystem that admits the introduction of the corresponding orthogonal coordinate system. We concretize geometry of the considered nanosystem using the developed above theory to the cases of a single-layer nanotube with a circular cross-section and a single-layer nanotube with an elliptic cross-section. We will find the transverse wavenumber spectrum in each case.

4. APPLICATION TO NANOSYSTEMS OF SPECIFIC CONFIGURATIONS

4.1 Nanotube in the form of a circular cylinder

We consider a ferromagnetic nanotube of a circular cross-section (circular cylinder) with internal radius a and external radius b. Cylindrical coordinate system (ρ, θ, z) corresponds to the symmetry of such problem. Here, magnetostatic potential (5) has the following form:

$$\Phi_0 = \left(A_1 J_n(k_\perp\rho) + A_2 N_n(k_\perp\rho)\right)\exp\left(i\left(n\theta + k_{\parallel} z\right)\right), \quad (12)$$

so that function

$$F(\rho, \theta, k_\perp) = \left(A_1 J_n(k_\perp\rho) + A_2 N_n(k_\perp\rho)\right)\exp(in\theta). \quad (13)$$

Boundary conditions (11) for such nanotube will be reduced to the following conditions on the internal and external surfaces of a nanotube:

$$\left.\frac{\partial F}{\partial\rho}\right|_{\rho = a,b} = 0. \quad (14)$$

We will substitute function F in the above written form into the boundary condition (14). Three unknowns – A_1, A_2 and $k_\perp$ – enter the obtained system of two equations. However, we can exclude one unknown dividing both equations of the system, for example, by constant A_1 and, thus, we obtain the total system of equations with respect to the variables $k_\perp$ and A_2/A_1

$$\begin{cases} J_n'(k_\perp a) + \dfrac{A_2}{A_1} N_n'(k_\perp a) = 0, \\ J_n'(k_\perp b) + \dfrac{A_2}{A_1} N_n'(k_\perp b) = 0. \end{cases} \quad (15)$$

For the case of a wide nanotube, so that $k_\perp a >> 1$, we can obtain the simplest expression for the transverse wavenumber spectrum using asymptotics of the Bessel functions. Indeed, at $k_\perp a >> 1$ we have

$$\Phi_0 \approx \frac{C}{\sqrt{\rho}} \sin\left(k_\perp \rho + \delta\right) \exp\left(i\left(n\theta + k_\parallel z\right)\right), \qquad (16)$$

where C is the normalization constant, δ is the initial phase. We will write the radial derivative from magnetic potential as

$$\frac{\partial \Phi_0}{\partial \rho} \approx \frac{k_\perp C}{\sqrt{\rho}} \left(\cos\left(k_\perp \rho + \delta\right) - \left(2k_\perp \rho\right)^{-1} \sin\left(k_\perp \rho + \delta\right)\right) \times \times \exp\left(i\left(n\theta + k_\parallel z\right)\right). \qquad (17)$$

For our approximation of a wide nanotube $2k_\perp \rho >> 1$ everywhere inside a nanotube. Because of the fact that $\cos(k_\perp \rho + \delta)$, $\sin(k_\perp \rho + \delta)$ are rapidly oscillating functions, we cannot directly neglect the term $(2k_\perp \rho)^{-1} \sin(k_\perp \rho + \delta)$. However, we can write with the accuracy to the second-order smallness

$$\cos\left(k_\perp \rho + \delta\right) - \frac{1}{2k_\perp \rho} \sin\left(k_\perp \rho + \delta\right) \approx \cos\left(k_\perp \rho + \frac{1}{2k_\perp \rho} + \delta\right), \qquad (18)$$

so that condition (14) will be approximately executed at

$$\cos\left(k_\perp a + \frac{1}{2k_\perp a} + \delta\right) = \cos\left(k_\perp b + \frac{1}{2k_\perp b} + \delta\right) = 0. \qquad (19)$$

Hence $\left(k_\perp b + \left(2k_\perp b\right)^{-1}\right) - \left(k_\perp a + \left(2k_\perp a\right)^{-1}\right) = \pi n$, where n is an arbitrary integer. Rewriting this expression as

$$k_\perp \left(b - a\right)\left(1 - \left(2k_\perp^2 ab\right)^{-1}\right) = \pi n \qquad (20)$$

and neglecting the term $\left(2k_\perp^2 ab\right)^{-1}$ in comparison with 1, we finally have the wavenumber spectrum in the form

$$k_\perp = \frac{\pi n}{b - a}. \qquad (21)$$

Thus, we have obtained in an implicit form the wavenumber spectrum for a ferromagnetic nanotube of a circular cross-section and written the spectrum in an explicit form for the case of a wide nanotube. We note that in the latter case, as seen from the relation (21), wavenumber spectrum becomes quasi-one-dimensional; and when the condition $k_\perp a >> 1$ holds, the pattern of spin waves in a nanotube becomes similar to the pattern of spin wave in a thin film.

4.2 Nanotube in the form of an elliptic cylinder

Now we consider a ferromagnetic nanotube with an elliptic cross-section (elliptic cylinder), whose semi-axes are equal to a_2, b_2 (for the external surface) and a_1, b_1 (for the internal surface). For such nanosystem we will introduce the coordinates of elliptic cylinder

$$\begin{cases} x = 0.5d\,\mathrm{ch}(u)\cos(v), \\ y = 0.5d\,\mathrm{sh}(u)\sin(v), \\ z = z. \end{cases} \qquad (22)$$

Equation u = const in these coordinates describes an elliptic cylinder with semi-axes $0.5d\,\mathrm{ch}(u)$ and $0.5d\,\mathrm{sh}(u)$. Thus, we can specify the surfaces which restrict the nanotube by equations $u = u_1$, $u = u_2$, at that $\mathrm{ch}(u_1) = 2b_1/d$, $\mathrm{sh}(u_1) = 2a_1/d$, $\mathrm{ch}(u_2) = 2b_2/d$, $\mathrm{sh}(u_2) = 2a_2/d$.

As known, the Mathieu functions are the solution of two-dimensional Helmholtz equation $\Delta_\perp F - k_\perp^2 F = 0$ in the elliptic coordinates

$$F\left(u, v, k_\perp\right) = \begin{cases} Ce_m\left(u,\alpha\right) ce_m\left(v,\alpha\right), \\ Se_m\left(u,\alpha\right) se_m\left(v,\alpha\right), \end{cases} \qquad (23)$$

where $\alpha = k_\perp^2 d^2/16$. Thus, general solution of (5) in this case has the following form:

$$\Phi_0 = \left(C_1 Ce_m\left(u,\alpha\right) ce_m\left(v,\alpha\right) + C_2 Se_m\left(u,\alpha\right) se_m\left(v,\alpha\right)\right) \times \times \exp\left(ik_\parallel z\right), \qquad (24)$$

so that function

$$F\left(u, v, k_\perp\right) = C_1 Ce_m\left(u,\alpha\right) ce_m\left(v,\alpha\right) + + C_2 Se_m\left(u,\alpha\right) se_m\left(v,\alpha\right). \qquad (25)$$

Boundary conditions (11) for an elliptic nanotube will be written as follows:

$$\left.\frac{\partial F}{\partial u}\right|_{u=u_1,u_2} = 0, \qquad (26)$$

hence we have

$$C_1 Ce'_m\left(u_1,\alpha\right) ce_m\left(v,\alpha\right) + C_2 Se'_m\left(u_1,\alpha\right) se_m\left(v,\alpha\right) = = C_1 Ce'_m\left(u_2,\alpha\right) ce_m\left(v,\alpha\right) + C_2 Se'_m\left(u_2,\alpha\right) se_m\left(v,\alpha\right) = 0. \qquad (27)$$

In order to satisfy condition (27) at any v, obviously, it is necessary to take $C_1 = 0$ or $C_2 = 0$. Thus, we obtain two classes of the solutions

$$\left[\begin{array}{l} \begin{cases} F\left(u, v, k_\perp\right) = C_1 Ce_m\left(u, k_\perp^2 d^2/16\right) ce_m\left(v, k_\perp^2 d^2/16\right), \\ Ce'_m\left(u_1, k_\perp^2 d^2/16\right) = Ce'_m\left(u_2, k_\perp^2 d^2/16\right) = 0, \end{cases} \\ \begin{cases} F\left(u, v, k_\perp\right) = C_2 Se_m\left(u, k_\perp^2 d^2/16\right) se_m\left(v, k_\perp^2 d^2/16\right), \\ Se_m{}'\left(u_1, k_\perp^2 d^2/16\right) = Se_m{}'\left(u_2, k_\perp^2 d^2/16\right) = 0. \end{cases} \end{array}\right. \qquad (28)$$

System (28) determines the desired transverse wavenumber spectrum for spin waves in a ferromagnetic nanotube of an elliptic cross-section.

5. CONCLUSIONS

Thus, we have developed the theory allowing to obtain the dispersion relation and transverse wavenumber spectrum for spin waves in a ferromagnetic cylindrical nanosystem of an arbitrary cross-section (subject to the possibility of introducing the corresponding orthogonal coordinate system). We have written the dispersion relation and system of equations for the transverse wavenumber spectrum in this case. The obtained dispersion

relation has the same form for all nanosystems of the above stated type and coincides with the known equation for a cylindrical nanowire. We have also written the dispersion relation for the case when transverse sizes of continuous regions of the ferromagnet are small (less or of the order of magnitude of the exchange length), so that one can neglect the transverse oscillations (and, therefore, in order to describe a spin wave it is not necessary to solve the system of equations for the transverse wavenumbers). We have applied the above described theory to the cases of a nanotube of a circular cross-section and a nanotube of an elliptic cross-section and have obtained the wavenumber spectrum for each of these cases. Wavenumber spectrum for a wide circular nanotube is also derived in the explicit form.

AKNOWLEDGEMENTS

The author of the paper expresses his gratitude to Dr. Phys.-Math. Sci., Professor, Corresponding Member of NAPSU Gorobets Yu.I. for the attention to the work, fruitful discussion and valuable comments.

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