JOURNAL OF NANO- AND ELECTRONIC PHYSICS
Vol. **6** No 2, 02011(5pp) (2014)
ЖУРНАЛ НАНО- ТА ЕЛЕКТРОННОЇ ФІЗИКИ
Том **6** № 2, 02011(5cc) (2014)

Contribution of the Magnetic Field of Eddy Currents to the Gilbert Damping Parameter

S.I. Denisov, T.V. Lyutyy, H.V. Babych, B.O. Pedchenko

Sumy State University, 2, Rimsky Korsakov Str., 40007 Sumy, Ukraine

(Received 29 January 2014; published online 20 June 2014)

We study the role of the magnetic field of eddy currents, which are induced in conducting single-domain particles of spherical form, in the magnetization dynamics. To describe the dynamic behavior of magnetization and electromagnetic field generating by the time-dependent magnetization, we use the coupled system of the Landau-Lifshitz-Gilbert (LLG) and Maxwell equations. Assuming that the magnetization direction depends on time in an arbitrary way, we find the solution of the Maxwell equations in the quasi-stationary approximation and calculate the averaged (over the particle volume) magnetic field of eddy currents. Considering this field as an extra contribution to the effective magnetic field acting on the particle magnetic moment, we derive the LLG equation in which the influence of eddy currents is completely accounted for by introducing an additional Gilbert damping parameter of electrodynamic origin.

Keywords: Conducting single-domain particles, Landau-Lifshitz-Gilbert equation, Maxwell equations, Quasi-stationary approximation, Eddy currents, Gilbert damping parameter.

PACS numbers: 75.78. – n, 41.20. – q

1. INTRODUCTION

Investigation of the magnetic properties of single-domain ferromagnetic particles and their ensembles is an important scientific problem. From the theoretical point of view, interest to such particles is conditioned by a number of physical phenomena, such, for example, as quantum tunneling of the magnetic moment [1], stochastic resonance [2], precessional switching of magnetization [3, 4] and switching by microwave radiation [5, 6]. On the other hand, single-domain particles find practical application (or they have a large application potential) in high-density data storage [7, 8], spintronics [9, 10], biomedicine [11-13], etc.

If the exchange interaction energy in a ferromagnet considerably exceeds the magnetic one, then the length of the magnetization vector can be considered constant. In this case, dependence of the direction of the magnetization vector on the coordinates and time is often described by the Landau-Lifshitz (LL) equation [14] or by the equivalent, but more preferred from the physical point view, Landau-Lifshitz-Gilbert (LLG) equation [15]. (Further, taking into account the equivalence of these equations, we will talk about the LLG equation, even if the LL equation was used in original works.) At sufficiently small (nanometer) sizes of ferromagnetic particles, a single-domain state characterized by the uniform distribution of magnetization is realized. As a consequence, LLG equation is greatly simplified; thereby it is widely used for the study of nonlinear dynamics of the magnetization in single-domain particles [16].

Since the considered particles have nanometer sizes, thermal fluctuations can play an important role in the description of their magnetic properties. For their account it is proposed in the work [17] to use the LLG equation with the effective magnetic field containing a random (vector) process of the white-noise type. This approach allowing to use power methods of the Langevin and Fokker-Plank equations was found to be so fruitful that currently stochastic LLG equation became one of the main tools of the investigation of magnetic fluctuations (see, for example, [18]). In particular, in the framework of the given approach we have studied a number of effects in the systems of single-domain particles, whose existence is directly connected with thermal fluctuations [19-24].

Recently, great attention is devoted to the investigations of nanocomposite materials [25, 26] including those which contain single-domain metallic particles. Magnetization dynamics in such particles is not already described by usual LLG equation, since in the effective magnetic field acting on the atomic magnetic moments it is necessary to take into account the magnetic field of eddy currents induced by the time-dependent magnetic induction. It is well known (see, for example, [27]) that in this case the LLG equation should be considered simultaneously with the Maxwell equations. In this case, using qualitative considerations, it is not difficult to find the order of magnitude of the magnetic field of eddy currents and estimate its influence on the magnetization dynamics. But analytical determination of this field is a rather serious problem. As far as we know, this problem has been most thoroughly analyzed in the work [28] within the quasi-stationary approximation. However, the authors of [28] have used an additional simplifying assumption which did not allow them to find the non-uniform magnetic field of eddy currents inside a particle and obtain the exact expression for the Gilbert damping parameter of electrodynamic origin. Since precise account of the conduction effects may play a significant role for the correct description of the magnetization dynamics in conducting particles (see below), finding of the induction electromagnetic field is of great importance. The present work is devoted to the analytical solution of this problem.

2. MODEL DESCRIPTION

In the present work we consider ferromagnetic particles of spherical form, whose radius R is supposed to be so small (its upper boundary does not usually exceed a few tens of nanometers) that a single-domain state is realized. In the simplest case this state can be characterized by the constant in magnitude magnetization vector $\mathbf{M} = \mathbf{M}(t)$ ($|\mathbf{M}| = M = \text{const}$), whose direction is changed in time according to the LLG equation [29]

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\mathbf{H}_{\rm eff} + \frac{\alpha}{M}\mathbf{M}\times\frac{d\mathbf{M}}{dt}. \qquad (2.1)$$

Here $\gamma(>0)$ is the gyromagnetic ratio, $\alpha(>0)$ is the Gilbert damping parameter, $\mathbf{H}_{\rm eff} = \mathbf{H}_{\rm eff}(t)$ is the effective magnetic field acting on the vector $\mathbf{M}$, sign $\times$ denotes the outer product. The effective magnetic field for conducting particles can be written in the form

$$\mathbf{H}_{\rm eff} = -\frac{\partial W_a}{\partial\mathbf{M}} + \overline{\boldsymbol{\mathcal{H}}}, \qquad (2.2)$$

where W_a is the volume energy density of magnetic anisotropy,

$$\overline{\boldsymbol{\mathcal{H}}} = \frac{1}{V}\int_V d\mathbf{r}\,\boldsymbol{\mathcal{H}}(\mathbf{r},t) \qquad (2.3)$$

is the magnetic field $\boldsymbol{\mathcal{H}}(\mathbf{r},t)$ inside a particle averaged over the particle volume $V = \left(\frac{4\pi}{3}\right)R^3$, $\mathbf{r}$ is the radius-vector of observation point in the Cartesian coordinate system, whose origin coincides with the particle center. It is convenient to represent the field $\boldsymbol{\mathcal{H}}(\mathbf{r},t)$ as a sum of the magnetic field $\mathbf{H} = \mathbf{H}(\mathbf{r},t)$ of eddy currents, which are induced by variable magnetic induction $\mathbf{B}_0 = \mathbf{H}_d + 4\pi\mathbf{M}$ ($\mathbf{H}_d = -(4\pi/3)\mathbf{M}$ is the demagnetizing magnetic field inside a spherical particle), and the external magnetic field $\mathbf{H}_0 = \mathbf{H}_0(t)$ modified by the magnetic fields of eddy currents. At $|\mathbf{H}_0| \ll |\mathbf{B}_0|$ we can neglect these fields, therefore we can write $\overline{\boldsymbol{\mathcal{H}}} = \overline{\mathbf{H}} + \mathbf{H}_0$. Introducing later the effective magnetic field

$$\mathbf{H}_{\rm eff}^{(0)} = -\frac{\partial W_a}{\partial\mathbf{M}} + \mathbf{H}_0 \qquad (2.4)$$

for non-conducting particles, the LLG equation (2.1) is re-written in the form

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times(\mathbf{H}_{\rm eff}^{(0)} + \overline{\mathbf{H}}) + \frac{\alpha}{M}\mathbf{M}\times\frac{d\mathbf{M}}{dt}. \qquad (2.5)$$

Thus, in the considered approximation all features of the magnetization dynamics in conducting particles are conditioned by the influence of the averaged magnetic field $\overline{\mathbf{H}}$. However, since, according to the law of electromagnetic induction, $\overline{\mathbf{H}}$ depends on $d\mathbf{M}/dt$, equation (2.5) is not closed. Therefore, in the general case, when $\overline{\mathbf{H}}$ is defined by all induction currents flowing in a particle, this equation should be solved simultaneously with the Maxwell equations. While numerical solution of the system of the LLG and Maxwell equations does not cause principal difficulties (see, for example, [30, 31]), analytical determination of the averaged magnetic field of eddy currents represents a certain problem. We know only a single work [28], in which system of the Maxwell equations in the quasi-stationary approximation is solved analytically for a spherical ferromagnetic particle. The authors of the mentioned work have calculated the magnetic field of eddy currents $\mathbf{H}(\mathbf{r},t)$ in the center of a particle ($\mathbf{r} = \mathbf{0}$) and supposed that $\overline{\mathbf{H}} = \mathbf{H}(\mathbf{0},t)$. However, as it will be shown below, $\mathbf{H}(\mathbf{r},t)$ strongly depends on $\mathbf{r}$, and $\mathbf{H}(\mathbf{0},t)$ gives only a qualitative estimate of the averaged magnetic field $\overline{\mathbf{H}}$.

For the quantitative determination of $\overline{\mathbf{H}}$ we will use the system of the Maxwell equations in the quasi-stationary approximation [32]. Inside a ball (when $r = |\mathbf{r}| \le R$) this system of equations has the form

$$\mathrm{rot}\,\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}(\mathbf{H} + \mathbf{B}_0), \qquad \mathrm{div}\,\mathbf{E} = 0,$$
$$\mathrm{rot}\,\mathbf{H} = \frac{4\pi}{c}\mathbf{j}, \qquad \mathrm{div}\,\mathbf{H} = 0, \qquad (2.6)$$

where $\mathbf{E} = \mathbf{E}(\mathbf{r},t)$ is the induction electric field strength, c is the light speed, $\mathbf{j} = \sigma\mathbf{E}$ is the electric current density, σ is the particle conductivity. For simplification we here assume that magnetic permeability of a particle, as well as the magnetic and dielectric permeability of the environment are equal to unit. In principle, for the determination of the electromagnetic field in a whole space the system of equations (2.6) should be supplemented by the system of the Maxwell equations for the quasi-stationary field at $r > R$ and the corresponding boundary conditions. However, later we will ascertain that in the considered case the averaged magnetic field $\overline{\mathbf{H}}$ of eddy currents inside a particle can be determined directly from (2.6).

In the quasi-stationary approximation, the electromagnetic field frequency ω should satisfy the conditions $\omega \ll c/R$ and $\omega \ll \sigma$. The first of these conditions provides smallness of the field change in the vicinity of a particle, and the second condition allows to neglect the bias current $(1/4\pi)\partial\mathbf{E}/\partial t$ in comparison with the conduction current $\mathbf{j}$. Assuming that σ has the meaning of stationary conductivity, we should require the fulfillment of additional condition $\omega \ll 1/\tau_0$, where τ_0 is the electron mean free time in a conductor (usually at room temperature $\tau_0 \sim 10^{-13}$ s). Since we consider the single-domain particles ($R < 10^2$ nm) of good conductors ($\sigma \sim 10^{16} - 10^{18}$ s^{-1}), the condition $\min\{c/R, \sigma, 1/\tau_0\} = 1/\tau_0$ takes place, according to which the Maxwell equations in the form of (2.6) are valid at $\omega\tau_0 \ll 1$.

If this condition holds, the first equation in (2.6) admits further simplification. Indeed, taking into account that $|\mathbf{H}| \sim \omega\sigma R^2 M/c^2$, we obtain the following estimate: $|\mathbf{H}|/|\mathbf{B}_0| \sim \omega\tau_0\eta$ ($\eta = \sigma R^2/\tau_0 c^2$). Since $\max\eta \sim 1$ and $\omega\tau_0 \ll 1$, it follows from here that we can neglect the field $\mathbf{H}$ in comparison with $\mathbf{B}_0$. Also considering the fact that $\mathbf{B}_0 = (8\pi/3)\mathbf{M}$ and $\mathbf{j} = \sigma\mathbf{E}$, equations (2.6) at $\omega\tau_0 \ll 1$ are reduced to

$$\mathrm{rot}\,\mathbf{E} = -\frac{8\pi}{3c}\frac{d\mathbf{M}}{dt}, \qquad \mathrm{div}\,\mathbf{E} = 0, \qquad (2.7a)$$

$$\mathrm{rot}\,\mathbf{H} = \frac{4\pi\sigma}{c}\mathbf{E}, \qquad \mathrm{div}\,\mathbf{H} = 0. \qquad (2.7b)$$

The main advantage of equations (2.7) consists in the following: equations (2.7a) for the induction electric field do not depend on the magnetic field strength of induction currents. This allows to solve firstly equations (2.7a), and then, using equations (2.7b), to obtain $\mathbf{H}$.

3. SOLUTION OF THE MAXWELL EQUATIONS

3.1 Induction electric field

Induction electric field induced by the time-dependent magnetization, is defined by equations (2.7a). Taking into account the relation $\mathrm{rot}(\mathbf{a}\times\mathbf{r}) = 2\mathbf{a}$, where $\mathbf{a}$ is the vector independent on the spatial variables, solution of equations (2.7a), in which a preferable direction is absent, can be searched in the form $\mathbf{E} = \mathbf{a}\times\mathbf{r}$. In this case, the second equation in (2.7a) is satisfied identically, and the first one gives $\mathbf{a} = -(4\pi/3c)d\mathbf{M}/dt$. Thus, induction electric field inside a conducting spherical particle is given by the formula

$$\mathbf{E} = -\frac{4\pi}{3c}\frac{d\mathbf{M}}{dt}\times\mathbf{r}. \tag{3.1}$$

According to (3.1), vector equation for the force lines of this field, $d\mathbf{r}\times\mathbf{E} = 0$, is equivalent to the following two scalar equations:

$$d\mathbf{r}\cdot\frac{d\mathbf{M}}{dt} = 0, \qquad d\mathbf{r}\cdot\mathbf{r} = 0, \tag{3.2}$$

where $d\mathbf{r}$ is the differential of the radius-vector $\mathbf{r}$ describing any field line, dot denotes the scalar product. The first equation in (3.2) shows that field lines lie in the planes perpendicular to the vector $d\mathbf{M}/dt$, and the second equation – that field lines represent the concentric circles, whose centers are located on a straight line passing through the origin of coordinates (particle center) in the direction of the vector $d\mathbf{M}/dt$ (see Fig. 1). Such structure of the field lines implies that boundary condition for the current density on the particle surface, $j_n = 0$ (index n denotes the normal component of the vector $\mathbf{j}$), holds automatically.

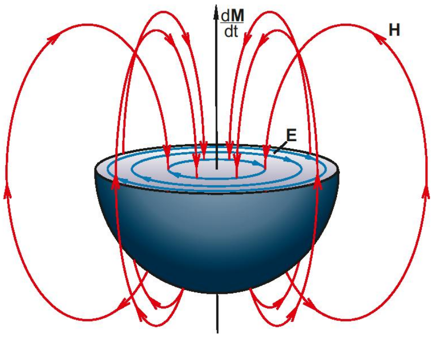

Fig. 1 – Schematic representation of the force lines of electric field induced by the time-dependent magnetization and force lines of magnetic field of eddy currents generated by the induction electric field. For illustrative purposes, we only show the electric field lines lying in the diametral plane and magnetic field lines crossing it at the fixed distance from the particle center

3.2 Magnetic field of eddy currents

Inside a particle, magnetic field of eddy currents, which are induced by the electric field (3.1), satisfies the system of equations (2.7b), and outside a particle – the same system of equations with $\sigma = 0$. Solution of these equations we will seek in the form of $\mathbf{H} = \operatorname{rot}\mathbf{A}$, where $\mathbf{A} = \mathbf{A}(\mathbf{r}, t)$ is the vector potential of magnetic field. Assuming that potential has the Coulomb calibration, $\operatorname{div}\mathbf{A} = 0$, and taking into account that in this case $\operatorname{rot}\operatorname{rot}\mathbf{A} = -\Delta\mathbf{A}$ (Δ is the Laplace operator), we obtain the vector Poisson equation

$$\Delta\mathbf{A} = \frac{16\pi^2\sigma}{3c^2}H(R-r)\frac{d\mathbf{M}}{dt}\times\mathbf{r} \tag{3.3}$$

($H(x) = 0$ at $x < 0$ and $H(x) = 1$ at $x \geq 0$) which determines the vector potential in t whole space.

It is well known (see, for example, [33]) that particular solution of this equation disappearing at $r\to\infty$ has the following form:

$$\mathbf{A} = -\frac{4\pi\sigma}{3c^2}\frac{d\mathbf{M}}{dt}\times\int_V\frac{\mathbf{r}'d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}. \tag{3.4}$$

Vector function of coordinates specified by the volume integral

$$\mathbf{I}(\mathbf{r}) = \int_V\frac{\mathbf{r}'d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}, \tag{3.5}$$

can be easily calculated. To this end, we re-write the desired integral in the spherical coordinate system and direct the z-axis of the rectangular coordinate system along the vector $\mathbf{r}$. In this case, $\mathbf{r} = r\mathbf{e}_z$ ($\mathbf{e}_z$ is the unit vector along the z-axis) and formula (3.5) is reduced to

$$\mathbf{I}(\mathbf{r}) = 2\pi\mathbf{e}_z\int_0^R\int_0^\pi\frac{r'^3\sin\theta\cos\theta\, dr' d\theta}{\sqrt{r^2+r'^2-2rr'\cos\theta}}. \tag{3.6}$$

Then, introducing a new integration variable $x = \cos\theta$ and using the tabulated integral [34]

$$\int\frac{xdx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b},$$

we obtain

$$\int_0^\pi\frac{\sin\theta\cos\theta\, d\theta}{\sqrt{r^2+r'^2-2rr'\cos\theta}} = \frac{2}{3}\begin{cases} r'/r^2 & (r'\leq r)\\ r/r'^2 & (r' > r).\end{cases} \tag{3.7}$$

Hence, integrating in (3.6) over r', we come to the following representation of the function $\mathbf{I}(\mathbf{r})$:

$$\mathbf{I}(\mathbf{r}) = \frac{2\pi}{15}\mathbf{r}\begin{cases}5R^2-3r^2 & (r\leq R)\\ 2R^5/r^3 & (r > R),\end{cases} \tag{3.8}$$

according to which we find from (3.4)

$$\mathbf{A} = -\frac{8\pi^2\sigma}{45c^2}\frac{d\mathbf{M}}{dt}\times\mathbf{r}\begin{cases}5R^2-3r^2 & (r\leq R)\\ 2R^5/r^3 & (r > R).\end{cases} \tag{3.9}$$

Finally, using formula $\mathbf{H} = \operatorname{rot}\mathbf{A}$ and easily verifiable relation

$$\operatorname{rot}[f(r)\,\mathbf{a}\times\mathbf{r}] = \left(2f(r) + r\frac{df(r)}{dr}\right)\mathbf{a} - \frac{1}{r}\frac{df(r)}{dr}(\mathbf{a}\cdot\mathbf{r})\mathbf{r}$$

($f(r)$ is an arbitrary function of r, vector $\mathbf{a}$ does not depend on the spatial variables), we obtain the magnetic field strength of eddy currents both inside a particle ($r\leq R$),

$$\mathbf{H} = -\frac{16\pi^2\sigma}{45c^2}\left[(5R^2-6r^2)\frac{d\mathbf{M}}{dt} + 3\left(\frac{d\mathbf{M}}{dt}\cdot\mathbf{r}\right)\mathbf{r}\right], \tag{3.10}$$

and outside a particle ($r > R$),

$$\mathbf{H} = \frac{16\pi^2\sigma R^5}{45c^2r^3}\left[\frac{d\mathbf{M}}{dt} - \frac{3}{r^2}\left(\frac{d\mathbf{M}}{dt}\cdot\mathbf{r}\right)\mathbf{r}\right]. \tag{3.11}$$

The last result, magnetic field outside a particle, can be also represented as the magnetic field of a point dipole

$$\mathbf{H} = -\frac{1}{r^3}\boldsymbol{\mu} + \frac{3}{r^5}(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r}, \tag{3.12}$$

where

$$\boldsymbol{\mu} = -\frac{16\pi^2\sigma R^5}{45c^2}\frac{d\mathbf{M}}{dt} \tag{3.13}$$

is the particle dipole magnetic moment conditioned by the flowing eddy currents, which are induced by the time-dependent magnetization $\mathbf{M}$. We also note that magnetic fields (3.10) and (3.11) have the rotational symmetry with respect to the (instantaneous) axis passing through the origin of coordinates parallel to the vector $d\mathbf{M}/dt$ (see Fig. 1).

4. EFFECTIVE LLG EQUATION

Now, using (3.10), we can find the averaged over the particle volume magnetic field of eddy currents $\bar{\mathbf{H}} = (1/V)\int_V d\mathbf{r}\,\mathbf{H}$. Taking into account that

$$\frac{1}{V}\int_V d\mathbf{r} = 1, \quad \frac{1}{V}\int_V d\mathbf{r}\, r^2 = \frac{3}{5}R^2,$$
$$\frac{1}{V}\int_V d\mathbf{r}\left(\frac{d\mathbf{M}}{dt}\cdot\mathbf{r}\right)\mathbf{r} = \frac{1}{5}R^2\frac{d\mathbf{M}}{dt} \qquad (4.1)$$

(these integrals can be easily calculated passing to the spherical coordinates), for the desired average field we obtain the following expression:

$$\bar{\mathbf{H}} = -\frac{32\pi^2\sigma R^2}{45c^2}\frac{d\mathbf{M}}{dt}. \qquad (4.2)$$

In accordance with the Lenz rule, direction of this field is opposite to the direction of the vector $d\mathbf{M}/dt$. We should also note that because of the nonuniformity of magnetic field of eddy currents, the field value in the particle center considerably exceeds the average value: $|\mathbf{H}(\mathbf{0},t)|/|\bar{\mathbf{H}}| = 2.5$.

Finally, substituting the average field (4.2) into the equation (2.5), we come to the effective LLG equation

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\mathbf{H}_{\text{eff}}^{(0)} + \frac{\alpha+\alpha_\sigma}{M}\mathbf{M}\times\frac{d\mathbf{M}}{dt}, \qquad (4.3)$$

which describes the magnetization dynamics in conducting spherical particles being in a single-domain sate. According to this equation, influence of the conductivity on the magnetization dynamics is completely accounted for by introducing an additional (relative to the Gilbert damping parameter α) Gilbert damping parameter

$$\alpha_\sigma = \frac{32\pi^2\sigma R^2\gamma M}{45c^2} \qquad (4.4)$$

of electrodynamic origin. We emphasize that this result is obtained using the condition of quasi-stationarity of electromagnetic field and approximation of static conductivity, which, as it was established in the second section, hold at $\omega \ll 1/\tau_0 \sim 10^{13}\,\text{s}^{-1}$. This means that, since intrinsic frequency of magnetization precision of ferromagnetic particles does not usually exceed the value of $10^{11}\,\text{s}^{-1}$, the effective LLG equation (4.3) is applicable in a wide frequency range including the ferromagnetic resonance region.

In conclusion, we discuss how important is accounting of conductivity in the description of the magnetization dynamics. It is seen from equation (4.3) that this accounting is necessary if electrodynamic damping parameter α_σ is comparable or exceeds the damping parameter α. Depending on the particle material and character of magnetic dynamics, the value of the latter parameter is usually in the range from 10^{-4} to 10^{-1} (for example, in garnets $\alpha \sim 10^{-4} - 10^{-3}$). For illustrative purposes, we will find α_σ for iron particles. Assuming that (in the CGS system) $\sigma = 10^{18}\,\text{s}^{-1}$, $4\pi M = 2.2\cdot 10^4$ Gs and $\gamma = 1.76\cdot 10^7\,\text{s}^{-1}\times Gs^{-1}$, we obtain from (4.4) $\alpha_\sigma \approx 2.4\cdot 10^{-6}R^2$, where R is measured in nanometers. Thus, for example, if $R = 10$ nm, then $\alpha_\sigma \approx 2.4\cdot 10^{-4}$. The given estimates show that damping parameters α_σ and α can be of the same order and, consequently, in these cases for the description of the magnetization dynamics one should use the effective LLG equation (4.3).

5. CONCLUSIONS

The influence of the conductivity of single-domain ferromagnetic particles of spherical form on the dynamics of their magnetization has been studied. Consideration has been performed in the framework of the model which uses the coupled system of the Landau-Lifshitz-Gilbert and Maxwell equations. Connection between these equations is carried out due to the fact that in the LLG equation the effective magnetic field contains averaged (over the particle volume) magnetic field of eddy currents which is derived from the Maxwell equations. In turn, the Maxwell equation describing the Faraday law of electromagnetic induction contains magnetization, whose dynamics obeys the LLG equation. The important feature of the Maxwell equations written in the quasi-stationary approximation consists in the fact that these equations for the considered problem geometry can be solved in the general case of an arbitrary dependence of magnetization direction on time. This allowed to find the exact expression for the magnetic field of eddy currents in the whole space and calculate its average value in the vicinity of a particle. Finally, using the last result, we have obtained the effective LLG equation describing the magnetization dynamics in conducting single-domain particles, in which influence of eddy currents is completely accounted for by introducing the additional Gilbert damping parameter. Analysis of the used approximations has shown that this equation describes both slowly and rapidly changing processes, whose characteristic frequencies do not exceed $10^{13}\,\text{s}^{-1}$.

AKNOWLEDGEMENTS

The present work has been performed under the partial financial support of the Ministry of Education and Science of Ukraine, Project No 0112U001383.

REFERENCES

1. E.M. Chudnovsky, J. Tejada, *Macroscopic Quantum Tunneling of the Magnetic Moment* (Cambridge University Press, Cambridge, 1998).
2. L. Gammaitoni, P. Hänggi, P. Jung, F. Marchesoni, *Rev. Mod. Phys.* **70**, 223 (1998).
3. M. Bauer, J. Fassbender, B. Hillebrands, R.L. Stamps, *Phys. Rev. B* **61**, 3410 (2000).
4. S. Kaka, S.E. Russek, *Appl. Phys. Lett.* **80**, 2958 (2002).
5. C. Thirion, W. Wernsdorfer, D. Mailly, *Nature Mater.* **2**, 524 (2003).
6. G. Woltersdorf, C.H. Back, *Phys. Rev. Lett.* **99**, 227207 (2007).
7. C.A. Ross, *Annu. Rev. Mater. Res.* **31**, 203 (2001).
8. B.D. Terris, T. Thomson, *J. Phys. D* **38**, R199 (2005).
9. I. Žutić, J. Fabian, S. Das Sarma, *Rev. Mod. Phys.* **76**, 323 (2004).
10. S. Maekawa (Ed.) *Concepts in Spin Electronics* (Oxford University Press: Oxford: 2006).
11. Q.A. Pankhurst, J. Connolly, S.K. Jones, J. Dobson, *J. Phys. D: Appl. Phys.* **36**, R167 (2003).
12. R. Hergt, S. Dutz, R. Müller, M. Zeisberger, *J. Phys.: Condens. Matter.* **18**, S2919 (2006).
13. S. Laurent, D. Forge, M. Port, A. Roch, C. Robic, L. Vander Elst, R.N. Muller, *Chem. Rev.* **108**, 2064 (2008).
14. L. Landau, E. Lifshitz, *Phys. Z. Sowjetunion* **8**, 153 (1935).

15. T.L. Gilbert, *IEEE Trans. Magn.* **40**, 3443 (2004).
16. G. Bertotti, I. Mayergoyz, C. Serpico, *Nonlinear Magnetization Dynamics in Nanosystems* (Elsevier, Oxford, 2009).
17. W.F. Brown Jr., *Phys. Rev.***130**, 1677 (1963).
18. W.T. Coffey, Yu.P. Kalmykov, J.T. Waldron, *The Langevin Equation*, 2nd ed. (World Scientific, Singapore, 2004).
19. S.I. Denisov, K.N. Trohidou, *Phys. Rev. B* **64**, 184433 (2001).
20. S.I. Denisov, T.V. Lyutyy, K.N. Trohidou, *Phys. Rev. B* **67**, 014411 (2003).
21. S.I. Denisov, T.V. Lyutyy, P. Hänggi, K.N. Trohidou, *Phys. Rev. B* **74**, 104406 (2006).
22. S.I. Denisov, T.V. Lyutyy, P. Hänggi, *Phys. Rev. Lett.* **97**, 227202 (2006).
23. S.I. Denisov, K. Sakmann, P. Talkner, P. Hänggi, *Phys. Rev. B* **75**, 184432 (2007).
24. S.I. Denisov, A.Yu. Polyakov, T.V. Lyutyy, *Phys. Rev. B* **84**, 174410 (2011).
25. S.S. Ray, M. Okamoto, *Prog. Polym. Sci.* **28** 1539 (2003).
26. F. Hussain, M. Hojjati, M. Okamoto, R.E. Gorga, *J. Compos. Mater.* **40**, 1511 (2006).
27. G. Bertotti, *Hysteresis in Magnetism* (Academic Press, San Diego, 1998).
28. E. Martinez, L. Lopez-Diaz, L. Torres, *J. Appl. Phys.* **99**, 123912 (2006).
29. M. Lakshmanan, *Phil. Trans. R. Soc. A* **369**, 1280 (2011).
30. I.D. Mayergoyz, C. Serpico, Y. Shimizu, *J. Appl. Phys.* **87**, 5529 (2000).
31. I. Cimrák, *Arch. Comput. Methods Eng.* **15**, 277 (2008).
32. L.D. Landau, E.M. Lifshitz, *Electrodynamics of Continuous Media*, 2nd ed. (Pergamon Press: Oxford: 1984).
33. A.D. Polyanin, *Handbook of Linear Partial Differential Equations for Engineers and Scientists*(Chapman & Hall/CRC Press: Boca Raton: 2002) Problem 8.2.2-1.
34. A.P. Prudnikov, Yu.A. Brychkov, O.I. Marichev, *Integrals and Series*, Vol. 1 (Gordon & Breach: New York: 1986).