# Peculiarities of the Dependence of the Strain Coefficient on the Deformation of Metal Films

K.V. Tyschenko<sup>1</sup>, L.V. Odnodvorets<sup>1</sup>, C.J. Panchal<sup>2</sup>, I.Yu. Protsenko<sup>1,\*</sup>

 Sumy State University, Rymsky-Korsakov Str., 2, 40007 Sumy, Ukraine
 Applied Physics Department, Faculty of Technology and Engineering, the M. S.University of Baroda, Vadodara, 390001 Gujarat, India

(Received 02 December 2012; published online 29 December 2012)

We propose a phenomenological theory to explain the physical nature of the maximum or minimum in the dependence of the instantaneous longitudinal strain coefficient,  $\gamma_{li}$ , versus the strain,  $\varepsilon_l$ , in double-layer films Fe/Cr, Cu/Cr and Fe/a-Gd. The theory is based on the analysis of extremum (maximum or minimum), which is obtained by simplifying the equation  $\partial \gamma_{li}/\partial \varepsilon_l = 0$ . It is concluded that the appearance of a maximum or minimum is caused by both non-linear deformation processes in  $\varepsilon_l$  and possible structural changes in the films.

Keywords: Strain coefficient, Double-layer films, Solid solution, Instantaneous strain coefficient.

PACS numbers: 68.60. Bs, 68.65.Ac

### 1. INTRODUCTION

Interest in the mechanical properties and thin-film materials due to the fact that they have a significant difference in comparison of bulk materials. The works [1-3] illustrate the most common problems that are constantly in view of researchers: studying stress-strain relations for metal films [1]; measurement of mechanical properties of thin films [2]; plastic deformation processes in bimetallic films [3] et al.

During the research of strain properties of singlelayer (Cr, Gd) and double-layer (Fe/Cr/Sub, Cu/Cr/Sub and Fe/a-Gd/Sub, where Sub - substrate) films we observed [4, 5] the effect of an abnormal increase in the instantaneous longitudinal strain coefficient, yli, under the strain  $\varepsilon_l$ . This effect is shown in Figs. 1 and 2 for Fe(20)/Cr(30)/Sub and Fe(50)/a-Gd (30)/Sub film systems, respectively (the layer thickness is in nm). Note that the maximum or minimum in the dependence of  $\gamma li$ on  $\varepsilon_l$  is observed not only in the dynamic mode of longitudinal deformation (in our experiments strain rate  $\Delta l/l$ varied from 0 to 0.1 %/sec, l – the initial length of the sample), but also under static loading. For seven times increase in the strain rate the mean strain coefficient, yli varied only within 5 %. In Ref. [5], it was noted that the strain value, for which a maximum and a minimum is observed, corresponds to a transition ( $\varepsilon_{ltr}$ ) from elastic or quasielastic to plastic deformation, respectively, implying a change in the deformation mechanism. Therefore, the appearance of maximum and minimum in the dependence of  $\gamma_{li}$  versus  $\varepsilon_l$  is partially not only due to the deformation, but also due to structural processes occurring witht the change in the deformation mechanism.

### 2. RESULTS AND DISCUSSIONS

To establish the conditions for the appearance of the maximum in the dependence of  $y_{li}$  versus  $\varepsilon_l$  we use the condition for the existence of extremum. Because the phase state of Cu/Cr/Sub and Fe/a-Gd /Sub film

systems correspond to the type "biplate" (which is confirmed by diffraction studies), whereas for Fe/Cr/Sub film systems a solid solution is formed throughout the sample (by work [6]), the equation for the dependence of  $\gamma_{li}$  versus all has a different form. For convenience of mathematical transformations, we get from  $\gamma_{li}$  to the  $\gamma_{li}^{\rho}$  (index " $\rho$ " means that the strain coefficient is expressed through the resistivity), between which there is a simple relationship:

$$\gamma_{li}^{\rho} \equiv \frac{d \ln \rho}{d \,\varepsilon_l} = \gamma_{li} - 1 - 2\mu,$$

where  $\mu$  is the Poisson's ratio.

It is easy to show that for the model of parallel connection of double layers

$$\rho = \frac{\rho_1 \rho_2 (d_1 + d_2)}{\rho_1 d_2 + \rho_2 d_1},$$

where  $d_k$  – thickness of separate layers (k = 1, 2).

In the first case of "biplate" the equation for  $\gamma_{li}^{\rho}$  has the form:

$$\begin{split} \gamma_{li}^{\rho} &= \frac{d \ln \rho}{d \varepsilon_{l}} = \gamma_{li1}^{\rho} + \gamma_{li2}^{\rho} - \frac{d_{1} \mu_{1} + d_{2} \mu_{2}}{d_{1} + d_{2}} - \\ &- \frac{\gamma_{li1}^{\rho} \rho_{1} d_{2} - \rho_{1} d_{2} \mu_{2} + \gamma_{li2}^{\rho} \rho_{2} d_{1} - \rho_{2} d_{1} \mu_{1}}{\rho_{1} d_{2} + \rho_{2} d_{1}} \end{split} . \tag{1}$$

Note that this approach is common in obtaining relations for thermal resistance coefficient [7] and Hall coefficient [8] etc. for double-layer films.

In the second case for film system as a solid solution the ratio for  $\gamma_{li}^{\rho}$  was obtained in [6] by using the ratio:

$$\rho = \rho_{res} + c_1 \rho_1 + c_2 \rho_2$$

where  $\rho_{res}$  – resudial resistivity,

\*

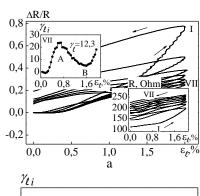
<sup>\*</sup> protsenko@aph.sumdu.edu.ua

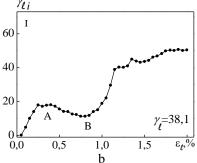
$$\gamma_{li}^{\rho} = \frac{\gamma_{li1}^{\rho}}{1 + c_2 \rho_2 / c_1 \rho_1} + \frac{\gamma_{li2}^{\rho}}{1 + c_1 \rho_1 / c_2 \rho_2} , \qquad (2)$$

where  $c_k$  is the common concentration of atoms in k-layer and where take into account that  $\gamma_{li}^{\rho_{res}} << \gamma_{li}^{\rho}$ .

From equation (1) and assuming that  $\partial \mu_k / \partial \varepsilon_l \approx 0$  and  $(d_1\mu_1 + d_2\mu_2)/(d_1 + d_2)$  is relatively small size (order unit), the extremum condition can be rewritten as:

$$\frac{1}{\rho_{1}} \frac{\partial^{2} \rho_{1}}{\partial \varepsilon_{l}^{2}} + \frac{1}{\rho_{2}} \frac{\partial^{2} \rho_{2}}{\partial \varepsilon_{l}^{2}} \cong \frac{\gamma_{li1}^{\rho}}{\rho_{1}} + \frac{\gamma_{li2}^{\rho}}{\rho_{2}} + \frac{\frac{\partial \gamma_{li1}^{\rho}}{\partial \varepsilon_{l}} \rho_{1} d_{2} + \frac{\partial \gamma_{li2}^{\rho}}{\partial \varepsilon_{l}} \rho_{2} d_{1}}{\rho_{1} d_{2} + \rho_{2} d_{1}} + \frac{\gamma_{li1}^{\rho} \rho_{1} d_{2} (\gamma_{li1}^{\rho} - 2\mu_{2}) + \gamma_{li2}^{\rho} \rho_{2} d_{1} (\gamma_{li2}^{\rho} - 2\mu_{1})}{\rho_{1} d_{2} + \rho_{2} d_{1}} + \frac{\gamma_{li1}^{\rho} \rho_{1} d_{2} (\gamma_{li1}^{\rho} - 2\mu_{2}) + \gamma_{li2}^{\rho} \rho_{2} d_{1} (\gamma_{li2}^{\rho} - 2\mu_{1})}{\rho_{1} d_{2} + \rho_{2} d_{1}}.$$
(3)





**Fig. 1** – The variation of  $\Delta R/R$  and  $\gamma_{li}$  versus  $\varepsilon_l$  for the Fe(20)/Cr(30)/Sub film system. R – resistance,  $\gamma_l$  – mean value of gauge factor. I, VII – number of deformation cycles "load-unload"

Assuming that the Poisson's coefficient depends on the deformation, i.e.  $\partial \mu_k / \partial \varepsilon_l \neq 0$ , we obtain an equation that is very similar to (3). Both of them take the following form:

$$\rho_2 \frac{\partial^2 \rho_1}{\partial \varepsilon_l^2} + \rho_1 \frac{\partial^2 \rho_2}{\partial \varepsilon_l^2} \cong \gamma_{li1}^{\rho} \rho_2 + \gamma_{li2}^{\rho} \rho_1, \tag{4}$$

if we consider that  $\gamma_{li}^{\rho}/\rho \sim 10^7 \, \mathrm{Ohm^{-1}} \, \mathrm{m^{-1}},$   $\gamma_{li}^{\rho} \cdot d/(d_1 + d_2) \sim 10 \, \mathrm{and} \, \gamma_{li}^{\rho} \cdot \rho d \sim 10^{-4} \, \mathrm{Ohm} \, \mathrm{m^2}.$ 

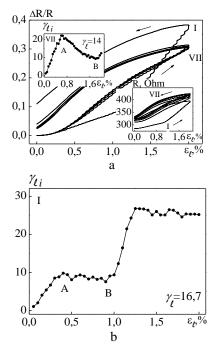
From equation (2), with the assumption that  $d\ln c_i/d\varepsilon = 0$ , we obtain the extremum condition similar to (4):

$$\frac{1}{\rho_1} \frac{\partial^2 \rho_1}{\partial \varepsilon_l^2} + \frac{1}{\rho_2} \frac{\partial^2 \rho_2}{\partial \varepsilon_l^2} \cong \gamma_{li1}^{\rho} c_1 + \gamma_{li2}^{\rho} c_2. \tag{5}$$

Equations (4) and (5) can be rewritten as follows:

$$\begin{split} \frac{\partial^{2} \rho_{1}}{\partial \varepsilon_{l}^{2}} &\cong \frac{\rho_{1}}{\rho_{2}} \left( \gamma_{li2}^{\rho} - \frac{\partial^{2} \rho_{2}}{\partial \varepsilon_{l}^{2}} \right) + \gamma_{li1}^{\rho} = C_{1}, \\ \frac{\partial^{2} \rho_{1}}{\partial \varepsilon_{l}^{2}} &\cong \rho_{1} \left( \gamma_{li1}^{\rho} c_{1} + \gamma_{li2}^{\rho} c_{2} - \frac{1}{\rho_{2}} \frac{\partial^{2} \rho_{2}}{\partial \varepsilon_{l}^{2}} \right) = C_{2}. \end{split}$$
 (6)

for the condition that  $\rho_2$  slightly depends on the deformation.



**Fig. 2** – Variation of  $\Delta R/R$ , and  $\gamma_{li}$  versus  $\varepsilon_l$  for Fe(50)/a-Gd(30)/Sub film system, a-amorphous phase

Analysis of equations (6) makes it possible to conclude, that if  $\frac{\rho_1}{\rho_2} \cdot \left| \frac{\partial^2 \rho_2}{\partial \varepsilon_l^2} \right| > \frac{\rho_1}{\rho_2} \gamma_{li2}^{\rho} + \gamma_{li1}^{\rho} \quad \text{or}$ 

$$\frac{\rho_1}{\rho_2} \cdot \left| \frac{\hat{\sigma}^2 \rho_2}{\hat{\sigma} \varepsilon_l^2} \right| > \rho_1 \left( \gamma_{li1}^{\rho} c_1 + \gamma_{li2}^{\rho} c_2 \right) \text{ then in the } \gamma_{li} \text{ versus } \varepsilon_l \text{ a}$$

maximum occurs if  $(\partial^2 \rho_1/\partial \varepsilon_l^2 < 0$ , point A in Fig. 1 and Fig. 2), and for the reverse inequalities a minimum occurs  $(\partial^2 \rho_1/\partial \varepsilon_l^2 > 0$ , point B in Fig. 1 and Fig. 2). We note that similar conclusions can be made with respect to the derivative  $\partial^2 \rho_2/\partial \varepsilon_l^2$ .

Under this condition, the dependence of the resistivity on the strain is nonlinear:

 $\rho\left(\varepsilon_{l}\right)=0.5C_{1}\varepsilon_{l}^{2}+A_{1}\varepsilon_{l}+B_{1}\text{ ("biplate"}\quad\text{film}\quad\text{system}$  type).

 $\rho \left( \varepsilon_l \right) = 0.5 C_2 \varepsilon_l^2 + A_2 \varepsilon_l + B_2 \ \ (\text{system, where solid solutions are formed}), \ \text{where} \ \ A_k = \left( \partial \rho \, / \, \partial \varepsilon_l \right)_{\varepsilon_l \to 0} - \text{sensitivity of the resistivity to strain at } \varepsilon_l \ \approx 0; \ B_k = \rho(0) - \text{the initial resistivity value}.$ 

### 3. CONCLUSIONS

Our analysis indicates that the appearance of a maximum or minimum in the variation of  $\gamma_{lm}$  versus  $\varepsilon_l$ is caused by the nonlinear variation of the resistivity that occurs under corresponding deformation or is the result of structural changes in the film system during the transition from elastic to plastic deformation (point A) or other deformation mechanism (point B). Comparison of the depending  $\gamma_{li}$  versus  $\varepsilon_l$  for the I and VII deformation cycles (Fig. 1, 2) leads to the conclusion that the intensity of non-linear processes are substantially independent of the number of deformation cycles, which means that, for small numbers of cycles have the elastic deformation (to A), the quasielastic (between points A and B) and plastic (after point B). At the V - VII deformation cycles occurs only plastic deformation with grain boundary sliding of grains, which causes the appearance of the maximum. Although this is only indirect conclusions, which are based on an analysis of resitometry dependencies.

Finally, we note the following fact. It is believed (see for example [9]) that at the plastic deformation of films  $\gamma_l^{\rho} \simeq 0$ , because, as the author consider, there is a slipping on the borders of grains, but individual grains are not deformed and in this case  $\mu_l \simeq 0.5$ . Then according to the ratio  $\gamma_l = \gamma_l^{\rho} + 1 + 2\mu_l$  it  $\gamma_l \simeq 2$ . In our case (Fig. 1, 2)  $\gamma_l$  much more than 2, because of our great contribution in the value of strain coefficient of scattering electrons at the grain boundary.

### ACKNOWLEDGEMENTS

K.V. Tyschenko, L.V. Odnodvorets and I.Yu. Protsenko are thankful to the Ministry of Education and Science, Young and Sport of Ukraine for financial assistance (Project No M/362, 31.08.2012), and C.J. Panchal are thankful to the Department of Science and Technology, Government of India for financial assistance under INDO-UKRAINE program of cooperation in science and technology.

# Особливості залежності коефіцієнта тензочутливості від деформації в металевих плівках

К.В. Тищенко<sup>1</sup>, Л.В. Однодворець<sup>1</sup>, С.J. Panchal<sup>2</sup>, І.Ю. Проценко<sup>1</sup>

<sup>1</sup> Сумський державний університет, вул. Римського-Корсакова, 2, 40007 Суми, Україна <sup>2</sup> Applied Physics Department, Faculty of Technology and Engineering, the M. S.University of Baroda, Vadodara, 390001 Gujarat, India

Нами запропонована феноменологічна модель для пояснення природи максимума або мінімума на залежності миттєвого значення коефіцієнта тензочутливості  $\gamma_{li}$  від деформації  $\varepsilon_l$  у двошарових плівках Fe/Cr, Cu/Cr та Fe/ $\alpha$ -Gd. Теорія основується на аналізі екстремума (максимума або мінімума), який отримується спрощенням рівняння  $\partial \gamma_{li}/\partial \varepsilon_l = 0$ . Зроблено висновок, що поява максимума або мінімума пов'язана як з нелінійними деформаційними процесами по  $\varepsilon_l$ , так і можливими структурними змінами у плівках.

**Ключові слова**: Коефіцієнт тензочутливості, Двошарові плівки, Твердий розчин, Миттєвий коефіцієнт тензочутливості.

# Особенности зависимости коэффициента тензочувствительности от деформации в металлических пленках

К.В. Тищенко<sup>1</sup>, Л.В. Однодворец<sup>1</sup>, С.J. Panchal<sup>2</sup>, І.Е. Проценко<sup>1</sup>

<sup>1</sup> Сумский государственный университет, ул. Римского-Корсакова, 2, 40007 Сумы, Украина <sup>2</sup> Applied Physics Department, Faculty of Technology and Engineering, the M. S.University of Baroda, Vadodara, 390001 Gujarat, India

Нами предложена феноменологическая модель для объяснения природы максимума или минимума на зависимости мгновенного значения коэффициента тензочувствительности  $y_{li}$  от деформации  $\varepsilon_l$  в двухслойных пленках Fe/Cr, Cu/Cr и Fe/a-Gd. Теория основана на анализе экстремума (максимума или минимума), который получается упрощением уравнения  $\partial y_{li}/\partial \varepsilon_l = 0$ . Сделано вывод, что появление максимума или минимума связано как с нелинейными деформационными процессами по  $\varepsilon_l$ , так и возможными структурными изменениями в пленках.

**Ключевые слова**: Коэффициент тензочувствительности, Двухслойные пленки, Твердый раствор, Мгновенный коэффициент тензочувствительности.

## REFERENCES

- J.N. Florando, W.D. Nix, J. Mech. Phys. Solids 53, 619 (2005).
- 2. E. Gacoin, C. Fretigny, A. Chatenauminois, A. Perriot, E. Barthel, *Tribol. Lett.* **21**, 245 (2006).
- K.S. Kumar, L. Reinbold, A.F. Bower, E. Chason, J. Mater. Res. 23, 2916 (2008).
- S.I. Protsenko, D.V. Velykodnyi, V.A. Kheraj, M.S. Desai, C.J. Panchal, I.Yu. Protsenko, J. Mater. Sci. 44, 4905 (2009)
- 5. I.P. Buryk, D.V. Velykodnyi, L.V. Odnodvorets, I.E. Protsenko,
- E.P. Tkach, *Techn. Phys.* 56, 232 (2011).
- L.V. Odnodvorets, S.I. Protsenko, O.V. Synashenko, D.V. Velykodnyi, I.Yu. Protsenko, Cryst. Res. Technol. 44, 74 (2009).
- 7. R. Dimmich, *Thin Solid Films* **158**, 13 (1988).
- 8. G. Reiss, K. Kapfberger, G. Meier, J. Vancea, H. Hoffman, J. Phys. Condens. Matter 1, 1275 (1989).
- 9. N.P. Klokova, *Tenzorezistory* (Moscow: Mashinostroenie: 1990) (in Russian).