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Active FEL-Klystrons as Formers of Femtosecond Clusters of Electromagnetic Field. Nonlinear Physics of the Transit Section

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The classification and the kinematic analysis of parametrical resonant interactions in the transit section of two-stream superheterodyne free electron laser are carried out. It is found out that realization of four types of parametrical resonant interactions is possible. A number of the investigated variants of interactions have plural character – hundreds and more harmonics connected with each other simultaneously participate in a three-wave parametric resonance. A cubically nonlinear multiharmonic theory of plural parametrical resonant interactions is constructed. It is established that such interactions can substantially influence the development of physical processes in the investigated system. It is offered to use the plural parametrical resonant interactions for the formation of a wide multiharmonic spectrum of waves in cluster two-stream superheterodyne free electron lasers.

Keywords: Free electron lasers, Three-wave parametric resonance, Two-stream instability.

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1. INTRODUCTION

The present work is the fifth part of the cycle of papers [1-4] devoted to the study of a new class of relativistic electronic devices, namely, the active FEL-klystrons which are intended for the formation of ultrashort clusters of electromagnetic field. The models of such devices based on the single-speed [1-3] and two-speed [4] relativistic electron beams are considered. General description of two-stream cluster FEL-klystrons is performed and key features of linear theory of multiharmonic processes in the transit section are discussed in [4]. In the given work we continue the analysis of the processes in the transit section of two-stream active FEL-klystron but within the nonlinear theory.

It is known that spatial charge waves (SCW) of different types can propagate in two-stream electron system [5-9]. Three-wave parametric resonances are realized between harmonics of such waves. In spite of the fact that this question was considered before in [10], a number of types of three-wave interactions was found to be not studied. We eliminate this shortage here.

Parametrical interactions of electron waves in the theory of two-stream FEL are not new and have been studied for not less than 30 years (for example, [11-21]). However, physical situation in the investigated model cardinally differs from that studied before. The explicitly expressed plurality is the specific character of the variety of three-wave resonances realized here. It appears that conditions of a three-wave parametric resonance can be simultaneously satisfied for a set of triples of interacting waves which are connected with each other via common waves. As a result, a number of interacting waves can be hundreds and more in some exceptional cases. It is evident that general picture of such plural interactions is really found to be rather complicated and interest. And this paper is devoted to the investigation of such interactions.

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2. THE MODEL. CONDITIONS FOR PARAMETRIC RESONANCES

Theoretical model of the transit section of the two-stream FEL is represented in Fig. 1. We consider the two-speed electron beam consisted of two partial ones 1 and 2 which are characterized by the velocities $\boldsymbol{\upsilon}_1$ and $\boldsymbol{\upsilon}_2$ ($\upsilon_1 - \upsilon_2 << \upsilon_1, \upsilon_2$) and the same plasma frequencies $\omega_{p1} = \omega_{p2} = \omega_p$. We assume that the beam is sufficiently wide, and therefore we can neglect the influence of the boundaries on the processes of wave interaction. We suggest that the beam moves in a focusing magnetic field $\boldsymbol{B}_0$ directed along the Z-axis. We do not consider the effects connected with quasi-static fields of spatial charge and neglect the thermal spread of electron velocities. We assume that SCW are multiharmonic. Then for the resulting electric field strength of SCW we have

$$\mathbf{E} = \sum_\chi \mathbf{E}_\chi \ , \ \mathbf{E}_\chi = \sum_{m=1}^{N} \left[E_{\chi,m} \exp(ip_{\chi,m}) + c.c. \right] \mathbf{e}_z \ . \quad (1)$$

In these correlations index χ characterizes the SCW type (as it will be shown later, χ takes values from 1 to 7, see Table 1); $p_{\chi,m} = \omega_{\chi,m} t - k_{\chi,m} z$ is the phase of the m-th harmonic of χ-th wave; $\omega_{\chi,m} = m \cdot \omega_{\chi,1}$ and $k_{\chi,m}$ are the frequency and wave number of the m-th harmonic of χ-th wave; N is the total number of harmonics which are considered during problem solving; m is the harmonic number; $\boldsymbol{e}_z$ is the unit vector of the Z-axis; notation "*c.c.*" denotes the "complex conjugate expression".

Presence of different SCW types in two-speed electron beam conditions a great number of interaction types of their harmonics. Dispersion laws of these waves are known [5, 6] and can be represented in the form of the following generalized dispersion relation

$$k_\chi = \frac{\omega_\chi}{\upsilon_0 (1 + \sigma_\chi \delta)} + r_\chi \frac{\omega_p}{\upsilon_0 \gamma_0^{3/2}} \ , \quad (2)$$

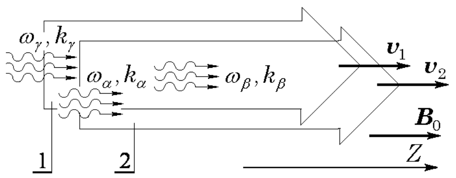

Fig. 1 – Theoretical model of multiharmonic transit section

Table 1 – Classification of SCW in two-stream system

Wave type (χ)	σ_χ	r_χ	Wave name
1	0	0	Growing wave ($\omega < \omega_{cr}$)
2	0	$+\sqrt{15}/2$	Slow wave ($\omega < \omega_{cr}$)
3	0	$-\sqrt{15}/2$	Fast wave ($\omega < \omega_{cr}$)
4	+1	+1	Slow wave of the first beam ($\omega > \omega_{cr}$)
5	+1	–1	Fast wave of the first beam ($\omega > \omega_{cr}$)
6	–1	+1	Slow wave of the second beam ($\omega > \omega_{cr}$)
7	–1	–1	Fast wave of the second beam ($\omega > \omega_{cr}$)

where $\upsilon_0 = (\upsilon_{01} + \upsilon_{02})/2$ is the average velocity of two-speed beam; $\gamma_0 = 1/\sqrt{1-(\upsilon_0/c)^2}$; $\delta = (\upsilon_{01} - \upsilon_{02})/2\upsilon_0$; σ_χ and r_χ are the sign functions whose values for different wave types are represented in Table 1.

Index $\chi = 1$ corresponds to the growing, 2 and 3 – to the slow and fast subcritical waves, i.e. waves whose frequency does not exceed the critical frequency

$$\omega_{cr} = \omega_p / (\sqrt{2}\delta\gamma_0^{3/2}) . \qquad (3)$$

As it follows from Table 1, subcritical region is described by the value of sign function $\sigma_\chi = 0$ (χ = 1, 2, 3). In this case, sign function $r_1 = 0$ describes the growing wave, $r_2 = +\sqrt{15}/2$ – the slow one, and $r_3 = -\sqrt{15}/2$ – the fast SCW. Index χ = 4, 5, 6 and 7 characterizes the overcritical slow and fast waves ($\omega > \omega_{cr}$). Here function $\sigma = -1$ characterizes waves of the first beam ($r_4 = +1$ – slow ($\chi = 4$), $r_4 = -1$ – fast ($\chi = 5$)). Function $\sigma = +1$ characterizes waves of the second beam ($r_6 = +1$ – slow ($\chi = 6$), $r_7 = -1$ – fast ($\chi = 7$)).

We take into account that three-wave parametrical interactions between wave harmonics are realized in plasma of the two-speed beam. We designate the frequencies and wave numbers of wave harmonics, which are involved into a parametrical-resonant triple chosen for the consideration, by the indexes α, β and γ (Fig. 1). Then condition of the parametrical resonance can be written in the form

$$p_{\alpha,m_\alpha} + \nu\, p_{\beta,m_\beta} = p_{\gamma,m_\gamma} \qquad (4)$$

or taking into account definition of the phase

$$m_\alpha \omega_{\alpha 1} + \nu\, m_\beta\, \omega_{\beta 1} = m_\gamma \omega_{\gamma 1} \,, \qquad (5)$$
$$k_{\alpha,m_\alpha} + \nu\, k_{\beta,m_\beta} = k_{\gamma,m_\gamma} \,. \qquad (6)$$

When analyzing three-wave parametrical resonances with participation of α, β and γ waves, it is necessary to single out physically different types of interactions. Since from the physical point of view α, β and γ waves are equivalent, an ambiguity can appear if separate different interaction types: at permutation of α, β and γ indexes nothing is physically changed, but mathematically we have different situations. To avoid this ambiguity in correlations (4)-(6), we take sign function ν to be equal to

$$\nu = -1. \qquad (7)$$

Then α wave, as it follows from equation (5), will have the largest frequency, γ wave – the smallest one, and β wave – an intermediate frequency. In connection with this, we will call wave with maximum frequency as α wave or signal one, wave with minimum frequency – as γ wave or pump, and wave with intermediate frequency – as β wave. During the analysis we will cast out cases at which, for example, wave frequency α will not be the maximum.

Taking into account generalized dispersion relation (2) and correlation (5), conditions of the parametrical resonance (6) can be written in the form

$$m_\alpha \omega_{\alpha 1} \frac{\sigma_\gamma - \sigma_\alpha}{1 + \sigma_\alpha \delta} - m_\beta \omega_{\beta 1} \frac{\sigma_\gamma - \sigma_\beta}{1 + \sigma_\beta \delta} = = \frac{\omega_p (1 + \sigma_\gamma \delta)(r_\gamma - r_\alpha + r_\beta)}{\delta \gamma_0^{3/2}} . \qquad (8)$$

Conditions (5) and (8) determine all possible types of parametrical resonant interactions within the studied model.

3. TYPES OF THREE-WAVE RESONANCES

Type 1: $\sigma_\alpha = \sigma_\beta = \sigma_\gamma = 0$, $r_\alpha = r_\beta = r_\gamma = 0$ – all α, β and γ waves are growing ones ($\chi = 1$) and belong to the subcritical region ($\omega \le \omega_{cr}$). Since α, β and γ waves belong to the same wave type, their first harmonics are equal: $\omega_{\alpha 1} = \omega_{\beta 1} = \omega_{\gamma 1}$. Then expression (5) is transformed to the form

$$m_\alpha - m_\beta = m_\gamma \,. \qquad (9)$$

We note, α, β and γ waves are characterized by sign functions $\sigma_\alpha = \sigma_\beta = \sigma_\gamma = 0$, $r_\alpha = r_\beta = r_\gamma = 0$, therefore for these waves condition (8) is found to be fulfilled.

Thus, condition (9), where m_α, m_β, m_γ are integer numbers, is the unique condition for the realization of three-wave resonances of such type. Obviously, it can be fulfilled for the majority triples of harmonics, and in this case such triples are connected with each other by common waves. They say about such situation that plural parametric resonances are realized in the system.

Type 2: resonant interaction of the growing, fast, and slow waves, whose frequencies do not exceed the critical one. Since $\omega_\chi \le \omega_{cr}$ for all waves, sign functions σ are equal to zero: $\sigma_\alpha = \sigma_\beta = \sigma_\gamma = 0$. Then correlation (8) takes the form

$$r_\gamma - r_\alpha + r_\beta = 0 \,. \qquad (10)$$

It follows from this correlation that realization of three variants of interaction is possible here: 1) interaction of the growing, fast, and slow waves; 2) interaction of the growing and two fast waves; 3) interaction of the growing and two slow waves.

We consider the first variant of interaction. Starting from correlation (10), we find that sign functions r can take the following values: 1) $r_\alpha = 0$, $r_\beta = +\sqrt{15}/2$, and $r_\gamma = -\sqrt{15}/2$; 2) $r_\alpha = 0$, $r_\beta = -\sqrt{15}/2$, $r_\gamma = +\sqrt{15}/2$. As seen, wave with minimum frequency can be both fast ($\chi = 2$) and slow ($\chi = 3$). In this case, condition (10) will be fulfilled for any wave harmonics of the given resonance type, since it does not depend on their numbers.

We consider condition of three-wave resonance for harmonic frequencies (5). As follows, if this condition is fulfilled, for example, for first ($m_\alpha = m_\beta = m_\gamma = m = 1$) harmonics

$$\omega_{\alpha1} - \omega_{\beta1} = \omega_{\gamma1}, \qquad (11)$$

it will hold for any triples of m-th harmonics

$$m\omega_{\alpha1} - m\omega_{\beta1} = m\omega_{\gamma1}. \qquad (12)$$

This means that plural parametric resonances between wave harmonics of different type are realized in the system.

Now we consider the second variant: interaction of the growing and two fast waves. For sign functions r, on the basis of correlation (10), we obtain the following: 1) $r_\alpha = -\sqrt{15}/2$, $r_\beta = -\sqrt{15}/2$, $r_\gamma = 0$; 2) $r_\alpha = -\sqrt{15}/2$, $r_\beta = 0$, $r_\gamma = -\sqrt{15}/2$. In this case, condition of three-wave resonance for harmonic frequencies (5) is transformed, as well as in the previous case, into correlations (11)-(12). Thus, such parametrical interactions also have plural behavior.

In the third variant of interaction, where growing and two slow waves participate, we find from condition (10) that functions r can take the following values: 1) $r_\alpha = +\sqrt{15}/2$, $r_\beta = +\sqrt{15}/2$, $r_\gamma = 0$; 2) $r_\alpha = +\sqrt{15}/2$, $r_\beta = 0$, $r_\gamma = +\sqrt{15}/2$. Condition of three-wave resonance for frequencies (5) is transformed, as well as in the previous case, into correlations (11)-(12). Thus, such parametrical interactions have plural behavior.

Type 3: subcritical and overcritical waves interact in three-wave parametric resonance. It follows from general considerations that two groups of interactions can be realized here. Resonances with participation of two subcritical and one overcritical waves form the first group. Resonant interactions with one subcritical and two overcritical waves belong to the second group.

Consider the first group of interactions. We assume that β and γ waves are subcritical ($\sigma_\gamma = \sigma_\beta = 0$), and α wave is overcritical one. Obviously that in this case the wave frequency α will be maximum. Then condition of parametric resonance (8) can be written in the form

$$\frac{m_\alpha \omega_{\alpha,1}}{\omega_{cr}} = -\frac{\left(r_\gamma - r_\alpha + r_\beta\right)}{\sqrt{2}\sigma_\alpha} > 1\,. \qquad (13)$$

In this correlation we have used the connection between plasma and critical (3) frequencies, as well as the condition that frequency of overcritical wave exceeds the critical frequency $m_\gamma\omega_{\gamma1}/\omega_{cr} > 1$.

Such group of resonant interactions principally can have the following values of sign functions r_χ:
1) $r_\gamma = 0$ or $r_\beta = 0$ (one of subcritical waves is growing);
2) $r_\gamma = \pm\sqrt{15}/2$, $r_\beta = \pm\sqrt{15}/2$ (both subcritical waves are not growing).

We consider in detail the first case of interaction. Taking into account that $r_\gamma = 0$, resonant condition (13) can be written as

$$-\frac{\left(-r_\alpha + r_\beta\right)}{\sqrt{2}\sigma_\alpha} > 1\,. \qquad (14)$$

Hence it appears that resonance is possible only in the case of different sign functions r_β and r_α (for example, $r_\alpha = -1$ is the fast overcritical SCW, $r_\beta = +\sqrt{15}/2$ is the slow subcritical SCW and when $\sigma_\alpha = -1$).

We pass to the consideration of the second group of interactions: parametric resonance of one subcritical and two overcritical waves. We consider the case when subcritical γ wave have minimum frequency and is the growing one ($\chi = 1$, $\sigma_\gamma = r_\gamma = 0$). Then, overcritical α wave have maximum frequency. In this case, condition of parametric resonance (8) takes the following form:

$$m_\alpha\omega_{\alpha1}\frac{\sigma_\alpha}{1+\sigma_\alpha\delta} - m_\beta\omega_{\beta1}\frac{\sigma_\beta}{1+\sigma_\beta\delta} = \frac{\omega_p\left(r_\alpha - r_\beta\right)}{\delta\gamma_0^{3/2}}. \qquad (15)$$

As follows from the obtained correlation, the given type of resonant interaction is possible in the case when both overcritical waves belong to the same beam: $\sigma_\alpha = \sigma_\beta$. Then condition (15) takes simpler form

$$\omega_\gamma = \sigma_\alpha\left(r_\alpha - r_\beta\right)\omega_p\left(1+\sigma_\alpha\delta\right)/\left(\delta\gamma_0^{3/2}\right), \quad \sigma_\alpha\left(r_\alpha - r_\beta\right) = +2. \qquad (16)$$

In this case, sign functions take values $\sigma_\alpha = +1$, $r_\alpha = +1$, $r_\beta = -1$. When wave with maximum frequency is the slow one and has negative energy, effect of explosive instability becomes possible [5, 6, 17].

Understandably that resonant interaction also takes place in the case when overcritical waves belong to different beams: $\sigma_\alpha = -\sigma_\beta$. Then condition (16) is written as

$$m_\alpha\omega_\alpha + m_\beta\omega_\beta \approx \frac{\sigma_\alpha\omega_p\left(r_\alpha - r_\beta\right)\left(1-\delta^2\right)}{\delta\gamma_0^{3/2}}, \quad \sigma_\alpha\left(r_\alpha - r_\beta\right) = +2. \qquad (17)$$

Inequality $\delta \cdot m_\gamma\omega_\gamma << \omega_{\alpha,\beta}$ was used here for simplification.

The above considered interactions are of an interest from the practical point of view. We suppose that two-speed beam is modulated on the frequency ω_γ at the transit section inlet of heterodyne FEL. This means that intensive low-frequency SCW propagates in such beam and, moreover, it rises due to the effect of two-stream instability. If weak high-frequency SCW α comes to the system inlet, then because of the above described effects of parametric three-wave resonances it is possible to transform the energy of low-frequency γ wave into the energy of high-frequency overcritical α wave. In essence, amplification of high-frequency signal of the frequency ω_α takes place due to low-frequency pump wave ω_γ by the mechanism of three-wave parametric resonance. β wave is excited because of interaction of γ and α waves.

Type 4: resonant interactions of three overcritical waves (ω_α, ω_β, $\omega_\gamma > \omega_{cr}$). Note, it follows from correlation (8) that parametric resonance when all three waves belong to the same beam ($\sigma_\alpha = \sigma_\beta = \sigma_\gamma$) cannot be realized. Only one of residual three combinations of sign functions σ satisfies the maximality condition of wave frequency α and minimality condition of wave frequency γ (7). Then

$$\sigma_\alpha = \sigma_\beta = -\sigma_\gamma, \tag{18}$$

γ wave belongs to one beam, α and β waves to another. Resonant condition (8) takes the form

$$m_\gamma \omega_{\gamma,1} = \frac{\sigma_\gamma\left(r_\gamma - r_\alpha + r_\beta\right)\omega_p\left(1-\delta^2\right)}{\left(2\delta\gamma_0^{3/2}\right)}. \tag{19}$$

To fulfill the condition $m_\gamma\omega_{\gamma,1} > \omega_{cr}$, it is necessary

$$\sigma_\gamma(r_\gamma - r_\alpha + r_\beta) = 3. \tag{20}$$

From condition (20) we find possible variants of sign functions for the considered case: 1) $r_\gamma = +1$, $r_\alpha = -1$, $r_\beta = +1$, $\sigma_\gamma = +1$, $\sigma_\beta = -1$, $\sigma_\alpha = -1$; 2) $r_\gamma = -1$, $r_\alpha = +1$, $r_\beta = -1$, $\sigma_\gamma = -1$, $\sigma_\beta = +1$, $\sigma_\alpha = +1$. Substituting (20) into (19) we obtain $\omega_{\gamma,m} = 3\omega_p(1-\delta^2)/2\delta\gamma_0^{3/2}$.

Characteristic feature of the interaction of the considered group of waves is the following: one of the frequencies (ω_γ) is defined by the system properties, it has a value close to the critical frequency (3), and it does not depend on the frequencies of two other waves with which it is in resonance. Considered types of resonant interactions are used in parametrical electron-wave superheterodyne FEL.

4. EFFECT OF PLURAL SUPERHETERODYNE RESONANCES

It follows from the above said that for waves, whose frequencies are less that the critical frequency, plural parametric resonances of the 1-st and 2-nd types are realized. Presence of a close coupling between waves of different interacting triples is the characteristic feature of such resonances. We note, the given phenomenon is not principally new for both theory of FEL and physical electronics in whole. In particular, interactions of any two parametrically coupled triples of waves (which can, in the general case, have different physical nature) via common wave are named the coupled parametric resonances [5, 6].

Thus, multiplicity of connections between different triples of wave harmonics is the key distinction of the above described version of coupled resonances from the traditionally studied ones in theory of FEL. Or, in other words, here, at least, two of interacting harmonics of each triple are common simultaneously for some other triples of waves. Due to this fact, many initially isolated triples of waves are combined into one big system, in which number of simultaneously occurring resonances can be equal to tens or even hundreds.

Further we note that the discussed in the present work phenomenon of plural parametric resonances has one more feature. Namely, in the considered two-stream system for the same SCW harmonics we have simultaneous overlap of two mechanisms of their amplification. The first mechanism is conditioned by the effect of two-stream instability, and due to realization of this effect we have amplification of SCW harmonics in the frequency range from the first harmonic to ω_{cr} (Fig. 6 in [4]). The second mechanism is determined by the realization of the discussed in the given work effect of plural parametric resonances. The result of such overlap cannot be interpreted neither by the properties of two-stream instability, nor by the properties of parametric resonance.

Situation when one of a triple of resonantly interacting waves receives additional amplification from any other amplification mechanism is named the effect of superheterodyne amplification [5, 6]. We note that in the general case the waves entering into composition of this triple can have different physical nature. In superheterodyne FEL (SFEL) where the effect of superheterodyne amplification is realized, these are two transverse electromagnetic waves (one of which, pump, can have the shape of magnetic undulator field) and one longitudinal SCW. The latter in SFEL is common one for three-wave and "overlaid" mechanisms [5, 6]. Characteristic feature of the model studied in the present work is the fact that all three waves which participate in three-wave interaction are longitudinal SCW. Here, all three waves from a triple simultaneously get an additional amplification from the "overlaid" mechanism, in the given case – two-stream instability. And since all triples of waves are coupled with each other, then by analogy with plural parametric resonance, here we can also say about plural superheterodyne resonances.

5. ABRIDGED EQUATIONS FOR COMPLEX AMPLITUDES OF WAVES

Now we perform the quantitative analysis of plural resonant interactions. To this end, we use the averaged characteristic method [5, 6] and make calculations in the cubic approximation over small parameter of the problem. We note that, as known, one of the features of this method is the fact that small parameter of the problem here is not proportional to the maximum amplitude of waves as it is traditionally taken in such problems of electrodynamics of plasma-like systems [7]. Exactly this circumstance allows to calculate the entire spectrum including its anomalous regions [5, 6]. As a result of sufficiently cumbersome calculations, we obtain a set of abridged equations for the harmonic amplitudes $E_{\chi,m}$, each of which is considered to be slowly varying along z-coordinate

$$C_{2,\alpha,m}\frac{d^2E_{\alpha,m}}{dz^2} + C_{1,\alpha,m}\frac{dE_{\alpha,m}}{dz} + D_{\alpha,m}E_{\alpha,m} = \\ = C_{3,\alpha,m}\left(E^*_{\beta,m}\delta_{\nu,+1} + E_{\beta,m}\delta_{\nu,-1}\right)E_{\gamma,m} + F_{\alpha,m};$$

$$C_{2,\beta,m}\frac{d^2E_{\beta,m}}{dz^2}+C_{1,\beta,m}\frac{dE_{\beta,m}}{dz}+D_{\beta,m}E_{\beta,m}=$$
$$=C_{3,\beta,m}\left(E^*_{\alpha,m}E_{\gamma,m}\delta_{\nu,+1}+E_{\alpha,m}E^*_{\gamma,m}\delta_{\nu,-1}\right)+F_{\beta,m};$$
$$C_{2,\gamma,m}\frac{d^2E_{\gamma,m}}{dz^2}+C_{1,\gamma,m}\frac{dE_{\gamma,m}}{dz}+D_{\gamma,m}E_{\gamma,m}= \quad (21)$$
$$=C_{3,\gamma,m}E_{\alpha,m}\left(E_{\beta,m}\delta_{\nu,+1}+E^*_{\beta,m}\delta_{\nu,-1}\right)+F_{\gamma,m}.$$

Coefficients in (21) are determined by the beam parameters, frequencies and wave numbers of the corresponding waves

$$D_{\chi,m}=-imk_\chi\left(1-\sum_{q=1}^{2}\left(\frac{\omega_p^2}{m^2(\omega_\chi-k_\chi\upsilon_{qz})^2\gamma_i^3}\right)\right); \quad (22)$$
$$C_{1,\chi,m}=\frac{\partial D_{\chi,m}}{\partial(-imk_\chi)},C_{2,\chi,m}=\frac{1}{2}\frac{\partial^2D_{\chi,m}}{\partial(-imk_\chi)^2};$$
$$C_{3,\alpha,m}=-k_\alpha\sum_{q=1}^{2}\left[\frac{\omega_p^2e/m_e}{\Omega_{\alpha,q}\Omega_{\beta,q}\Omega_{\gamma,q}\gamma_q^6m^2}\times\right.$$
$$\left.\times\left(\frac{k_\alpha}{\Omega_{\alpha,q}}+\frac{k_\beta}{\Omega_{\beta,q}}+\frac{k_\gamma}{\Omega_{\gamma,q}}-\frac{3\upsilon_q\gamma_q^2}{c^2}\right)\right];\; C_{3,\beta,m}=-\frac{k_\beta C_{3,\alpha,m}}{k_\alpha};$$
$$C_{3,\gamma,m}=-\frac{k_\gamma C_{3,\alpha,m}}{k_\alpha};\;\Omega_{\chi,q}=\omega_\chi-k_\chi\upsilon_q;\;\gamma_q=\frac{1}{\sqrt{1-(\upsilon_q/c)^2}}.$$

In correlations (21) $\delta_{\nu,\pm1}$ is the Kronecker symbol; $F_{\chi,m}=F_{\chi,m}(\mathbf{E}_\alpha,\mathbf{E}_\beta,\mathbf{E}_\gamma)$ are the functions which take into account cubic nonlinear terms, including those connected with parametrical resonant interactions in the investigated system. These functions are rather cumbersome, therefore we do not write them in explicit form.

Expression for $D_{\chi,m}$ (22) is the dispersion function for m-th harmonic of χ-th wave. As it is known, types of the longitudinal waves which propagate in the system are determined by the solutions of dispersion equation $D(\omega_\chi,k_\chi)=0$. Correlation (2) is the solution of this dispersion equation.

6. NONLINEAR AMPLITUDE ANALYSIS

Now, we will perform the numerical analysis of the influence of parametrical resonant interactions of the first and the second types (see Section 3) on the development of two-stream instability by using the obtained correlations (21).

We assume that monochromatic growing wave whose frequency is much less than the critical one (3) is formed on the inlet of the transit section. We consider the case when fast and slow waves at the inlet into investigated system are absent. Then only plural parametric resonances of the first type take place in the transit section. Multiharmonic growing SCW whose spectrum is illustrated in Fig. 2 is formed as a result of such interaction. The spectrum is shown for the case when at the system inlet ($z=0$) amplitude of the first harmonic of the growing wave is equal to 10 V/cm and its frequency is 25 times less than the critical one ($\omega_{cr}/\omega_{\alpha1}=25$), other harmonics are equal to zero. 50 harmonics were taken into account in our calculations.

As it was expected, this spectrum has "anomalous" region from 1-st to 15-th harmonic. Here higher harmonic has larger amplitude. Moreover, harmonic whose frequency is equal to the optimal one has maximum amplitude. The spectrum width, as seen (Fig. 2), is determined by the frequency of the 1-st harmonic of the growing wave ω_1 and frequency ω_{min} which corresponds to the harmonic with minimum amplitude. We also note that frequency ω_{min}, as it follows from Fig. 2, is higher that the critical one $\omega_{min}>\omega_{cr}$.

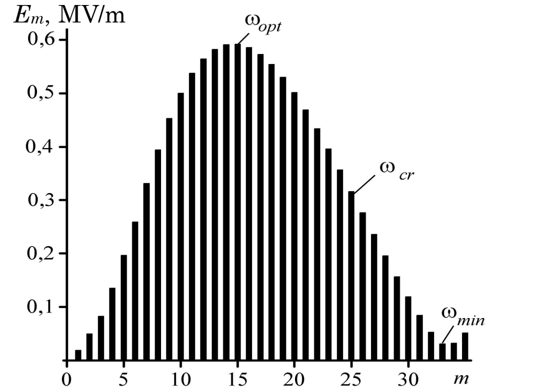

Fig. 2 – Dependence of the harmonic amplitude E_m of the growing SCW on the harmonic number m. Frequency of the 1-st harmonic is $\omega_1=3{,}1\cdot10^{11}$ s^{-1}. Spectrum is plotted for the longitudinal coordinate $z=110$ cm. Calculations are performed for the following parameters: $\omega_p=1{,}5\cdot10^{11}$ s^{-1}; $\gamma_0=4{,}5$; $\Delta\gamma=0{,}5$

Now consider how the shape of the growing wave spectrum is changed under the influence of plural parametric resonances of different type waves. We assume that harmonic amplitudes of fast (wave type 3) and slow (wave type 2) waves, which are in a parametric resonance with 20-*th* harmonic of the growing wave, are equal to 0,5 V/cm. In calculations, we take into account the influence of 50 harmonics of each interacting wave. Other parameters are the same as in the case of Fig. 2. The results of such calculations are shown in Fig. 3. It follows from the comparison of Fig. 2 and Fig. 3 that resonant interactions of longitudinal waves of different types considerably influence the formation of multiharmonic growing SCW. Maximum of the spectrum is on 19-*th* harmonic now. This is conditioned by parametric resonance of different longitudinal waves which takes place, first of all, between twenties harmonics of the corresponding wave types, as well as with the fact that increment of growth for 19-*th* harmonic is higher than for 20-*th*. Spectrum shape is also significantly changed (compare Fig. 2 and Fig. 3). Region of "anomalous spectrum" is increased from 15 harmonics in Fig. 2 to 19 harmonics in Fig. 3. We also have to note that in this case maximum amplitude of the spectrum considerably increased from 0,6 MV/m in Fig. 2 to 6 MV/m in Fig. 3 (10-fold increase!).

Thus, parametric resonance of longitudinal waves of different types allows to substantially change the shape of wide multiharmonic spectrum of the growing longitudinal SCW. Due to this fact, the given effect can be used for the formation of wide multiharmonic spectrum of waves in two-stream electron system with specified parameters, and then for the formation of cluster electromagnetic waves in multiharmonic superheterodyne two-stream FEL.

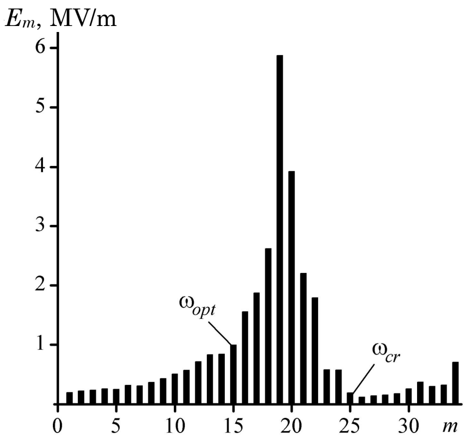

Fig. 3 – Dependence of the harmonic amplitude E_m of the growing SCW on the harmonic number m at z = 109 cm

7. CONCLUSIONS

Thus, classification and kinematic analysis of all possible variants of three-wave parametrical resonant interactions in plasma of relativistic two-stream electron beam is carried out in the work. It is established that realization of four groups of parametrical resonant interactions is possible here. A number of investigated variants of parametrical resonant interactions has plural character: hundreds and more coupled with each other harmonics participate simultaneously in three-wave resonance.

Multiharmonic cubically nonlinear theory of plural parametrically resonant interactions in two-speed relativistic electron beam is also constructed in the work. It is shown that such resonances considerably influence the development of physical processes in the studied system. It is proposed to use the considered modes for the formation of wide multiharmonic wave spectrum in two-stream electron system, as well as in cluster superheterodyne two-stream FEL.

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