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ACTIVE FEL-KLYSTRONS AS FORMERS OF FEMTOSECOND CLUSTERS OF ELECTROMAGNETIC FIELD. DESCRIPTION OF MODELS ON THE BASIS OF SECTIONS OF 'ORDINARY' FEL

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Physical processes of ultrashort electromagnetic cluster formation in multiharmonic parametrical free electron lasers are analyzed. The conditions, which are necessary for formation of such clusters, are found out. Two formation variants, which differ with input electromagnetic signal spectra and multiharmonic pump magnetic field spectra, are studied. Possibility of the ultrashort electromagnetic field cluster formation in the multiharmonic parametrical free electron lasers is shown.

Keywords: *FREE ELECTRON LASERS, ULTRASHORT ELECTROMAGNETIC CLUSTERS, MULTIHARMONIC INTERACTIONS.*

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1. INTRODUCTION

The present paper is the third part of the work, two of which were published in [1, 2]. In [1] general qualitative description of a new class of relativistic electron devices – multiharmonic FEL-klystrons aimed for the formation of femtosecond clusters of electromagnetic field is carried out. The discussion of a number of possible theoretical models of such free electron lasers (FEL) as well as their design schemes is also performed in [1].

In contrast to the paper [1], which is rather general, in [2] the main attention is given to the detailed description of one of the particular versions of such systems. Their distinctive feature is the use, as the design basis, of multiharmonic sections of traditional ("ordinary") parametrical FEL. Moreover, a number of production questions is also discussed in [2]. In particular, we have examined the theoretical "broad-beam" model, according to which a beam can be considered as transversely unbounded one if certain conditions hold. The models of multiharmonic magnetic undulator as well as the key FEL sections (modulation and terminal, respectively) are proposed. The system of nonlinear reduced equations for the complex harmonic amplitudes of interacting waves is also derived.

In the given paper, relied on the previous works, we have carried out the computer simulation of physical processes occurring during the formation of ultrashort electromagnetic clusters in the section based on the "ordinary" parametrical FEL. It is shown that such devices can be effective formers of ultrashort, including femtosecond, clusters of electromagnetic field.

2. THE MODEL OF MULTIHARMONIC FORMER OF FEMTOSECOND ELECTROMAGNETIC CLUSTERS BASED ON "ORDINARY" FEL

Design features of the former of femtosecond electromagnetic clusters based on multiharmonic parametrical FEL are stated in the work [2]. In the present paper, to describe physical processes in the FEL-former we use the theoretical model shown in Fig. 1. Here the transversely unbounded beam 1 moves along the z-axis in the transverse linearly polarized field of multiharmonic magnetic undulator. Design of such undulator is described in detail in [2]. Magnetic field of the undulator we represent in the following form:

$$\vec{B}_2 = \sum_{n_2=1}^{N} \left[B_{2,n_2} \exp(i n_2 p_2) + c.c. \right] \vec{e}_y , \tag{1}$$

where $B_{2,n2}$ is the complex amplitude of the pump field n_2-th harmonic magnetic induction; $n_2 = 1, 2, ..., N$ are the numbers of harmonics; $p_2 = k_2 z$ is the phase of the pump field 1-st harmonic; $k_2 = 2\pi/\lambda_2$ is the wave number; $\vec{e}_y$ is the unit vector of the y-axis.

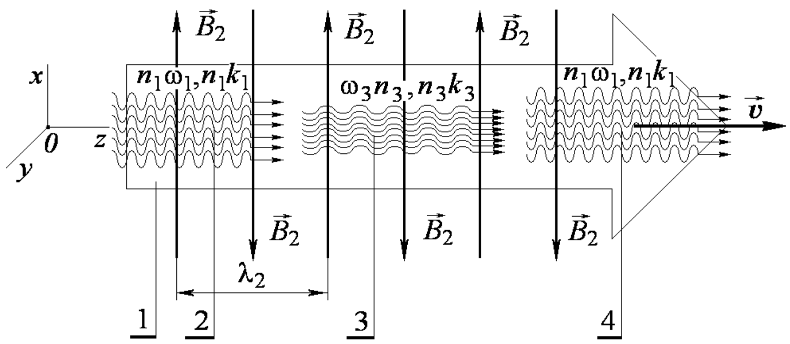

Fig. 1 – Model of the FEL-former of femtosecond electromagnetic clusters. Here 1 – relativistic electron beam; 2 – multiharmonic electromagnetic signal (cluster signal wave) at the system input with the spectrum $n_1\omega_1$, n_1k_1; 3 – multiharmonic spatial charge wave (cluster SCW) with the spectrum $n_3\omega_3$, n_3k_3; 4 – multiharmonic electromagnetic signal (cluster signal wave) at the system output; $\vec{B}_2$ is the magnetic field induction of multiharmonic pump field with the spatial period λ_2

We assume that to the input of multiharmonic FEL-former the following multiharmonic electromagnetic signal comes:

$$\vec{E}_1 = \sum_{n_1=1}^{N} \left[E_{1,n_1} \exp(i n_1 p_1) + c.c. \right] \vec{e}_x , \tag{2}$$

where $E_{1,n1}$ is the amplitude of the electric field intensity of the n_1-th harmonic of the signal field; $n_1 = 1, 2, ..., N$ are the numbers of harmonics; $p_1 = \omega_1 t - s_1 k_1 z$ is the phase of the 1-st harmonic of the signal field; ω_1, k_1 are the frequency and wave number of the 1-st harmonic; $s_1 = \pm 1$ is the sign function; $\vec{e}_x$ is the unit vector along the x-axis.

As a result of interaction of the pump (1) and signal (2) fields, multiharmonic SCW is exited

$$\vec{E}_3 = \sum_{\chi} \vec{E}_{3,\chi} = \sum_{n_3=1}^{N} \left[E_{3,\chi,n_3} \exp(ip_{3,\chi,n_3}) + c.c.\right]\vec{e}_z , \tag{3}$$

where $\chi = \pm 1$ is the sign denoting type of the SCW field ($\chi = +1$ corresponds to the slow SCW, and $\chi = -1$ – to the fast one); $E_{3,\chi,n3}$ is the amplitude of the electric field intensity of the n_3-th harmonic of the χ-th SCW field; $n_3 = 1, 2, ..., N$ are the numbers of harmonics; $p_{3,\chi,n3} = n_3\omega_3 t$ - $k_{3,\chi,n3}$ is the phase; $n_3\omega_3$ and $k_{3,\chi,n3}$ are the frequency and wave number of the n_3-th harmonic of the χ-th SCW field; $\vec{e}_z$ is the unit vector along the z-axis.

We assume that wave interaction in the system is quasi-stationary and steady-state, i.e. all transient processes are over long ago. In this case, we use the boundary condition

$$E_{3,\chi,n_3}\big|_{z=0} = 0, \quad E_{1,n_1}\big|_{z=0} = E_{10,n_1} . \tag{4}$$

We consider that the Compton mode of multiple parametrically resonant interaction [3] is realized in the system. This means that system in such state does not discern slow and fast SCW

$$k_{3,\chi,n_3}(\chi = +1) \approx k_{3,\chi,n_3}(\chi = -1) \approx n_3 k_3 . \tag{5}$$

Conditions for the realization of such multiple degenerate parametric resonance we choose in the form of [3-5]

$$n_1\omega_1 \approx n_2\omega_3, \quad n_3 k_3 \approx \mathrm{n}_1 k_1 + n_2 k_2 . \tag{6}$$

We also take into account the three-wave parametric resonant interactions between harmonics of the same waves.

It is easy to see that, for example, in the particular case $n_1 = n_2 = n_3$ and under the fulfillment of the condition of negligible dispersion (5), resonance condition (6) can hold simultaneously for any quantity of harmonics. This fact is the main idea of the mechanism of multiple three-wave parametric resonances on harmonics. For the first time it was formulated in our work [6] for the case of two-stream FEL and here, as it was mentioned above, it is generalized for the case of a single-beam “ordinary” FEL.

3. BASIC REDUCED EQUATIONS FOR THE AMPLITUDES OF WAVE HARMONICS

As the basis we choose the set of Maxwell equations and relativistic quasi-hydrodynamic equation of a beam motion. Then we use the proposed above theoretical models, standard statement of the problem, and methods of the theory of hierarchical vibrations and waves described, for example, in the monographs [3-5]. As a result of sufficiently cumbersome analytical transformations for the complex amplitudes of field harmonics (2) and (3), we obtain the following set of the reduced equations in cubically non-linear approximation:

$$K_{1,n_1} \frac{d^2 E_{1,n_1}}{dz^2} + K_{2,n_1} \frac{dE_{1,n_1}}{dz} + D_{1,n_1} E_{1,n_1} = K_{3,n_1} B_{2,n_1} E_{3,n_1} + F_{1,n_1} , \tag{7}$$

$$C_{1,n_3}\frac{d^2E_{3,n_3}}{dz^2}+C_{2,n_3}\frac{dE_{3,n_3}}{dz}+D_{3,n_3}E_{3,n_3}=C_{3,n_3}E_{1,n_1}B^*_{2,n_2}+C_{4,n_3}\left\langle E_3E_3^{\mathrm{int}}\right\rangle_{n_3p_3}+F_{3,n_3}.$$

Here

$$D_{1,n_1}=\left(n_1{}^2/c^2\right)\left[k_1^2c^2-\omega_1^2-\left(\omega_p^2/n_1{}^2\bar{\gamma}\right)\right],\quad D_{3,n_3}=-i(n_3k_3)\cdot\left(1-\frac{\omega_p^2}{\Omega_3^2n_3{}^2\bar{\gamma}^3}\right),$$

$$K_{1,n_1}=\partial^2D_{1,n_1}/\partial(-in_1k_1)^2/2,\quad C_{1,n_3}=\partial^2D_{3,n_3}/\partial(-in_3k_3)^2/2,$$

$$K_{2,n_1}=\partial D_{1,n_1}/\partial(-in_1k_1),\quad C_{2,n_3}=\partial D_{3,n_3}/\partial(-in_3k_3),$$

$$K_{3,n_1}=\frac{\omega_p^2\omega_1e}{2n_1{}^2\Omega_3m_e\bar{\gamma}^4k_1k_2c^3}\left(\frac{\upsilon\,\bar{\gamma}^2}{c^2}-\frac{k_3}{\Omega_3}\right),\quad C_{3,n_3}=\sum_{q=1,2}\left(\frac{\omega_p^2ek_3}{n_3{}^2\Omega_3^2m_e\bar{\gamma}^2c^2k_2}\left(\frac{\upsilon}{c}-\frac{k_3c}{\omega_3}\right)\right),$$

$$C_{4,n_3}=\frac{3\omega_p^2ek_3}{in_3\Omega_3^3m_e\bar{\gamma}^6}\cdot\left(\frac{k_3}{\Omega_3}-\frac{\upsilon\,\bar{\gamma}^2}{c^2}\right),\quad \bar{\gamma}=\frac{1}{\sqrt{1-(\upsilon/c)^2}},\quad \omega_p^2=\frac{4\pi\bar{n}e^2}{m_e},$$

$\Omega_\chi=\omega_\chi-k_\chi\bar{\upsilon}$, index χ takes the values 1 and 3.

We also used here the notations

$$\langle\ldots\rangle_{n_\chi p_\chi}=\frac{1}{(2\pi)^3}\int_0^{2\pi}\left(\ldots\ \exp(-in_\chi p_\chi)\,dp_1dp_2dp_3\right),$$

$$E^{\mathrm{int}}_{\chi n_\chi}=\sum_{m=1}^{N}\left[E_{\chi n_\chi}\frac{\exp(in_\chi p_\chi)}{in_\chi}+c.c.\right],\ (\chi=1,3).$$

In equations (7) $F_{1,n1}$ and $F_{3,n3}$ are the functions, which contain the cubic non-linear terms of the following form:

$$F_{1,n_1}=K_{5,n_1}E'^{\,\mathrm{int}}_{3,n_1}B_{2,n_1}+K_{6,n_1}B_{2,n_1}\left\langle E_3E_3^{\mathrm{int}}\right\rangle_{n_1p_3}+K_{7,n_1}B_{2,n_1}\left\langle E_3^{\mathrm{int}}E_3^{\mathrm{int}}\right\rangle_{n_1p_3}+$$
$$+K_{8,n_1}\left\langle E_1E_1^{\mathrm{int}}E_1^{\mathrm{int}}\right\rangle_{n_1p_1}+\left\langle E_1\sum_{l=1}^{N}\left(K_{9,n_1,l}E_{3,l}B_{2,l}e^{ilp_1}+c.c\right)\right\rangle_{n_1p_1}+$$
$$+E_{1,n_1}\sum_{l=1}^{N}\left(K_{10,n_1,l}\left|E_{3,l}\right|^2+K_{11,n_1,l}\left|B_{2,l}\right|^2\right).\tag{8}$$

$$F_{3,n_3}=C_{5,n_3}E'_{1,n_3}B^*_{2,n_3}+C_{6,n_3}\left\langle E_3E_3^{\mathrm{int}}E_3^{\mathrm{int}}\right\rangle_{n_3p_3}+$$
$$+\left\langle E_3\sum_{l=1}^{N}\left(C_{7,n_3,l}E_{1,l}B^*_{2,l}e^{ilp_3}+c.c\right)\right\rangle_{n_3p_3}+E_{3,n_3}\sum_{l=1}^{N}\left(C_{8,n_3,l}\left|E_{1,l}\right|^2+C_{9,n_3,l}\left|B_{2,l}\right|^2\right)+$$
$$+C_{10,n_3}\left\langle E'_3E_3^{\mathrm{int}}\right\rangle_{n_3p_3}+C_{11,n_3}\left\langle E_3E'^{\,\mathrm{int}}_3\right\rangle_{n_3p_3}+C_{12,n_3}\left\langle E_3^{\mathrm{int}}E'^{\,\mathrm{int}}_3\right\rangle_{n_3p_3}.\tag{9}$$

In correlations (8)-(9) we used the designation

$$E'_\chi=\sum_{m=1}^{N}\left[\frac{dE_{\chi,m}}{dz}\exp(imp_\chi)+c.c.\right].$$

Coefficients C and K depend on the wave numbers, frequencies, constant components of the velocity $\bar{\upsilon}$ and concentration $\bar{n}$ of an electron beam. We supplement the set of equations (7) with the equations for constant components

$$\frac{d\bar{\upsilon}}{dz} = V_1\left\langle E_1' \, E_1\right\rangle_0 + \sum_{l=1}^{N}\left(V_{2,l}E_{3,l}E_{1,l}^{*}B_{2,l} + c.c\right)$$
$$+ V_{3,3}\left\langle E_3' \, E_3\right\rangle_0 + V_{4,3}\left\langle E_3 E_3^{\text{int}} E_3^{\text{int}}\right\rangle_0 + V_{5,3}\left\langle E_3 E_3 E_3^{\text{int}}\right\rangle_0 . \tag{10}$$

$$\frac{d\bar{n}}{dz} = N_1\left\langle E_1' \, E_1\right\rangle_0 + \sum_{l=1}^{N}\left(N_{2,l}E_{3,l}E_{1,l}^{*}B_{2,l} + c.c\right) +$$
$$+ N_{3,3}\left\langle E_3' \, E_3\right\rangle_0 + N_{4,3}\left\langle E_3 E_3^{\text{int}} E_3^{\text{int}}\right\rangle_0 + N_{5,3}\left\langle E_3 E_3 E_3^{\text{int}}\right\rangle_0 . \tag{11}$$

Coefficients V and N depend on the wave numbers, frequencies, constant components of the velocity $\bar{\upsilon}$ and concentration $\bar{n}$ of an electron beam.

Equations (7)-(11) describe the dynamics of the multiple parametrically resonant interaction of wave harmonics in the working region of multiharmonic FEL-former. Then, using the set of equations (7)-(11) as the base one and the boundary conditions (4), we will analyze the formation process of femtosecond electromagnetic clusters.

4. FEATURES OF THE FORMATION OF FEMTOSECOND CLUSTERS OF ELECTROMAGNETIC FIELD

Features of the formation of femtosecond clusters of electromagnetic field are considered in the works [1-7]. Here we consider some details connected with the formation of such clusters in one-stream parametrical FEL.

It is clear that in a perfect case at the output of multiharmonic cluster FEL-former we should obtain periodic series of delta-functions, which have, for example, the following form:

$$E(t) = A \cdot T \cdot (-\delta(t - T/4 + nT) + \delta(t - 3T/4 + nT)) . \tag{12}$$

Here δ is the Dirac delta-function; T is the period; n is an integer number; A is a certain factor. In the essence, such sequence of delta-functions $E(t)$ is a complex periodic multiharmonic signal and it can be expanded into the Fourier series

$$E(t) = E_0 + \sum_{m=1}^{\infty}\left(E_m \exp(im(2\pi/T)\cdot t) + c.c.\right), \tag{13}$$

where

$$E_m = (1/T)\int_{-T/2}^{T/2} E(t)\cdot \exp(-im(2\pi/T)\cdot\tau)d\tau \tag{14}$$

is the complex amplitude of the m-th harmonic. For the considered case, when $E(t)$ is determined by the correlation (12) it is not difficult, using expression (14), to calculate the complex amplitudes E_m. As a result, we obtain

$$E_m = 2iA\cdot\sin(m\pi/2) . \tag{15}$$

That is $E_0 = 0$, $E_1 = +\,2iA$, $E_2 = 0$, $E_2 = -\,2iA$, $E_4 = 0$, $E_5 = +\,2iA$, $E_6 = 0$, $E_7 = -\,2iA$, etc. It follows from the correlation (15) that 1) amplitudes of all even harmonics are equal to zero; 2) magnitudes of odd harmonics are the same and equal, in the given case, to $2A$; 3) phases of the 1-st, 5-th, 9-th, etc harmonics are equal to $+\,\pi/2$; 4) phases of the 3-d, 7-th, 11-th, etc harmonics are equal to $-\,\pi/2$.

Thus, in order to obtain the series of femtosecond clusters of electromagnetic field (12) it is necessary to form multiharmonic signal, whose phases and amplitudes would satisfy the condition (15).

Let us analyze the equations (7), which describe the dynamics of complex amplitudes of the signal and SCW in the cubically non-linear approximation in multiharmonic parametrical FEL from the point of view of the possibility in principle to solve the above stated problem. We confine ourselves by the linear terms and will seek solutions of such system in the form of $E_{1,m} \sim E_{01,m}\exp(\alpha_m z)$ and $E_{3,m} \sim E_{03,m}\exp(\alpha_m z)$ taking into account that multiply three-wave parametrical resonances between harmonics of the signal waves, SCW, and undulator magnetic field ($n_1 = n_2 = n_3 = m$) are realized in the system. As a result, we obtain the following set of equations with respect to the initial amplitudes $E_{01,m}$ and $E_{03,m}$:

$$\begin{gathered}\left(K_{1,m}\alpha_m^2 + K_{2,m}\alpha_m\right)E_{01,m} - K_{3,m}B_{2,m}E_{03,m} = 0\,, \\ -\,C_{3,m}B_{2,m}^{*}E_{01,m} + \left(C_{1,m}\alpha_m^2 + C_{2,m}\alpha_m\right)E_{03,m} = 0\,.\end{gathered} \tag{16}$$

System (16) will have nontrivial solutions if its determinant will equal to zero

$$\left(K_{1,m}\alpha_m^2 + K_{2,m}\alpha_m\right)\left(C_{1,m}\alpha_m^2 + C_{2,m}\alpha_m\right) - K_{3,m}C_{3,m}\left|B_{2,m}\right|^2 = 0\,. \tag{17}$$

It is not difficult to find approximate solutions of the equation (17)

$$\alpha_m = \alpha_{0,m} + \alpha_{1,m} + \ldots\,, \tag{18}$$

where

$$\alpha_{0,m} = \sqrt{\frac{K_{3,m}C_{3,m}\left|B_{2,m}\right|^2}{\left(C_{2,m}K_{2,m}\right)}} \approx \sqrt{\frac{\omega_p}{2k_{2,m}c\gamma^{3/2}}}\,\frac{|\,eB_{2,m}\,|}{m_e c\bar{\upsilon}} \tag{19}$$

is the increment of parametric instability, which coincides with the increments obtained in [3, 4, 8-12];

$$\alpha_{1,m} = -\alpha_{0,m}^2\left(K_{1,m}/K_{2,m} + C_{1,m}/C_{2,m}\right)/\,2\,. \tag{20}$$

Using expressions for the coefficients K and C (see comments to the system (7)), it is not difficult to ascertain that $\alpha_{0,m}$ is purely real value, and $\alpha_{1,m}$ is purely imagine one. This means that the real part of the increment $\alpha_{0,m}$ is responsible for the change in the magnitude of the complex amplitude of the m-th harmonic, and $\alpha_{1,m}$ – for the change in the initial phase of the complex amplitude. Thus, multiharmonic FEL-former based on the parametrical FEL can change both the magnitudes and phases of the harmonic amplitudes. Correlations (19) and (20) allow to roughly estimate these changes. More exact results we can obtain using system (7). So, the former based on parametrical FEL can form ultrashort electromagnetic field pulses.

Now we will study the formation dynamics of the power ultrashort electromagnetic cluster using equations (7) in the system, whose parameters are represented in Table 1. We consider two variants: the cases when electromagnetic signal on the FEL-former input has a) normal spectrum (higher harmonic has smaller amplitude) and b) anomalous spectrum (higher harmonic has larger amplitude).

Table 1 *– Parameters of the FEL-former*

Wavelength of the 1-st signal harmonic	0,37 cm
Value of the relativistic factor of the electron beam	6,6
Plasma frequency of the beam	$1{,}2\cdot10^{11}$ s^{-1}
Spatial period of the undulator	13,3 cm

5. CASE OF THE NORMAL SPECTRUM OF THE INPUT ELECTROMAGNETIC SIGNAL

We consider the case when electromagnetic signal with the spectrum where higher harmonics have smaller amplitude (see further Fig. 4a) comes to the input of the FEL-former. As it is known, in the cluster amplitudes of the odd harmonics should be equal. Therefore, if at the system input higher harmonics have smaller amplitude, they should be more amplified, i.e. starting from the relation (19) they should have higher increments of growth $\alpha_{0,m}$. This requirement can be performed using multiharmonic undulator with anomalous spectrum. In Fig. 2a we show the magnetic field spectrum of such undulator, and in Fig. 2b – dependence of the increment of growth $\alpha_{0,m}$ on the harmonic number for the case of the normal spectrum of the input electromagnetic signal.

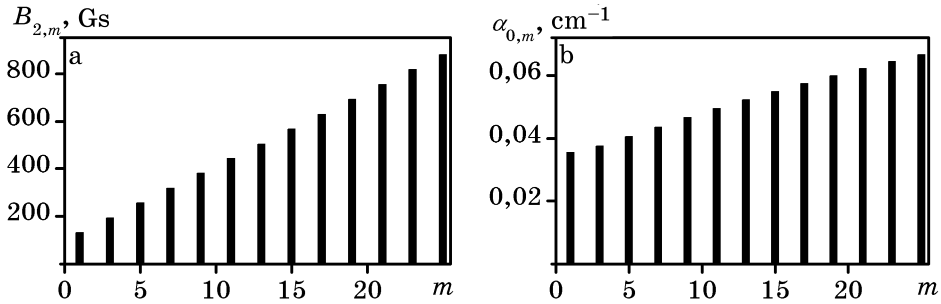

Fig. 2 *– Dependences of the harmonic amplitudes of the magnetic field induction $B_{2,m}$ of the undulator (a) and the increment of growth $\alpha_{0,m}$ (b) on the harmonic number for the case of the normal spectrum of the input electromagnetic signal*

As seen, the given problem is solved because of the use of undulator with the spectrum where higher harmonics have larger amplitude. We have also to note that even harmonics of the undulator are equal to zero. This means that amplification will be absent for even harmonics of the signal, and such harmonics will be absent as well at the FEL-former output. This allows to fulfill one of the conditions of the cluster formation (see comments to the relation (15)).

Then, using the set of equations (7) we will analyze the dynamics of the amplitudes and phases of harmonics in the FEL-former, in which magnetic field spectrum of the undulator is represented in Fig. 2a, taking into account cubic nonlinearities.

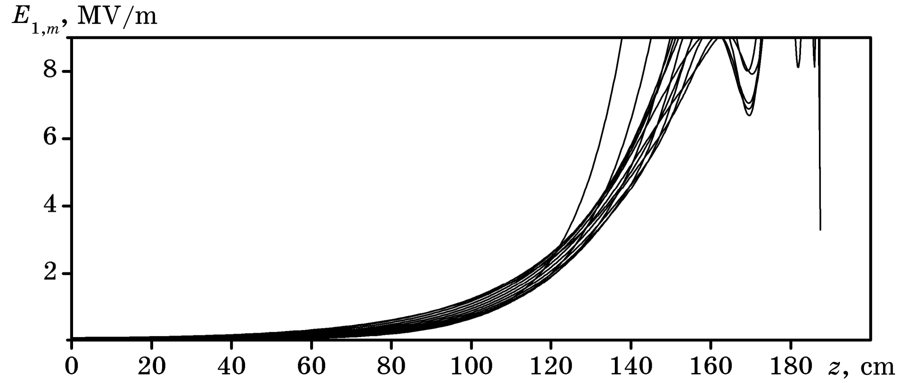

***Fig. 3** – Dependences of the magnitude of harmonic complex amplitudes of the signal wave $E_{1,m}$ on the longitudinal coordinate z*

The dependences of the magnitude of harmonic complex amplitudes of the signal wave $E_{1,m}$ on the longitudinal coordinate z are represented in Fig. 3. As seen, magnitudes of complex amplitudes of odd harmonics exponentially increase, and in the region with coordinate $z = 118$ cm they are approximately equal. Fig. 4 illustrates the transformation and amplification of the complex multiharmonic electromagnetic signal in the cluster parametrical FEL. Fig.4a shows the spectrum of the input electromagnetic signal where higher harmonics have smaller amplitude, and Fig. 4b – the spectrum of the same signal in the point with coordinate $z = 118$ cm. As follows from Fig. 4, in the point $z = 118$ cm amplitudes of all odd harmonics are approximately equal. Comparison of the figures 4a and 4b shows that 10^2-10^3-fold amplification of the signal harmonics takes place in the system.

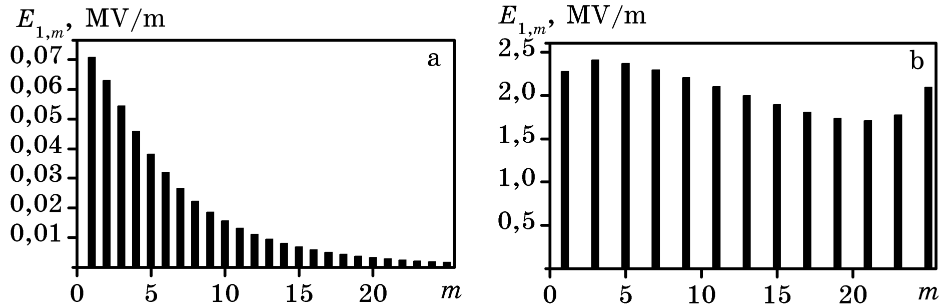

***Fig. 4** – Dependences of the magnitudes of harmonic complex amplitudes of the electromagnetic signal $E_{1,m}$ on the harmonic number for the longitudinal coordinate at the system input $z = 0$ cm (a) and for $z = 118$ cm (b)*

Fig. 5 shows the dependences of the initial phases of harmonic complex amplitudes of multiharmonic electromagnetic wave versus the longitudinal coordinate. As seen, in the region with longitudinal coordinate $z = 118$ cm initial phases of the 1-st, 5-th, 9-th, etc harmonics are approximately equal to $+\ \pi/2$ (curves 1 in Fig. 4), and initial phases of the 3-d, 7-th, 11-th, etc harmonics are equal to $-\ \pi/2$ (curves 2 in Fig. 4).

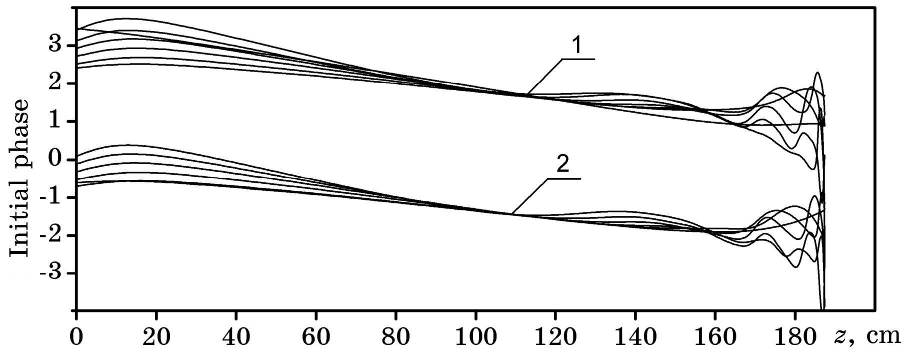

Fig. 5 *– Dependences of the initial phases of harmonic complex amplitudes of the signal wave* $E_{1,m}$ *on the longitudinal coordinate z. Curves 1 correspond to the 1-st, 5-th, 9-th, etc harmonics; curves 2 correspond to the 3-d, 7-th, 11-th, etc harmonics*

Thus, multiharmonic parametrical FEL forms in the point of $z = 118$ cm cluster of electromagnetic field, whose harmonics satisfy the condition (15). Dependence of the energy flux density of electromagnetic signal on the ratio t/T, where t is the time, T is the period of the 1-st harmonic, is represented in Fig. 6. As it was assumed, electromagnetic signal has the form of a short cluster with duration of $2{,}5 \cdot 10^{-13}$ s. If suggest that the beam cross-section area of the studied system is equal to 1 cm^2, instantaneous power of such a cluster achieves the value of 1 GW. Thus, multiharmonic parametrical FEL can form power narrow clusters of electromagnetic field.

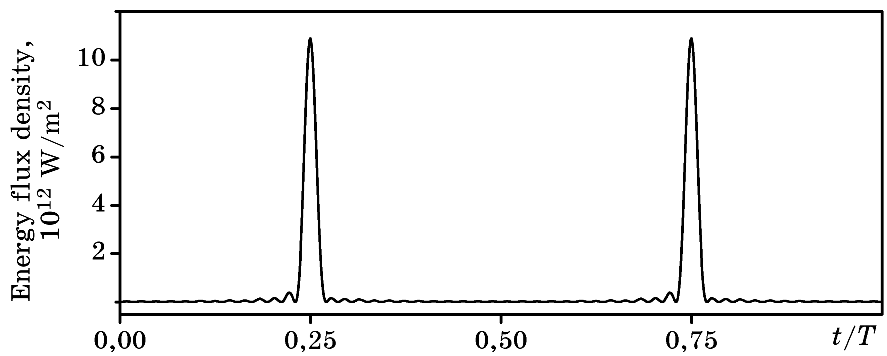

Fig. 6 *– Dependence of the energy flux density of electromagnetic signal on the normalized time t/T at z = 118 cm*

6. CASE OF THE ANOMALOUS SPECTRUM OF THE INPUT ELECTROMAGNETIC SIGNAL

Production of multiharmonic undulators, in which higher harmonics of the magnetic field have larger amplitude than the lower ones do (see Fig. 2a), is a rather complicated technological problem. Therefore, in a number of cases it is easier to produce at the FEL-former input the electromagnetic signal with anomalous spectrum (higher harmonic has larger amplitude) and to use multiharmonic undulator with usual spectrum, in which higher harmonics have smaller amplitude. We will study exactly this case in the given Section.

When using the multiharmonic undulators, where higher harmonics have smaller amplitude (see Fig. 7a), increments of growth, as it follows from the relation (19), will have the similar spectrum in such system (Fig. 7b).

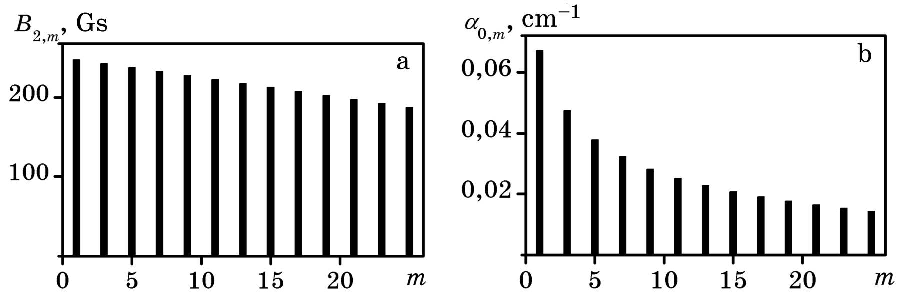

Fig. 7 *– Dependence of the harmonic amplitudes of the magnetic field induction of undulator $B_{2,m}$ (a) and the increment of growth $\alpha_{0,m}$ (b) on the harmonic number for the case of the anomalous spectrum of the input electromagnetic signal*

As in the previous case (Fig. 2a), even harmonics of undulator are equal to zero. Because of this fact, even harmonics of the electromagnetic field will be absent as well, i.e. one of the conditions of the electromagnetic field short cluster formation (see comments to the correlation (15)) will hold.

Using the set of cubically non-linear equations (7), we will investigate the dynamics of the harmonic amplitudes and phases in the FEL-former, in which the magnetic field spectrum of undulator is represented in Fig. 7a.

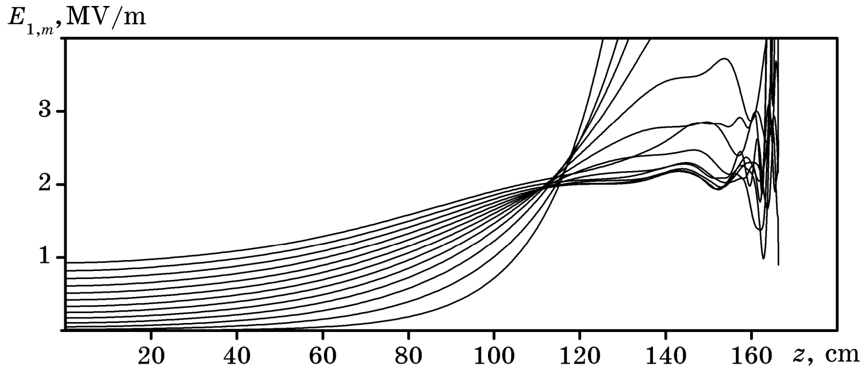

Fig. 8 *– Dependences of the magnitude of harmonic complex amplitudes of the signal wave $E_{1,m}$ on the longitudinal coordinate z*

The dependences of the magnitude of harmonic complex amplitudes of the signal wave $E_{1,m}$ on the longitudinal coordinate z are illustrated in Fig. 8. As seen, magnitudes of complex amplitudes of odd harmonics exponentially increase, and in the region with coordinate $z = 118$ cm they are approximately equal. The dependences of the initial phases of harmonic complex amplitudes of multiharmonic electromagnetic wave versus the longitudinal coordinate are shown in Fig. 9. As seen from the figure, in the region with longitudinal coordinate $z = 120$ cm initial phases of the 1-st, 5-th, 9-th, etc harmonics are approximately equal to $+\pi/2$ (curves 1 in Fig. 9), and initial phases of the 3-d, 7-th, 11-th, etc harmonics are equal to $-\pi/2$ (curves 2 in Fig. 9).

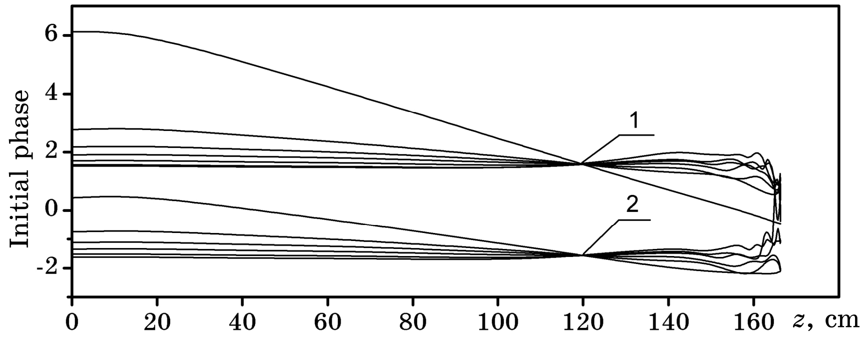

Рис. 9 *– Dependences of the initial phases of harmonic complex amplitudes of the signal wave $E_{1,m}$ on the longitudinal coordinate z. Curves 1 correspond to the 1-st, 5-th, 9-th, etc harmonics; curves 2 correspond to the 3-d, 7-th, 11-th, etc harmonics*

So, multiharmonic parametrical FEL forms in the point of $z = 118$-120 cm cluster of electromagnetic field, whose harmonics satisfy the condition (15). Dependence of the energy flux density of electromagnetic signal on the ratio t/T, where t is the time, T is the period of the 1-st harmonic, in the point of $z = 119$ cm is illustrated in Fig. 10. As it was assumed, electromagnetic signal has the form of a short power cluster. Comparing Fig. 9 and Fig. 5 we see that parameters of electromagnetic field clusters obtained in both cases are almost the same.

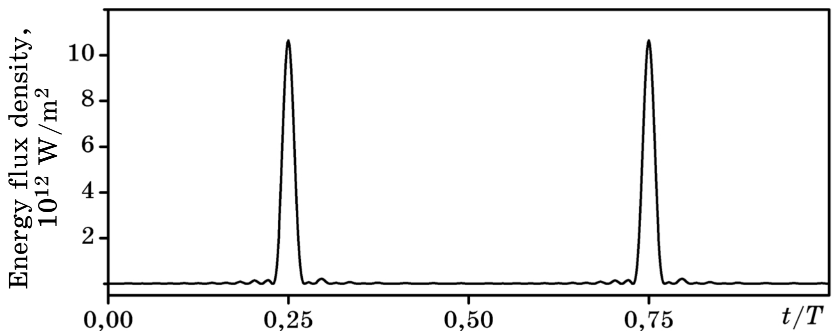

Fig. 10 *– Dependence of the energy flux density of electromagnetic signal on the normalized time t/T at $z = 119$ cm*

7. CONCLUSIONS

Thus, analysis of the physical processes during the formation of ultrashort electromagnetic clusters in devices based on multiharmonic parametrical FEL is carried out in the work. Conditions necessary for the electromagnetic field short cluster formation are found out. Two variants of the cluster formation which differ in the input spectrum of multiharmonic electromagnetic signal are considered. Possibility of the formation of electromagnetic field ultra-short clusters in the systems of the type of multuharmonic parametrical FEL is shown.

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