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ACTIVE FEL-KLYSTRONS AS FORMERS OF FEMTOSECOND CLUSTERS OF ELECTROMAGNETIC FIELD. DESCRIPTION OF THE MODELS BASED ON "ORDINARY" FEL SECTIONS

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The qualitative properties discussion of the femtosecond active cluster FEL-klystrons, which are designed on the basis of "ordinary" FEL sections, is performed. Theoretical models of two different types of such devices are proposed. The main difference between them consists in the acceleration block arrangement. Namely, in the model with intermediate acceleration, a part of the acceleration sections is placed between the modulation and energy-transformation sections. The formulation of the problem is done. The basic system of truncated equations (in the cubic-nonlinear approximation) for the complex amplitudes of harmonics of resonantly-interacting waves is obtained.

Keywords: *FREE ELECTRON LASERS, FEMTOSECOND CLUSTERS OF ELECTROMAGNETIC FIELD, ACTIVE KLYSTRONS.*

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1. INTRODUCTION

The given work is the second part of the paper [1], where general qualitative description of a new class of relativistic electron devices, namely, active FEL-klystrons aimed for the formation of femtosecond clusters of electromagnetic field is carried out. The discussion of a number of the theoretical models of such free electron lasers (FEL) and the design schemes of their realization is performed in [1].

In contrast to the paper [1], where we have emphasized the general description of active FEL-klystrons as a class of devices, here we focus on more detailed analysis of only two particular versions. Their distinctive feature lies in the use of multiharmonic sections of traditional ("ordinary") parametric FEL as the basis.

The given work consists of two parts. In the first part (presented), the discussion of the chosen for study design schemes and their theoretical models is performed. In the second one, the results of the nonlinear multiharmonic numerical and analytical analysis of these theoretical models are described.

2. DESIGN SCHEMES OF THE STUDIED VERSIONS OF ACTIVE FEL-KLYSTRONS

Examples of design schemes of the discussed FEL-klystrons are illustrated in Fig. 1 and Fig. 2. As seen from the figures, the key difference between them consists, first of all, in the realization of the electron beam acceleration section. Moreover, in the version presented in Fig. 1 the acceleration block 3 as a whole is placed traditionally, i.e., between the injector 1 and the multiharmonic signal input system 5.

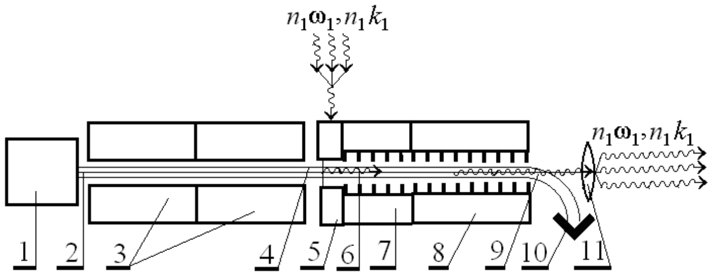

Fig. 1 *– Design scheme of the active FEL-klystron with "ordinary" arrangement of acceleration sub-blocks: 1 – electron injector; 2 – injected electron beam; 3 – sub-blocks of the electron accelerator; 4 – accelerated electron beam; 5 – input system of the multiharmonic signal 6 with the spectrum* $n_1\omega_1$, n_1k_1 *(*n_1 *are the numbers of harmonics of the input signal;* ω_1 *and* k_1 *are the cyclic frequency and the wave number of the input signal); 7 – the first section of the multiharmonic pump system; 8 – the second section of the multiharmonic pump system; 9 – output cluster (multiharmonic) electromagnetic signal; 10 – recuperation system and electron collector; 11 – output system of the cluster (multiharmonic) electromagnetic signal*

At the same time, in the second version presented in Fig. 2 the arrangement of the acceleration system blocks looks slightly paradoxical. Namely, the acceleration block is divided into two sub-blocks 3 and 8. Moreover, only the sub-block 3 (see Fig. 2), as well as in the first case, is disposed traditionally (i.e., as in the case illustrated in Fig. 1). The second one 8 is placed between two sections of the multiharmonic pump system 6 and 10.

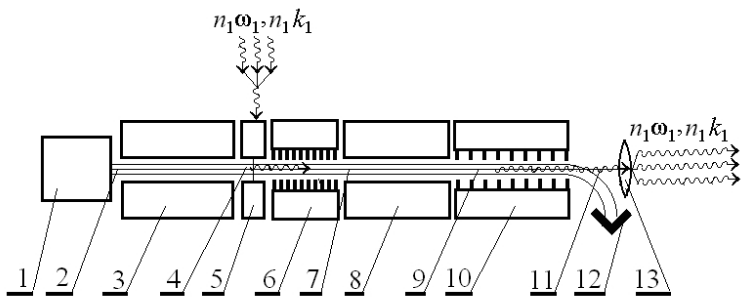

Fig. 2 *– Design scheme of the active FEL-klystron with intermediate acceleration of an electron beam: 1 – electron injector; 2 – injected electron beam; 3 – the first sub-block of the electron accelerator; 4 – accelerated in the sub-block 3 electron beam; 5 – input system of the multiharmonic signal with the spectrum* $n_1\omega_1$, n_1k_1*; 6 – the first section of the multiharmonic pump system; 7 – modulated in the section 6 electron beam; 8 – the second sub-block of the electron accelerator; 9 – accelerated in the sub-block 8 modulated electron beam; 10 – the second section of the multiharmonic pump system; 11 – output cluster (multiharmonic) electromagnetic signal; 12 – recuperation system and electron collector; 13 – output system of the cluster (multiharmonic) electromagnetic signal*

Both versions of the active FEL-klystrons shown in Fig. 1 and Fig. 2, as it was noted, are the particular realizations of the generalized design scheme discussed in the first part of the given work [1]. Therefore their basic working principles, mainly, coincide with the described ones. In this connection, here we will restrict ourselves only to the brief comments of one of the most specific aspects concerning the design of the pump system in Fig. 1.

At first sight, in the design illustrated in Fig. 1 the "true klystron" nature of the construction is not observed: both sections of multiharmonic pump 7 and 8 look as they formally form the "joint" one. However, the more detailed analysis shows that in the general case it is not always true. First of all, because both sections function differently.

The first of them (section 7 in Fig. 1) acts as the key element in parametrically-resonant system of multiharmonic beam modulation (see Fig. 12 in [1]). Correspondingly, both the frequency spectrum of the pump field and distribution of the field amplitude are matched here starting from its main destination. Moreover, it is assumed in many cases that a special absorbing insert (see, for example, Fig. 12 in [1]) is placed between the first and the second pump sections as an effective measure of suppression of self-excitation of the given FEL-former. Total absorption of the spent electromagnetic signal occurs there, and in the section 8 (Fig. 1) generation of the cluster electromagnetic signal starts from zero value of its field. Or, in other words, two pump sections 7 and 8 are separated both functionally and structurally.

The second pump section 8 (see Fig. 1) plays the key role in the process of energy extraction, i.e., functionally it belongs to the formation system of the cluster signal wave. And, correspondingly, structurally its optimization is realized exactly for this function. For example, isochronization of interaction can be foreseen there, and due to this the essential increase in the electron efficiency of interaction as well. Within the framework of the studied here models we will use the known type of synchronization, which consists in the application of the optimal longitudinal electric field to the interaction region (methods and results of isochronization of interaction in "ordinary" FEL see, for example, in [2, 3]).

In other respects, under self-evidence of the klystron idea itself, to our opinion it is reasonable to drop the more detailed description of the basic working principles of the studied FEL-klystron.

3. ABOUT "ACCURATE" AND "ROUGH" THEORETICAL MODELS

Before we start to describe the theoretical model of the devices illustrated in Fig. 1 and Fig. 2 in a whole, we make some methodological remarks.

First of all, we note that two fundamentally different types of theoretical models are known in the theory of FEL [2-10]. Models of the first type are conditionally named the "accurate" ones. Their most characteristic feature is the achievement of the maximum possible (for the current level of theory development) adequacy level of the developed theoretical models with the studied experimental systems. It is obvious that being the most exact ones, they automatically become the most complicated and narrowly applied in practice. That is, the given theoretical model is applicable for acceptable quantitative description of certain FEL designs, which contain specific design solutions for pump systems, electrodynamic signal systems, electron paths for the electron beam transport, etc. Theoretical models of "ordinary FEL" with magnetoundulator pump (H-ubitrons according to our classification) [8-10] can be a striking example. Advantage of such “accurate” models is a possibility of taking into account a quantity of “fine physical and design details”, which condition, for example, the multimode property of beam and electromagnetic wave processes in the FEL interaction region of the totally specific geometry, etc. However, their disadvantages, “as properly”, are the continuation of the advantages. Such complicated and very informative models are found to be the most effective ones in situations, when the preproject analysis has been already done and the optimal design solution has been chosen. And the problem of the theoretical analysis in this case consists, first of all, in the following: as closely as possible to work out all key physical and technological elements of the chosen design solution by the numerically-analytical simulation of all processes in the system. We have to note that experiments in the field of FEL technique are always expensive and possible mistakes while designing run into money. This is the essence of practical importance of the discussed here type of models.

But what should we do in the cases when the problem is to perform just the preproject analysis? When, before start the abovementioned “exact analysis” it is necessary to estimate beforehand, at least approximately, the prospects of different design solutions from the list of possible “applicants” and chose the solution, which will be considered as the optimal one? Obviously, it is almost impossible to realize this using the “exact” models, since each of them is very complicated and narrow-specialized. And a number of possible design solutions, which should be estimated, is too large. As a result, amount of required work becomes impossibly bulk. Moreover, significant amount of the detailed information, which can be obtained in this case, is found to be an excess one in practice. Indeed, why do we have information about the details of the mode structure of electromagnetic field in the interaction region, when we are not sure at all that either design is possible to function, if proceed from the basic physical principles? Slightly “rough” but more universal models are more preferable in such situations. That is, the necessary basic characteristics can be obtained in a simpler way, and then one can specify the physical and technological peculiarities of the chosen solution using more precise models. For example, it is possible to describe the most of pump systems, which are really used in FEL (including the mentioned in the first part of [1] H-ubitron, electromagnetic waveguide, laser, delayed, plasma, etc.), within the single model of artificial magnetodielectric. Such general approach in the 70-th-80-th was developed by the Soviet scientific school [2, 4, 6] and was later widely used in many problems of the preproject analysis.

However, we note that just such multivariant problems were stated above both in the given (second) part and the first part of the work [1]. The essence of each such problem consists in the following: to perform the approximate quantitative and qualitative assessment of a whole range of design versions of active cluster FEL-klystrons within some universal multifunction model. Within such analysis, first of all, one should ascertain when and at what conditions the use of either design schemes is found to be the reasonable or even optimal (from the point of view of the criteria formulated by requirements specification). For example, the instantaneous power of output femtosecond cluster, mean power of generated cluster wave, electron efficiency, and dimensions of FEL part of the analyzed system can be such criteria in our case. It is also obvious that to perform such full-fledged analysis for all described in [1] possible design schemes of active klystrons based on “ordinary FEL" within the present (which is the volume-limited) work is the evidently unreal problem. Therefore in the present and subsequent parts of the given investigation using ideology of the stated above "approximate but sufficiently universal" approach we will confine ourselves to studying only some of them, which are the most promising for practice.

4. TRANSVERSELY UNBOUNDED MODEL OF AN ELECTRON BEAM

According to the foresaid logic, we will carry out the substantiation of such "approximate but sufficiently universal" model of an electron beam, which later will be widely used in the analysis.

We note that nowadays the transversely unbounded model of relativistic electron beam by the majority of scientists is considered to be the obsolete one and that, which can not adequately describe the physics of real processes in FEL. At the same time, the author of monograph [3] stated that in spite of the obvious roughness of such simplified models, they for certain describe the key peculiarities of basic working mechanism in FEL obtained in experiments. More detailed analysis shows that, in fact, the abovementioned stereotype is found to be partly true. Namely, in the case of the relatively narrow "moderately current" FEL, which, as known, now are the most "popular" test subjects in the given field [11, 12], the model of transversely unbounded beam is really slightly applicable. In the case of the high-current systems with wide beams the situation is not so unambiguous and evident. The fact is that there is a sufficiently extensive region of combinations of the beam parameters (such as, plasma density, transverse dimension and geometric configuration, wave number band of the spatial charge waves (SCW), etc.), when use of more full and perfect transversely unbounded high-current models is found to be almost unreasonable. First of all, because at the sharp increase in the total amount of calculations, as it was pointed above, we, however, do not obtain the equivalent compensation in the form of principally new knowledge. Majority of the most significant for practice results here can be obtained in a simpler way using the much more simple and sufficiently substantiated transversely unbounded model.

Let us indicate in an outline the key landmarks of the criterion analysis, which allows to "legalize" application of the transversely unbounded models for studying the physics of the processes in high-current FEL, which we are interested in.

As known, the influence of real transverse boundedness of a beam comes to manifestation of three basic effects [13]. The first effect consists in the appearance of "sagging" of the beam Coulomb field outside its transverse boundary and connected with it the transverse electron energy spread in the radius. This, in turn, leads to the appearance of the radial-nonuniform field reduction of SCW inside a beam. This phenomenon in traditional microwave electronics [14, 15] is called the beam wave depression. It can be conveniently described using the so-called reduced plasma frequency ω_{pr}. The second effect consists in the appearance of multimode property of a beam that in general case comes to the appearance of additionally (with respect to the longitudinal Langmuir waves) transverse and mixed transverse-longitudinal electron waves. And, finally, effects of the third group are connected with certain "deformation" of the dispersion laws of longitudinal waves due to the boundary influence.

As known, dependence of the reduced frequency on the equilibrium radius R_b of a charge-compensated beam can be roughly estimated as [14]

$$\omega_{pr} \approx R_r \frac{\omega_p}{\gamma^{1/2}} , \tag{1}$$

where for the coefficient of plasma reduction $R_r(R_b)$ we can accept

$$R_r \sim 1 - \exp\left\{-0.7\omega_3 R_b / v_0\right\} , \tag{2}$$

$\omega_p / \gamma^{1/2} = \sqrt{4\pi e^2 n_e / m_e \gamma}$ is the “relativistic” plasma frequency of a beam; e and m_e are the charge and the rest mass of the electron; v_0 is its unperturbed axial velocity; n_e is the electron plasma density of an equilibrium beam; $\gamma = E/(m_e c^2)$ is the general relativistic factor; E is the beam energy. It is easy to see that at

$$R_b >> 1/k_3 \approx v_0 / \omega_3 , \tag{3}$$

where $k_3 \approx \omega_3 / v_0$ is the wave number of SCW in a beam, we can neglect the influence of the plasma reduction effect. Numerical evaluations show that in typical for the high-current FEL situations the condition (3) can hold, for example, in the case of the uniform cylindrical beams [13, 16].

Specific character of the chosen in the given work model consists in the fact that at the fulfillment of the condition (3) we can neglect the influence of the mentioned effect of multimode property of a beam. This is conditionned by the following: the studied cluster FEL-klystron as the system with multiple three-wave parametric resonators in the region of energy extraction also operates as the peculiar active filter of working SCW. As such in the given case only the longitudinal slow and fast SCW of the frequency ω_3 and their harmonics act. At the same time, the effect of excitation of transverse and transverse-longitudinal waves has the non-resonant nature. We have to note that the foresaid does not guarantee that such marked coupling does not appear within the theory of higher approximations due to the nonlinear coupling of waves. However, as it follows from the analysis, in the considered here (and later in other parts of the work) cubic approximation such effects do not play an essential role.

And finally, a few words about the influence of the “deformation” effect of dispersion properties of longitudinal SCW conditioned by the transverse boundaries. As a simple analysis shows, the corresponding “deformation” corrections to the dispersion laws obtained within the model of transverse-longitudinal cylindrical beam are proportional to ω_p [16]. As a rule, in the case of wide high-current beams they are found to be less (in any case they do not exceed) than the separation in wave vectors of slow and fast SCW. On the other hand, it is known from the FEL theory [2] that the influence of this separation on the processes in FEL physically appears only in the case when the half-width of SCW resonance line during the resonance interaction becomes less. This case corresponds to the so-called Raman interaction in FEL [2]. Otherwise, when this half-width is more than the separation value, as if the system “pays no heed” to this separation, i.e., “does not see” separate fast and slow SCW. This is the Compton mode of FEL operation [2].

It follows from the aforesaid that under the condition of fulfillment of other mentioned criteria, we can neglect the influence of the "deformation" effect of the dispersion law of longitudinal SCW under realization of the Compton mode. The Raman mode of FEL operation is found to be more sensitive to the influence of the given "deformation" effect. However, even in this case it is possible to select sufficiently wide variability intervals of the parameters, at which we also can use the dispersion laws obtained within the theory of transversely unbounded beam.

We have to add to the aforesaid that width of the Compton resonance line should not be large enough. Namely, it should be less than the "distance" to the nearest transverse and transverse-longitudinal beam mode.

As other criteria we choose the well-known in the FEL theory [2] assumptions, namely, the transverse dimension of the beam is much more than the Debye screening radius, the period of plasma oscillations is much less than the electron transit time of the interaction region, etc. Moreover, we consider that the beam moves in an axial uniform magnetic field, the value of which in accordance to the introduced in [2, 4] classification is assumed to be small (the so-called case of a weak magnetic field). The latter physically implies the smallness of a cyclotron frequency in the SCW frequency scale.

Thus, an electron beam is supposed to be cold, spatially unbounded, sufficiently wide, transversely uniform, relativistic, high-current and satisfying the all abovementioned validity criteria of transversely unbounded models.

5. MODEL OF THE MULTIHARMONIC (CLUSTER) MAGNETIC UNDULATOR

We accept the design solution proposed in [1] for the case of linear-polarized multiharmonic (cluster) magnetoundulator pump (Fig. 15 in [1]) as the basic one. Namely, we assume that the given undulator differs from the standard ones, which are well-known in FEL technique [2-10, 12], first of all, by the fact that here the width of each magnetic pole is found to be much less than the distance to the adjacent port: $2d << \lambda_2$ (see Fig. 3).

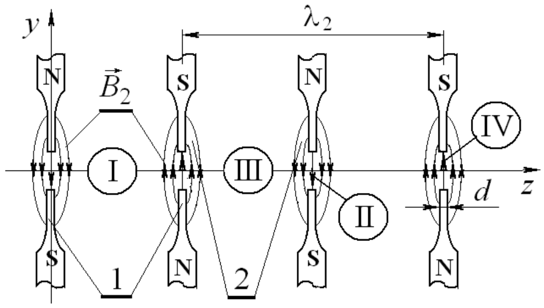

Fig. 3 *– Configuration of multiharmonic field of magnetic undulator (H-ubitron) of the pump system: 1 – magnetic poles; 2 – clusters of magnetic field; I and III are the magnetic field regions between the adjacent ports; II and IV are the magnetic field regions between poles 1; λ_2 is the undulator period; d is the width of magnetic pole; $\vec{B}_2$ are the force lines of the magnetic field induction*

Further we exercise a formal description of the field presented in Fig. 3 using the method described in the monograph [2]. The latter, in turn, is a planar periodic version of another method (which is the well-known in betatron theory [18, 19]) of approximation of magnetic field formed by a couple of cylinder magnetic poles. In accordance with the described in [2] algorithm for y-component of magnetic field B_{2y} in II and IV regions (see Fig. 3) we obtain the following approximations:

$$B_{2y} = \begin{cases} B_{2m}, & |z| \le d/2, \\ B_{2m}\left(\dfrac{d}{|z|}\right)^n, & |z| > d/2, \end{cases} \tag{4}$$

where B_{2m} is the maximum value of the y-component of magnetic field and

$$n = \lambda_2/2d \tag{5}$$

is the form-factor.

Then we will use the Maxwell equations, expressions (4) and periodicity of the given cluster magnetic field. After some simple transformations for its components $\vec{B}_2 = \{B_{2x}, B_{2y}, B_{2z}\}$ we obtain the following result:

$$B_{2x} = 0,$$

$$B_{2y} = \begin{cases} B_{2m}; & \lambda_2 j \le z \le d + \lambda_2 j; \\ K_1 B_{2m}\left\{\left(\dfrac{d}{z-\lambda_2 j}\right)^n - \left(\dfrac{d}{d+\frac{1}{2}\lambda_2 - z - \lambda_2 j}\right)^n\right\}; & \lambda_2 j + d \le z \le \frac{1}{2}\lambda_2 + \lambda_2 j; \\ -B_{2m}; & \lambda_2 j + \frac{1}{2}\lambda_2 \le z \le d + \frac{1}{2}\lambda_2 + \lambda_2 j; \\ -K_1 B_{2m}\left\{\left(\dfrac{d}{z-\frac{1}{2}\lambda_2 - \lambda_2 j}\right)^n - \left(\dfrac{d}{d+\lambda_2(1-j)-z}\right)^n\right\}; & \lambda_2 j + d + \frac{1}{2}\lambda_2 \le z \le \lambda_2 (j+1); \end{cases} \tag{6}$$

$$B_{2z} = y\frac{\partial B_{2y}}{\partial z},$$

where $K_1 = (1 - (2d/\lambda_2)^{\,n})^{-1}$. We remind that here λ_2 is the repetition period of magnetic clusters; $j = 0, 1, 2, \ldots \infty$ is the current undulation period of the cluster magnetic field (see Fig. 3).

Illustrative quantitative examples of three types of spatial configuration of the studied field at different values of the form-factor are presented in Fig. 4. The more detailed analysis of the obtained results (4)-(6) shows that

1. in general case the modeled cluster undulator magnetic field contains both the transverse and longitudinal components;
2. the transverse component is almost the transversely uniform, while the longitudinal one is found to be the transversely nonuniform.
3. the presence and the value of the transversely nonuniform longitudinal component of the field appears only far from the XZ-plane, namely, in the immediate vicinity of pole surfaces in the gap. In the case when the beam diameter is appreciably less than the gap width, we can neglect the influence of the longitudinal component of the field.

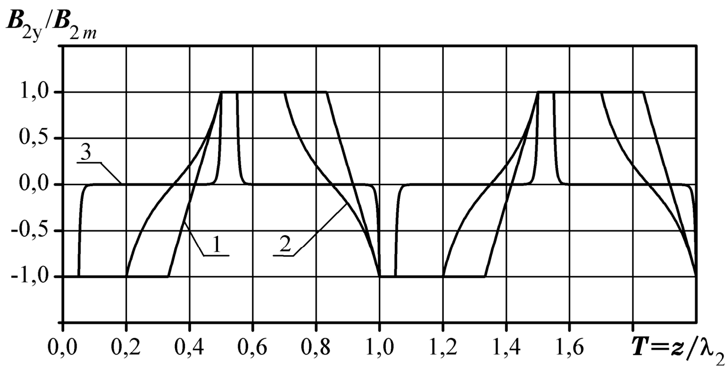

***Fig. 4** – Illustrative examples of various spatial configurations of the undulator magnetic field for different values of the form-factor. Here: curve 1 corresponds to the form-factor $n = 1{,}5$; curve 2 – $n = 2{,}5$; curve 3 – $n = 10$; λ_2 is the period of magnetic field; B_{2m} is the maximum value of the y-component of magnetic field*

Thus, taking into account the aforesaid for the vector of magnetic field induction of the cluster undulator presented in Fig. 3 we can write

$$\vec{B}_2 \approx B_{2y}\vec{e}_y \ . \tag{7}$$

Then we expand the obtained expressions (6), (7) into the Fourier series and find the desired quantity of the cluster linearly-polarized field in terms of its spatial harmonics

$$\vec{B}_2 = \sum_{n_2=1}^{N} \left[B_{2,n_2} \exp\left(in_2 p_2\right) + c.c. \right] \vec{e}_y \ , \tag{8}$$

where $B_{2,n2}$ is the complex amplitude of magnetic field induction of the n_2-th harmonic of the pump field,

$$B_{2,n_2} = \frac{1}{\lambda_2} \int_0^{\lambda_2} B_2 \exp(-in_2 k_2 z) dz \ , \tag{9}$$

$n_2 = 1, 2, ..., N$ are the numbers of harmonics; $p_2 = k_2 z$ is the phase of the 1-st harmonic of the pump field; $k_2 = 2\pi/\lambda_2$ is the wave number; $\vec{e}_y$ is the unit vector along the y-axis. We have to note that in case of use of representations (8), (9) in the theory of isochronous FEL with optimal variation of wiggler, one should take into account the possible slow dependence of its period λ_2 (or that is the same, the wave number k_2) on the z-coordinate.

Further in the present work and in all subsequent its parts, we accept the representations for the cluster magnetic field of the form of (7), (8) as the basic ones.

6. MODELS OF THE MODULATOR, END AND ACCELERATION SECTIONS OF THE FEL-KLYSTRON

We consider that the section of multiharmonic cluster FEL, the idea of which is illustrated in Fig. 5, is used as the modulator. Here the transversely unbounded beam 1 moves along the z-axis with velocity v_0 through the linearly-polarized cluster magnetoundulator (H-ubitron) pump and its representation we choose in the form of (7)-(9).

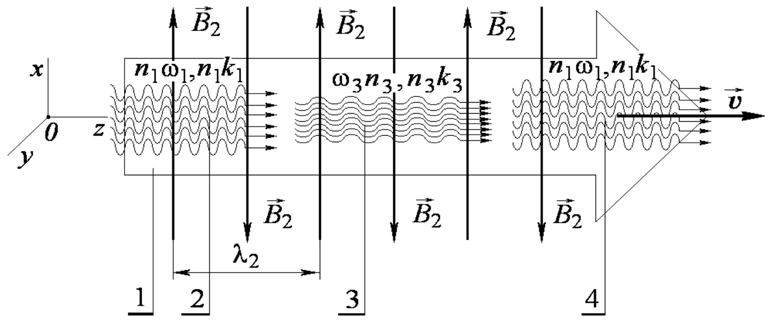

Fig. 5 *– Theoretical model of the multiharmonic section of modulator of a cluster FEL-klystron: 1 – relativistic electron beam; 2 – multiharmonic electromagnetic signal (cluster signal wave) at the system input with the spectrum $n_1\omega_1$, n_1k_1; 3 – multiharmonic SCW (cluster SCW) with the spectrum $n_3\omega_3$, n_3k_3; 4 – multiharmonic electromagnetic signal (cluster signal wave) at the system output; $\vec{B}_2$ is the induction vector of multiharmonic pump magnetic field with the spatial period λ_2*

We assume that to the input of the modulator presented in Fig. 4 the following multiharmonic electromagnetic signal (cluster wave) comes

$$\vec{E}_1 = \sum_{n_1=1}^{N} \left[E_{1,n_1} \exp\left(i n_1 p_1 \right) + c.c. \right] \vec{e}_x , \qquad (10)$$

where $E_{1,n1}$ is the amplitude of the electric field intensity of the n_1-th harmonic of a signal field; $n_1 = 1, 2, ..., N$ are the numbers of harmonics; $p_1 = \omega_1 t - s_1 k_1 z$ is the phase of the 1-st harmonic of a signal field; ω_1, k_1 are the frequency and the wave number of the 1-st harmonic; $s_1 = \pm 1$ is the sign function; $\vec{e}_x$ is the unit vector along the x-axis.

As a result of interaction of the pump (4) and signal (10) fields the SCW spectrum is excited in the system

$$\vec{E}_3 = \sum_{\chi} \vec{E}_{3\chi} = \sum_{n_3=1}^{N} \left[E_{3,\chi,n_3} \exp(i p_{3,\chi,n_3}) + c.c. \right] \vec{e}_z , \qquad (11)$$

where $\chi = \pm 1$ is the sign denoting the type of the SCW field ($\chi = +1$ corresponds to the slow SCW and $\chi = -1$ – to the fast one); $E_{3,\chi,n3}$ is the amplitude of the electric field intensity of the n_3-th harmonic of the χ-th SCW field; $n_3 = 1, 2, ..., N$ are the numbers of harmonics; $p_{3,\chi,n3} = n_3\omega_3 t - k_{3,\chi,n3} z$ is the phase; $n_3\omega_3$, $k_{3,\chi,n3}$ are the frequency and the wave number of the n_3-th harmonic of the χ-th SCW field; $\vec{e}_z$ is the unit vector along the z-axis.

We assume that the wave interaction in the system is quasi-stationary and steady-state, i.e., all transient processes are over long ago. In this case we use the boundary condition

$$E_{3,\chi,n_3}\Big|_{z=0} = 0, \quad E_{1,n_1}\Big|_{z=0} = E_{10,n_1}\,. \tag{12}$$

We consider that the Compton mode of multiple parametrically resonance interaction is realized in the system. This means that system in such state does not discern slow and fast SCW

$$k_{3,\chi,n_3}\left(\chi = +1\right) \approx k_{3,\chi,n_3}\left(\chi = -1\right) \approx n_3 k_3\,. \tag{13}$$

We choose the conditions for realization of such multiple degenerate parametric resonance in the form [2, 4, 6]

$$n_1\omega_1 \approx n_3\omega_3, \quad n_1 k_1 \approx n_2 k_2 + n_3 k_3\,. \tag{14}$$

We also take into account the three-wave parametric resonance interactions between the harmonics of the same waves.

It is easy to see that, for example, in particular case $n_1 = n_2 = n_3$ and under the fulfillment of the condition of negligible dispersion (13), the resonance condition (14) can hold simultaneously for any quantity of harmonics. Exactly this fact is the main idea of the mechanism of multiple three-wave parametric resonances on harmonics. For the first time it was formulated in our work [17] for the case of two-stream FEL and here, as it was mentioned, it is generalized for the case of a single-beam "ordinary" FEL.

Model of the terminal section of the studied FEL-klystron is shown in Fig. 6. As seen, it is similar to the described above modulator model presented in Fig. 5. Observed non-essential (for calculation) differences here concern only three things. The first one consists in the choice of the boundary conditions in another than (12) form, namely

$$E_{3,n_3}\Big|_{z=L_{s1}} = E_{3L_{s1},n_3}, \quad E_{1,n_1}\Big|_{z=L_{s1}} = 0\,. \tag{15}$$

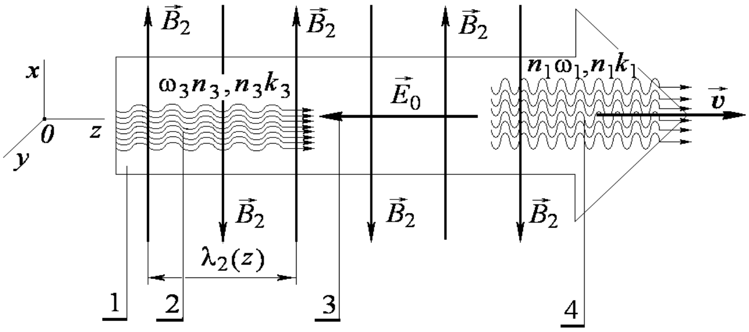

Fig. 6 – *Theoretical model of the multiharmonic terminal section of a cluster FEL-klystron: 1 – relativistic electron beam; 2 – multiharmonic SCW (cluster SCW) with the spectrum $n_3\omega_3$, n_3k_3; 3 – longitudinal electric upthrust field with intensity $\vec{E}_0$; 4 – multihar-monic electromagnetic signal (cluster signal wave) at the system output with the spectrum $n_1\omega_1$, n_1k_1; $\vec{B}_2$ is the induction vector of multiharmonic magnetic pump field with the spatial period λ_2*

Moreover, here in order to increase the electron efficiency the possibility in principle of isochronization of the interaction process using two well-known in the theory of "ordinary" FEL methods [2-6] is provided for. The first of them consists in the introduction of the pulling electric upthrust field (see Fig. 6) into the interaction region

$$\vec{E}_0 = -E_0 \vec{e}_z \,. \tag{16}$$

In this case by varying the upthrust value (16) along the z-coordinate the maximum of the electron efficiency is achieved [2, 4, 6]. In the second case, as it was noted above, the variation of the undulator period $\lambda_2 = 2\pi/k_2$ (the method of a variable wiggler [2-6]) is realized. As the experiment showed, both methods are found to be sufficiently effective in practice. In the following part of the given work we, however, confine ourselves only to the discussion of the version with optimal electric field (16).

And, finally, we briefly consider the model of the acceleration section introduced between the modulator and the terminal section in the design version of the FEL-klystron with intermediate acceleration (Fig. 2, position 8). In the general case, physics of the processes in the working volume of such section is found to be complicated. This is defined, first of all, by the fact that in the given case the matter is the acceleration of sufficiently strongly modulated multiharmonic electron beam. In the following part of the given work we, however, confine ourselves only by the discussion of the simplest model with pulling longitudinal uniform (or quasi-uniform) electric field

$$\vec{E}_{ac} = -E_{ac} \vec{e}_z \,. \tag{17}$$

Physical features of the process of intermediate acceleration, which appear in the case if use for acceleration the longitudinally non-uniform electric fields, especially those that contain the periodic longitudinal component, will be considered in other works of the authors.

7. BASIC REDUCED EQUATIONS FOR THE AMPLITUDES OF WAVE HARMONICS

As the basis we choose the set of Maxwell equations and relativistic quasi-hydrodynamic equation of a beam motion. Then we use the proposed above theoretical models, standard statement of the problem and methods of the theory of hierarchical vibrations and waves described, for example, in the monographs [2, 4, 6]. As a result of sufficiently cumbersome analytical transformations for the complex amplitudes of field harmonics (8), (10), (11) in cubically non-linear approximation we obtain the following set of the so-called reduced equations:

$$K_{1,n_1} \frac{d^2 E_{1,n_1}}{dz^2} + K_{2,n_1} \frac{dE_{1,n_1}}{dz} + D_{1,n_1} E_{1,n_1} = K_{3,n_1} B_{2,n_1} E_{3,n_1} + F_{1,n_1} \,, \tag{18}$$

$$C_{1,n_3} \frac{d^2 E_{3,n_3}}{dz^2} + C_{2,n_3} \frac{dE_{3,n_3}}{dz} + D_{3,n_3} E_{3,n_3} = C_{3,n_3} E_{1,n_1} B^*_{2,n_2} + C_{4,n_3} \left\langle E_3 E_3^{\text{int}} \right\rangle_{n_3 p_3} + F_{3,n_3} \,.$$

Here

$$D_{1,n_1} = \left({n_1}^2 / c^2 \right) \left[k_1^2 c^2 - \omega_1^2 - \left(\omega_p^2 / {n_1}^2 \bar{\gamma} \right) \right] \,, \quad D_{3,n_3} = -i(n_3 k_3) \cdot \left(1 - \frac{\omega_p^2}{\Omega_3^2 {n_3}^2 \bar{\gamma}^3} \right),$$

$$K_{1,n_1} = \partial^2 D_{1,n_1} / \partial\left(-in_1k_1\right)^2 / 2\,,\ C_{1,n_3} = \partial^2 D_{3,n_3} / \partial\left(-in_3k_3\right)^2 / 2\,,$$

$$K_{2,n_1} = \partial D_{1,n_1} / \partial\left(-in_1k_1\right),\ C_{2,n_3} = \partial D_{3,n_3} / \partial(-in_3k_3)\,,$$

$$K_{3,n_1} = \frac{\omega_p^2\omega_1 e}{2{n_1}^2\Omega_3 m_e\bar{\gamma}^4 k_1k_2c^3}\left(\frac{\bar{\upsilon}\,\bar{\gamma}^2}{c^2} - \frac{k_3}{\Omega_3}\right),\ C_{3,n_3} = \sum_{q=1,2}\left(\frac{\omega_p^2 e k_3}{{n_3}^2\Omega_3^2 m_e\bar{\gamma}^2c^2k_2}\left(\frac{\bar{\upsilon}}{c} - \frac{k_3c}{\omega_3}\right)\right),$$

$$C_{4,n_3} = \frac{3\omega_p^2 e k_3}{in_3\Omega_3^3 m_e\bar{\gamma}^6}\cdot\left(\frac{k_3}{\Omega_3} - \frac{\bar{\upsilon}\,\bar{\gamma}^2}{c^2}\right),\ \bar{\gamma} = \frac{1}{\sqrt{1-\left(\bar{\upsilon}/c\right)^2}}\,,\ \omega_p^2 = \frac{4\pi\bar{n}e^2}{m_e}\,,$$

$\Omega_\chi = \omega_\chi - k_\chi\bar{\upsilon}$, index χ takes the values 1 and 3.
We also used here the notations

$$\left\langle\ldots\right\rangle_{n_\chi p_\chi} = \frac{1}{\left(2\pi\right)^3}\int_0^{2\pi}\left(\ldots\ \exp(-in_\chi p_\chi)\,dp_1dp_2dp_3\right),$$

$$E^{\text{int}}_{\chi n_\chi} = \sum_{m=1}^{N}\left[E_{\chi n_\chi}\frac{\exp(in_\chi p_\chi)}{in_\chi} + c.c.\right],\ (\chi = 1,3\,).$$

In equations (18) $F_{1,n1}$, $F_{3,n3}$ are the functions, which contain the cubic non-linear terms of the following view:

$$F_{1,n_1} = K_{5,n_1}E'^{\,\text{int}}_{3,n_1}B_{2,n_1} + K_{6,n_1}B_{2,n_1}\left\langle E_3E_3^{\text{int}}\right\rangle_{n_1p_3} + K_{7,n_1}B_{2,n_1}\left\langle E_3^{\text{int}}E_3^{\text{int}}\right\rangle_{n_1p_3} +$$
$$+K_{8,n_1}\left\langle E_1E_1^{\text{int}}E_1^{\text{int}}\right\rangle_{n_1p_1} + \left\langle E_1\sum_{l=1}^{N}\left(K_{9,n_1,l}E_{3,l}B_{2,l}e^{ilp_1} + c.c\right)\right\rangle_{n_1p_1} +$$
$$+E_{1,n_1}\sum_{l=1}^{N}\left(K_{10,n_1,l}\left|E_{3,l}\right|^2 + K_{11,n_1,l}\left|B_{2,l}\right|^2\right). \qquad (19)$$

$$F_{3,n_3} = C_{5,n_3}E'_{1,n_3}B^*_{2,n_3} + C_{6,n_3}\left\langle E_3E_3^{\text{int}}E_3^{\text{int}}\right\rangle_{n_3p_3} +$$
$$+\left\langle E_3\sum_{l=1}^{N}\left(C_{7,n_3,l}E_{1,l}B^*_{2,l}e^{ilp_3} + c.c\right)\right\rangle_{n_3p_3} + E_{3,n_3}\sum_{l=1}^{N}\left(C_{8,n_3,l}\left|E_{1,l}\right|^2 + C_{9,n_3,l}\left|B_{2,l}\right|^2\right) +$$
$$+C_{10,n_3}\left\langle E'_3E_3^{\text{int}}\right\rangle_{n_3p_3} + C_{11,n_3}\left\langle E_3E'^{\,\text{int}}_3\right\rangle_{n_3p_3} + C_{12,n_3}\left\langle E_3^{\text{int}}E'^{\,\text{int}}_3\right\rangle_{n_3p_3}. \qquad (20)$$

In correlations (19)-(20) we used the designation

$$E'_\chi = \sum_{m=1}^{N}\left[\frac{dE_{\chi,m}}{dz}\exp(imp_\chi) + c.c.\right].$$

Coefficients C and K depend on the wave numbers, frequencies, constant components of the velocity $\bar{\upsilon}$ and concentration $\bar{n}$ of an electron beam. We supplement the set of equations (18) by the equations for constant components

$$\frac{d\bar{\upsilon}}{dz} = V_1\left\langle E'_1\ E_1\right\rangle_0 + \sum_{l=1}^{N}\left(V_{2,l}E_{3,l}E^*_{1,l}B_{2,l} + c.c\right)$$
$$+V_{3,3}\left\langle E'_3\ E_3\right\rangle_0 + V_{4,3}\left\langle E_3E_3^{\text{int}}E_3^{\text{int}}\right\rangle_0 + V_{5,3}\left\langle E_3E_3E_3^{\text{int}}\right\rangle_0. \qquad (21)$$

$$\frac{d\bar{n}}{dz} = N_1 \left\langle E_1' \, E_1 \right\rangle_0 + \sum_{l=1}^{N} \left(N_{2,l} E_{3,l} E_{1,l}^{*} B_{2,l} + c.c \right) +$$
$$+ N_{3,3} \left\langle E_3' \, E_3 \right\rangle_0 + N_{4,3} \left\langle E_3 E_3^{\text{int}} E_3^{\text{int}} \right\rangle_0 + N_{5,3} \left\langle E_3 E_3 E_3^{\text{int}} \right\rangle_0 . \qquad (22)$$

Coefficients V and N depend on the wave numbers, frequencies, constant components of the velocity $\bar{\upsilon}$ and concentration $\bar{n}$ of an electron beam.

Equations (18)-(22) describe the dynamics of the multiple parametrically resonance interaction of wave harmonics in the working regions both of the modulator and the terminal section of the FEL-klystron. And both for the model presented in Fig. 6 and the model shown in Fig. 5. Then using the set of equations (18)-(22) as the base one and the boundary conditions (12) and (15) we will carry out the comparative analysis of the formation process of femtosecond electromagnetic clusters using the design versions of the cluster FEL-klystron illustrated in Fig. 1 and Fig. 2. Results of the performed quantitative analysis are described in the following part of the given work.

8. CONCLUSIONS

Thus, the qualitative discussion of the features of femtosecond active cluster FEL-klystrons based on “ordinary” parametrical FEL is carried out in the work. Theoretical models of two types of such devices, which differ by the acceleration block arrangement of an electron beam, are proposed. In one of the model (the model with intermediate acceleration) a part of acceleration sections is placed between the sections of beam modulation and energy extraction. The model of multiharmonic magnetic undulator is described. Statement of the problem is performed and the basic system of reduced equations in cubically non-linear approximation for the complex amplitudes of harmonics of the resonantly interacting waves in the studied femtosecond active cluster FEL-klystrons is obtained.

REFERENCES

1. V.V. Kulish, A.V. Lysenko, A.Ju. Brusnik, *J. Nano- Electron. Phys.* **2** No2, 96 (2010).
2. V.V. Kulish, *Hierarchical methods: Undulative electrodynamic systems, Vol.2* (Dordrecht/Boston/London: Kluwer Academic Publishers: 2002).
3. T.C. Marshall, *Free electron laser* (New York, London: Mac Millan: 1985).
4. V.V. Kulish, *Methods of averaging in nonlinear problems of relativistic electrodynamics* (Atlanta: World Federation Publishers: 1998).
5. C. Brau, *Free electron laser* (Boston: Academic Press: 1990).
6. V.V. Kulish, *Hierarchic Methods: Hierarchy and Hierarchic Asymptotic Methods in Electrodynamics, Vol.1* (Dordrecht/Boston/London: Kluwer Academic Publishers: 2002).
7. H.P. Freund, T.M. Antonsen, *Principles of Free Electron Lasers* (Springer: Berlin-Heidelberg-New York-Tokyo: 1996).
8. E.L. Saldin, E.V. Scheidmiller, M.V. Yurkov, *The physics of Free Electron Lasers* (Springer: Berlin-Heidelberg-New York-Tokyo: 2000).
9. T. Shiozawa, *Classical Relativistic Electrodynamics: Theory of Light Emission and Application to Free Electron Lasers* (Springer: Berlin-Heidelberg-New York-Tokyo: 2004).
10. P. Schmuser, M. Ohlus, J. Rossbach, *Ultraviolet and Soft X-Ray Free Electron Lasers: Introduction to Physical Principles, Experimental Results, Technological Challenges* (Springer: Berlin-Heidelberg-New York-Tokyo: 2008).

11. C. Hernandez-Garcia, M.L. Stutzman, P. G. O'Shea, *Phys. Today* **61** No2, 44 (2008).
12. M.J. Kelley, G.R. Neil, *The Landolt-Bornstein Database – Advanced Materials and Technologies Laser Physics and Applications Laser Systems. Part 2*, **12**, 189 (Springer: Berlin Heidelberg: 2008).
13. A.A. Rukhadze, L.S. Bogdankevich, S.E. Rosinskiy, V.G. Rukhlin, *Fizika sil'notochnyh relyativistskih elektronnyh puchkov* (M.: Atomizdat: 1980).
14. D.I. Trubetskov, A.E. Khramov, *Lektsii po sverhvysokochastotnoi elektronike dlya fizikov, Vol.1* (M.: FIZMATLIT: 2003).
15. D.I. Trubetskov, A.E. Khramov, *Lektsii po sverhvysokochastotnoi elektronike dlya fizikov, Vol.2* (M.: FIZMATLIT: 2003).
16. R. Devidson, *Teoriya zaryazhennoi plazmy* (M.: Mir: 1978).
17. V.V. Kulish, O.V. Lysenko, V.I. Savchenko, I.G. Majornikov *Laser Phys.* **15**, 1629 (2005).
18. Yu.A. Bystrov, I.A. Ivanov, *Uskoritel'naya tekhnika i rentgenovskie pribory* (M.: Vysshaya shkola: 1983).
19. L.D. Landau, E.M. Lifshitz, *Teoriya polya* (M.: Nauka: 1974).