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ELECTROMAGNETIC WAVE SCATTERING BY THE COATED IMPEDANCE CYLINDER

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In this work the boundary conditions for the impedance circular cylinder coated by a low contrast dielectric thin layer are derived. Expression for the reduced impedance of the cylinder is obtained. Conditions and applicability limits of the proposed approach are defined. Influence of the coating impedance on the reduced impedance of the cylinder is investigated.

Keywords: *SCATTERING, DIELECTRIC LAYER, IMPEDANCE CYLINDER, BOUNDARY CONDITIONS, REDUCED IMPEDANCE.*

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1. INTRODUCTION

Solution of the scattering problems for convex bodies coated by thin dielectric layer is of a great practical importance. While solving these problems, the approach emerged from [1] and studied in detail in [2] is usually used to obtain the boundary conditions. In the present work we apply the approach proposed in Ref. [3] and used by the author to derive the reduced boundary conditions for a plane unlimited perfectly conducting surface coated by thin dielectric layer. The feature of this approach is the fact that it allows to obtain the reduced impedance, i.e., the impedance of homogeneous cylinder without coating but with regard to its influence. In this work we show, that the given method without significant modifications can be used not only for plane but also for cylinder surfaces, including those with non-zero impedance coated by thin dielectric layer. The reduced boundary conditions for the impedance cylinder coated by a low contrast thin dielectric layer are derived and the applicability limits of such approach are defined.

2. OBTAINING OF THE REDUCED IMPEDANCE BOUNDARY CONDITIONS

To derive the reduced boundary conditions we consider the problem represented in Fig. 1. Homogeneous cylinder of the radius ρ_1 has the surface impedance η. Leontovich-Schukin impedance boundary conditions hold on the cylinder surface $\rho = \rho_1$

$$E_{\varphi}(\rho_1+) = -\eta H_z(\rho_1+)\,, \tag{1}$$

where ρ_1+ denotes the outer cylinder surface $\rho = \rho_1$.

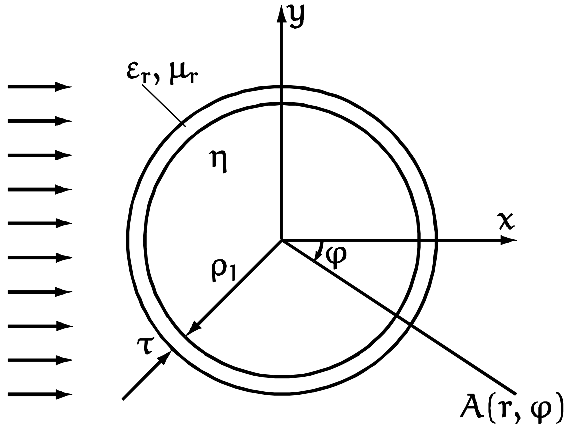

Fig. 1 – The problem geometry

Thin dielectric layer of the thickness τ with permittivity ε_r and magnetic conductivity μ_r is placed on the impedance cylinder surface $\rho = \rho_1$. Under the stipulation that $\delta = k_0\sqrt{\varepsilon_r\mu_r}\tau << 1$ (low contrast coating), where k_0 is the wave number, the dielectric field can be expanded in series of the layer thickness

$$E_\varphi(\rho_1+) = E_\varphi(\tau-) - \tau\frac{\partial}{\partial\rho}E_\varphi(\tau-)\,. \tag{2}$$

Here we are confined by two expansion terms that is completely acceptable for practical calculations.

From the Maxwell equations for complex amplitudes it is possible to find the ratio between the field components in the cylindrical coordinate system

$$\frac{1}{\rho}E_\varphi + \frac{\partial}{\partial\rho}E_\varphi - \frac{1}{\rho}\frac{\partial}{\partial\varphi}E_\rho = i\omega\mu_r H_z\,, \tag{3}$$

where ω is the circular frequency. Hence we write

$$\tau\frac{\partial}{\partial\rho}E_\varphi(\tau-) = i\omega\mu_r\tau H_z(\tau-) + \frac{\tau}{\rho}\frac{\partial}{\partial\varphi}E_\rho(\tau-) - \frac{\tau}{\rho}E_\varphi(\tau-)\,, \tag{4}$$

where τ denotes the interior of the interface dielectric layer – vacuum.

Substituting (4) into (2) we obtain (at $\rho = \rho_1$)

$$E_\varphi(\rho_1+) = \left(\frac{\rho_1+\tau}{\rho_1}\right)E_\varphi(\tau-) - i\omega\mu_r\tau H_z(\tau-) - \frac{\tau}{\rho_1}\frac{\partial}{\partial\varphi}E_\rho(\tau-)\,. \tag{5}$$

In order to find the boundary conditions on the outer surface of the dielectric it is necessary to take into account that since in an ambient space $\varepsilon_r = 1$

$$\varepsilon_r E_\rho\Big|_{\rho=\tau-} = E_\rho\Big|_{\rho=\tau+} \Rightarrow E_\rho(\tau-) = \frac{1}{\varepsilon_r}E_\rho(\tau+)\,, \tag{6}$$

and taking into consideration the continuity of the field tangential components we obtain from (5)

$$E_\varphi(\rho_1+) = \left(\frac{\rho_1+\tau}{\rho_1}\right) E_\varphi(\tau+) - i\omega\mu_r \tau H_z(\tau+) - \frac{\tau}{\rho_1\varepsilon_r}\frac{\partial}{\partial\varphi} E_\rho(\tau+)\,. \tag{7}$$

Expression (7) connects the field on the outer boundary of impedance surface ρ_1+ with the field on the outer boundary of dielectric field $\tau+$. Since the boundary conditions (1) hold on the impedance surface, substituting (7) into (1) we have

$$-\eta H_z(\rho_1+) = \left(\frac{\rho_1+\tau}{\rho_1}\right) E_\varphi(\tau+) - i\omega\mu_r \tau H_z(\tau+) - \frac{\tau}{\rho_1\varepsilon_r}\frac{\partial}{\partial\varphi} E_\rho(\tau+) \tag{8}$$

Equation (8) represents the equivalent boundary condition on the outer surface of the layer.

Let us to apply this condition back to the surface $\rho = \rho_1$. For this purpose we assume that now there is a vacuum between $\rho = \rho_1$ and $\rho = \tau$. We perform the inverse expansion of the field into the Taylor series and express the field component on the interior of the interface vacuum-vacuum τ through the components on the outer boundary $\rho = \rho_1$. Finally we obtain

$$E_\varphi(\tau-) = E_\varphi(\rho_1+) + \tau \frac{\partial}{\partial\rho} E_\varphi(\rho_1+) \tag{9}$$

Since for vacuum $\mu_r = 1$, expression (3) can be written as

$$\frac{1}{\rho} E_\varphi + \frac{\partial}{\partial\rho} E_\varphi - \frac{1}{\rho}\frac{\partial}{\partial\varphi} E_\rho = i\omega H_z\,, \tag{10}$$

and, correspondingly,

$$\tau \frac{\partial}{\partial\rho} E_\varphi(\rho_1+) = i\omega\tau H_z(\rho_1+) + \frac{\tau}{\rho}\frac{\partial}{\partial\varphi} E_\rho(\rho_1+) - \frac{\tau}{\rho} E_\varphi(\rho_1+)\,. \tag{11}$$

Substituting (11) into (9), we have (at $\rho = \rho_1$)

$$E_\varphi(\tau-) = \left(\frac{\rho_1-\tau}{\rho_1}\right) E_\varphi(\rho_1+) + i\omega\tau H_z(\rho_1+) + \frac{\tau}{\rho_1}\frac{\partial}{\partial\varphi} E_\rho(\rho_1+)\,. \tag{12}$$

Taking into consideration the continuity of the tangential components on the interface vacuum-vacuum, we find from (12)

$$E_\varphi(\tau+) = \left(\frac{\rho_1-\tau}{\rho_1}\right) E_\varphi(\rho_1+) + i\omega\tau H_z(\rho_1+) + \frac{\tau}{\rho_1}\frac{\partial}{\partial\varphi} E_\rho(\rho_1+)\,. \tag{13}$$

Now consider expression (5) obtained by the direct expansion into the Taylor series. If layer τ is filled with vacuum, then taking into account the continuity of the tangential components on the boundary $\tau\big|_-^+$ we obtain from (5)

$$E_\varphi(\tau+) = \left(\frac{\rho_1}{\rho_1+\tau}\right)E_\varphi(\rho_1+) + \left(\frac{\rho_1}{\rho_1+\tau}\right)i\omega\tau H_z(\tau+) + \left(\frac{\tau}{\rho_1+\tau}\right)\frac{\partial}{\partial\varphi}E_\rho(\tau+)\,. \quad (14)$$

Comparing (13) and (14), we see that these expressions are equal if

$$\begin{aligned} &H_z(\tau+) = \left(\frac{\rho_1+\tau}{\rho_1}\right)H_z(\rho_1+);\\ &\frac{\partial}{\partial\varphi}E_\rho(\tau+) = \left(\frac{\rho_1+\tau}{\rho_1}\right)\frac{\partial}{\partial\varphi}E_\rho(\rho_1+);\\ &\left(\frac{\rho_1}{\rho_1+\tau}\right) = \left(\frac{\rho_1-\tau}{\rho_1}\right). \end{aligned} \quad (15)$$

We note, that the third equality in (15) will strictly hold only in the case when $\tau = 0$. However, it will be held approximately if $\tau << \rho_1$, that is the applicability condition of the given approach to the cylinder structures.

Substituting (15) into (8), we find the expression

$$E_\varphi(\tau+) = \left(\frac{\rho_1}{\rho_1+\tau}\right)\left(i\omega\mu_r\tau\left(\frac{\rho_1+\tau}{\rho_1}\right)-\eta\right)H_z(\rho_1+) + \frac{\tau}{\rho_1\varepsilon_r}\frac{\partial}{\partial\varphi}E_\rho(\rho_1+)\,. \quad (16)$$

Substituting (16) into (13), we finally obtain the reduced boundary conditions on the surface $\rho = \rho_1$

$$\begin{aligned} &\left(\frac{\rho_1-\tau}{\rho_1}\right)E_\varphi(\rho_1+) =\\ &= \left(\left(\frac{\rho_1}{\rho_1+\tau}\right)\left(i\omega\mu_r\tau\left(\frac{\rho_1+\tau}{\rho_1}\right)-\eta\right)-i\omega\tau\right)H_z(\rho_1+) + \left(\frac{1-\varepsilon_r}{\rho_1\varepsilon_r}\right)\tau\frac{\partial}{\partial\varphi}E_\rho(\rho_1+). \end{aligned} \quad (17)$$

It is important to emphasize that the boundary conditions (17) are assigned on the surface of homogeneous impedance cylinder without coating though they take into account its influence.

3. SOLUTION OF THE SCATTERING PROBLEM OF ELECTROMAGNETIC WAVE ON A CYLINDER

Consider the irradiation of the cylinder of radius ρ_1 by a plane H_z-polarized wave propagating in the XY-plane perpendicular to the cylinder axis. As it is known, the field components on the surface and outside the cylinder (at $\rho \geq \rho_1$) are defined as

$$H_z = \sum_{n=0}^{\infty}\varepsilon_n(-i)^n\{J_n(k_0\rho) + R_nH_n^{(1)}(k_0\rho)\}\cos n\varphi\,, \quad (18)$$

$$E_\rho = -i\frac{Z_0}{k_0\rho}\sum_{n=0}^{\infty}\varepsilon_n(-i)^n n\{J_n(k_0\rho) + R_nH_n^{(1)}(k_0\rho)\}\sin n\varphi\,, \quad (19)$$

$$E_{\phi} = -iZ_0 \sum_{n=0}^{\infty} \varepsilon_n (-i)^n \{J'_n(k_0\rho) + R_n H'^{(1)}_n(k_0\rho)\} \cos n\varphi \,. \tag{20}$$

where $\varepsilon_0 = 1$, $\varepsilon_n = 2$, $J_n, J'_n, H^{(1)}_n, H^{(1)'}_n$ are the Bessel and Hankel functions and their derivatives, respectively, and Z_0 is the free space impedance.

Substituting expressions (18)-(20) into the boundary condition (17) on the cylinder surface (at $\rho = \rho_1$), we obtain

$$\begin{aligned} &-iZ_0\left(\frac{\rho_1-\tau}{\rho_1}\right)\left(J'_n(k_0\rho_1) + R_n H'^{(1)}_n(k_0\rho_1)\right) = \\ &= \left(\frac{\rho_1}{\rho_1+\tau}\left(i\omega\mu_r\tau\left(\frac{\rho_1+\tau}{\rho_1}\right)-\eta\right)-i\omega\tau\right)\left(J_n(k_0\rho_1)+R_nH^{(1)}_n(k_0\rho_1)\right)- \\ &-\left(\frac{1-\varepsilon_r}{\rho_1\varepsilon_r}\right)\frac{\tau n^2 iZ_0}{k_0\rho_1}\left(J_n(k_0\rho_1)+R_nH^{(1)}_n(k_0\rho_1)\right). \end{aligned} \tag{21}$$

Hence the factor R_n is equal to

$$\begin{aligned} R_n = &-\left(i\varepsilon_r k_0\rho_1^3 J'_n(k_0\rho_1) - i\varepsilon_r k_0\rho_1\tau^2 Z_0 J'_n(k_0\rho_1) - in^2\tau^2 Z_0 J_n(k_0\rho_1) + \right.\\ &+ i\varepsilon_r n^2\rho_1\tau Z_0 J_n(k_0\rho_1) - i\varepsilon_r k_0\rho_1^2\tau^2\omega J_n(k_0\rho_1) - i\varepsilon_r k_0\rho_1^3\tau\omega J_n(k_0\rho_1) - \\ &- \varepsilon_r k_0\eta\rho_1^3 J_n(k_0\rho_1) + i\varepsilon_r\mu_r k_0\rho_1^3\tau\omega J_n(k_0\rho_1) + i\varepsilon_r\mu_r k_0\rho_1^2\tau^2\omega J_n(k_0\rho_1) + \\ &\left.+ i\varepsilon_r n^2\tau^2 Z_0 J_n(k_0\rho_1) - in^2\rho_1\tau Z_0 J_n(k_0\rho_1)\right)\Big/\left(i\varepsilon_r k_0\rho_1^3 H'^{(1)}_n(k_0\rho_1) - \right.\\ &- i\varepsilon_r k_0\rho_1\tau^2 Z_0 H'^{(1)}_n(k_0\rho_1) - in^2\tau^2 Z_0 H^{(1)}_n(k_0\rho_1) + i\varepsilon_r n^2\rho_1\tau Z_0 H^{(1)}_n(k_0\rho_1) - \\ &- i\varepsilon_r k_0\rho_1^2\tau^2\omega H^{(1)}_n(k_0\rho_1) - i\varepsilon_r k_0\rho_1^3\tau\omega H^{(1)}_n(k_0\rho_1) - \varepsilon_r k_0\eta\rho_1^3 H^{(1)}_n(k_0\rho_1) + \\ &+ i\varepsilon_r\mu_r k_0\rho_1^3\tau\omega H^{(1)}_n(k_0\rho_1) + i\varepsilon_r\mu_r k_0\rho_1^2\tau^2\omega H^{(1)}_n(k_0\rho_1) + i\varepsilon_r n^2\tau^2 Z_0 H^{(1)}_n(k_0\rho_1) - \\ &\left.- in^2\rho_1\tau Z_0 H^{(1)}_n(k_0\rho_1)\right). \end{aligned} \tag{22}$$

H_z-component of the field in far zone can be written as

$$H_z = \sum_{n=0}^{\infty} \varepsilon_n (-i)^n \{J_n(k_0\rho) + R_n H^{(1)}_n(k_0 r)\} \cos n\varphi \,, \tag{23}$$

where r is the radius-vector to the point of observation.

In Fig. 2 we present the directional patterns of the H_z-component of a complete field calculated using formula (23). Obtained results at the relatively small coating thickness ($\tau \le 0{,}04\rho_1$) with a graphical accuracy coincide with the results calculated by the formula in [4].

It is important to note that taking into account the assumption about the smallness of δ, we are restricted by the maximum permissible frequency, for which the given approach (at the prescribed values of τ, ε_r and μ_r) can be applied. Calculations show that the value of δ should be $\le 0{,}5$.

Expression (22) is obtained from the boundary conditions (17) applied to homogeneous impedance uncoated cylinder but with the modified impedance considering the influence of dielectric layer. On the other hand, it is known that for the plane H_z-polarized wave incident on the impedance uncoated cylinder of the radius ρ_1, R_n can be written as

$$R_n = -\frac{J_n^{'}(k_0\rho_1) - i\eta_p J_n(k_0\rho_1)}{H_n^{'(1)}(k_0\rho_1) - i\eta_p H_n^{(1)}(k_0\rho_1)} \tag{24}$$

Equating expressions (24) and (22) for the reduced impedance we get

$$\eta_p = \frac{1}{(-\tau^2 + \rho_1^2)k_0\rho_1 Z_0\varepsilon_r}(-i\rho_1^3\varepsilon_r k_0\omega\tau + i\rho_1^2\varepsilon_r\mu_r k_0\omega\tau^2 + i\rho_1^3\varepsilon_r\mu_r k_0\omega\tau + \\ + i\rho_1\varepsilon_r\tau n^2 Z_0 - i\rho_1\tau n^2 Z_0 - i\rho_1^2\varepsilon_r k_0\omega\tau^2 - \rho_1^3\varepsilon_r k_0\eta - i\tau^2 n^2 Z_0 + i\varepsilon_r\tau^2 n^2 Z_0). \tag{25}$$

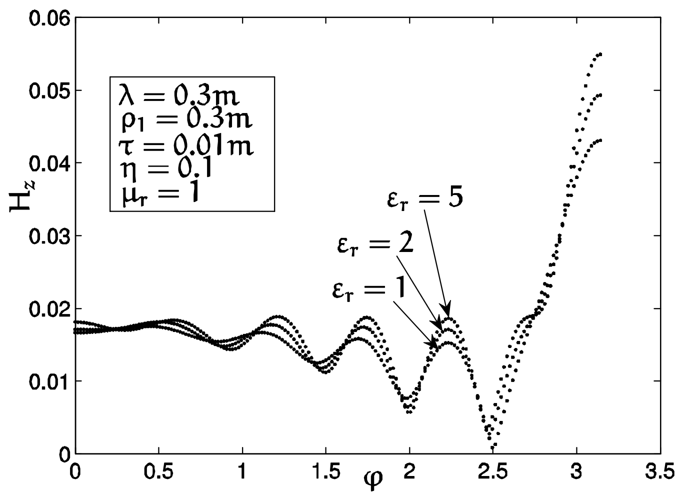

Fig. 2 *– H_z-component of a complete field in far zone at the incidence of H_z-polarized plane wave on the impedance cylinder*

In Fig. 3 we show the dependence of the reduced impedance of the cylinder on the coating impedance. An important fact is the following: though both the impedance of the cylinder and the coating impedance have real values, the reduced impedance represents purely imaginary quantity.

In conclusion we would like to dwell on the availability of asymptotic solution for the field scattered by the impedance circular cylinder with dielectric coating. In principle, it is possible to directly apply the approach described in [5] to the expression (23) where R_n is given in the form of (22). However for the circular cylinder, it is obviously simpler to use the reduced impedance and obtain the asymptotic solution of the scattering problem for homogeneous impedance cylinder, which will be completely equivalent to the solution for the coated impedance cylinder.

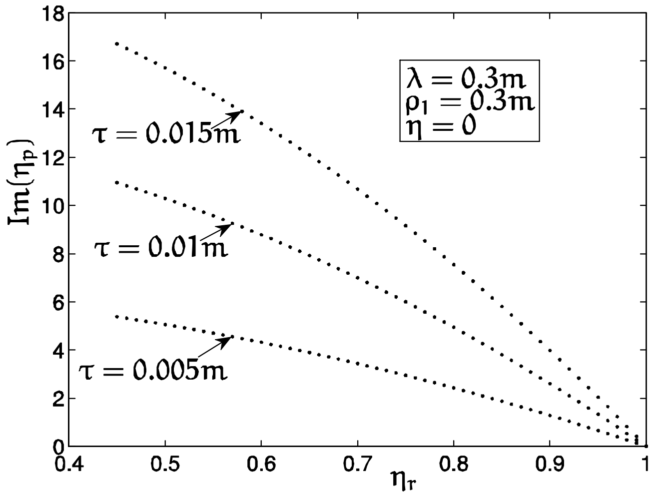

Fig. 3 *– Dependence of the reduced impedance of the cylinder on the impedance of dielectric coating at different coating thickness*

4. CONCLUSIONS

The reduced boundary conditions for the impedance cylinder coated by a low contrast thin dielectric layer are obtained. The condition, under which the mentioned approach can be applied to cylindrical structures, is defined. The expression for the calculation of the directional pattern of a complete field for homogeneous impedance cylinder with the reduced impedance is derived. Shown, that the given solution completely corresponds to the solution for the corresponding coated impedance cylinder. This approach can be easily applied to circular impedance cylindrical structures coated by some low contrast dielectric layers.

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