ANALYSIS OF THE THERMAL CONDITIONS OF PULSE IMPACT AVALANCHE TRANSIT-TIME DIODES

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The nonstationary problem of the thermal process for the pulse mode of impact avalanche transit-time diodes is solved. The one-dimensional thermal model of such diodes, which takes into account the heterogeneity of the thermal power distribution and the heat spreading along the heat sink in the operating temperature range, is considered. The numerical calculation results of the average temperature of the active layer and the thermal resistance on the pulse parameters, as well as on the geometric and thermophysical diode parameters, are presented.

**Keywords:** THERMAL CONDUCTIVITY, THERMAL DIFFUSIVITY, PULSE DURATION, PULSE ON/OFF RATIO, THERMAL RESISTANCE.

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1. INTRODUCTION

Maximum output powers of semiconductor devices are restricted, first of all, by heating of the active regions of semiconductor up to the temperatures enough for the thermal destruction of these regions. Use of the pulse mode allows to increase the instantaneous values of the microwave device output capacity; and the smaller pulse duration, the greater this increase. Rigorous solution of the nonstationary thermal problems for the specific structures is mathematically extremely complicated and can be found with computer only. For a number of simplified models the analytical time dependences of the active region temperature of impact avalanche transit-time (IMPATT) diodes are obtained [1, 2]. However, these dependences are useless for strictly quantitative estimates, since they do not take into account the essential factor, namely, the temperature variation of the thermal conductivity in the operating temperature range 300-500 K, which is quite substantial.

In the present paper the determination of the temperature characteristics of IMPATT diodes is carried out using the analytical approach [3]. Results of such analysis comprise a wide range of parameters and are found to be comparable with experimental study results of the temperature characteristics.

2. STATEMENT OF THE PROBLEM

IMPATT diodes assembled using the active region directly on the heat sink have negligible thermal resistance of the p-n junction and can be represented by the following thermal model: the finite-length cylinder (active region) on the semi-infinite heat sink. The considered model is presented in Fig. 1.
The semiconductor structure 1 is fixed on the copper heat-eliminating substrate 2 from one side, and from another side it is connected with the flange 4 of the diode body 5 using the wired plate 3. The calculation model is reduced to the following:

- pulse power supply of the diode is applied to the p-n junction 00 (see Fig. 1);
- periodic supply mode with the constant values of the pulse power $P_p$, pulse duration $T_p$ and period $T$ is considered.

Considered model satisfies the relation $T_p << \tau_1 << T$, where $\tau_i = r^2/\lambda_i$ ($i = 1, 2$) is the relaxation time of heat spreading through the passive heat sink; $r$ is the cross-section radius of the diode; $\alpha$ are the thermal diffusivities; the heat-eliminating substrate III is considered as a half-space with the given values of thermal conductivity coefficient $\lambda$, material density $\rho$, thermal diffusivity $\alpha = 1/c\rho$, where $c$ is the heat capacity of the material. In Table 1 we present the thermal characteristics of some materials taken from [4].

**Table 1 – Thermal characteristics of some materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$, g/cm$^3$</th>
<th>$c$, J/g·deg</th>
<th>$\lambda$, W/cm·deg 300 K</th>
<th>$\alpha$, cm$^2$/s</th>
<th>$\sigma$, W·s/cm$^2$·deg 300 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si-n</td>
<td>2.3</td>
<td>0.76</td>
<td>1.1</td>
<td>0.7</td>
<td>0.63</td>
</tr>
<tr>
<td>Si-n$^+$</td>
<td>2.3</td>
<td>0.76</td>
<td>0.8</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>Cu</td>
<td>8.9</td>
<td>0.39</td>
<td>4.0</td>
<td>3.8</td>
<td>1.14</td>
</tr>
</tbody>
</table>

In semiconductor microwave devices the active region size of the p-n junction ($l_1 + l_n = l$) is substantially less than the size of the heat sink. In 8-mm range the radius of the p-n junction is $r_{p,n} = 80$ μm and the active region length is $l = 1$ μm. Minimum relaxation times $\tau_a$, which determine the time of the main temperature rise of the active layer under pulse switching-on of the power supply, correspond to the active region. Further temperature establishment, connected with the heat diffusion into the distant from the active region elements of the semiconductor structure and the heat sink, occurs much slower. Therefore, there is no need to calculate the whole process of the temperature establishment in the system, and it is enough to know the law of the temperature rise during the time $t < \tau_a$. 
Under assumptions mentioned above one can use the one-dimensional model of the diode and the heat sink (see Fig. 1). In this case the boundary problem, which describes the thermal process in the diode and the heat sink, is stated in the following way. Let \( T_1 \) and \( T_2 \) are the excess temperatures of the diode and the heat sink, respectively, in comparison with the environmental temperature (which is conditionally accepted to be zero). These values should satisfy the equations of parabolic type

\[
\frac{\partial^2 T_1}{\partial x^2} - \frac{1}{a_1} \frac{\partial T_1}{\partial t} = -\frac{Q(t)}{\lambda_1}, \quad 0 < x < l, \tag{1}
\]

\[
\frac{\partial^2 T_2}{\partial x^2} - \frac{1}{a_2} \frac{\partial T_2}{\partial t} = 0, \quad x < 0, \tag{2}
\]

the initial conditions at \( t = 0 \)

\[
T_1 (x, 0) = T_2 (x, 0) = 0 , \tag{3}
\]

and the boundary conditions

\[
\left. \frac{\partial T_1}{\partial x} \right|_{x=0} = 0, \quad T_1 (0, t) = T_2 (0, t) , \tag{4}
\]

\[
\lambda_1 \left. \frac{\partial T_1}{\partial x} \right|_{x=l} = \lambda_2 \left. \frac{\partial T_2}{\partial x} \right|_{x=-l}, \quad T_2 \big|_{x=-l} = 0 , \tag{5}
\]

where \( a_1 \) and \( a_2 \) are the thermal diffusivities and \( \lambda_1 \) and \( \lambda_2 \) are the thermal conductivity coefficients of the diode and the heat sink, respectively; \( l \) is the thickness of the active layer (diode); \( Q(t) \) is the thermal power density.

3. CALCULATION OF THE THERMAL CHARACTERISTICS OF IMPATT DIODES

At first, the investigated process of the active region temperature rise under action of the single-pulse power \( Q(t) \) has the form

\[
Q(t) = U \begin{cases} 1, & 0 \leq t \leq \tau_p, \\ 0, & t < 0, \\ 0, & t > \tau_p, \end{cases}
\tag{6}
\]

where \( \tau_p \) is the pulse duration.

Further, we assume that the coefficients \( \lambda_n \) and \( a_n \) \((n = 1, 2)\) do not depend on the temperature, i.e., the original problem (1)-(5) is considered in linear approximation.

Solution of the problem (1)-(5) is based on the operation method [6]. Let \( \overline{T_1 (x, p)}, \overline{T_2 (x, p)}, \overline{Q (p)} \) are the Laplace transformations of the functions \( \overline{T_1 (x, t)}, \overline{T_2 (x, t)}, \overline{Q (t)} \), namely,

\[
\overline{T_1 (x, p)} = \int_0^{\infty} T_1 (x, t) e^{-pt} dt , \tag{7}
\]
$$\overline{T_2}(x, p) = \left[ T_2(x,t) e^{-\beta t} dt \right],$$  \hspace{1cm} (8)

$$\overline{Q}(p) = \left[ Q(t) e^{-\beta t} dt \frac{Q_0 (1 - e^{-\beta t})}{\beta} \right].$$  \hspace{1cm} (9)

Then, applying the Laplace transformations to equations (1) and (2) and taking into account the initial conditions (3) we obtain

$$\frac{d^2 \overline{T_1}}{dx^2} - \frac{p}{\lambda_1} \overline{T_1} = \overline{Q}, \quad 0 < x < l,$$  \hspace{1cm} (10)

$$\frac{d^2 \overline{T_2}}{dx^2} - \frac{p}{\lambda_2} \overline{T_2} = 0, \quad x < 0.$$  \hspace{1cm} (11)

Further, applying the Laplace transformations to the boundary conditions (4) and (5) we find

$$\lim_{x \to l} \frac{dT_1}{dx} = 0, \quad \overline{T_1}(0, p) = \overline{T_2}(0, p),$$  \hspace{1cm} (12)

$$\lim_{x \to 0} \lambda_1 \frac{dT_1}{dx} = \lambda_2 \lim_{x \to 0} \frac{dT_2}{dx}, \quad \overline{T_2}(x, p) \bigg|_{x=\infty} = 0.$$  \hspace{1cm} (13)

Thus, using the Laplace transformations in time the original problem for the partial differential equations is reduced to the boundary problem for the system of the ordinary second-order differential equations.

Solving the problem (10)-(13), expression for the temperature of the active region (diode) averaged over its thickness was obtained

$$T_{1av}(t) = \frac{U \tau l^2}{3T \lambda_1} + \frac{a_1 U \tau}{2 \lambda_1} \left[ 1 + \frac{\gamma}{T} (2T - \tau) \right] -$$

$$- \sum_{n=0} \frac{Q_n e^{\frac{\gamma n}{T} \tau}}{\sqrt{a_1} \lambda_1 \lambda_2} \frac{a_n + c b}{\sqrt{a_1} \lambda_1}, \quad 0 \leq t \leq 2\pi.$$  \hspace{1cm} (14)

Knowing the average temperature of the active layer, the thermal resistance can be calculated in accordance with (3)

$$R(t) = \frac{l}{2\pi \lambda_1 r^2} T_{1av}(t),$$  \hspace{1cm} (15)

where \(l\) and \(r\) are the thickness and the cross-section radius of the diode, respectively.

Thus, we have obtained formulas, which describe the thermal process in the diode under periodic pulse action subject to the geometric and thermophysical diode parameters.
Calculation results of the normalize average temperature \( T = T_{1av}/a_1 U \tau \) of the active layer and the thermal resistance \( R(t) \) for the time moment \( t = \tau \) (\( \tau \) is the pulse duration) at different values of pulse parameters are presented in Fig. 2.

![Graph](image)

**Fig. 2 – Dependence of the normalized temperature \( T \) of the active layer on the pulse on/off ratio**

As seen from Fig. 2 the normalized average temperature tends to 1 with the increase of the pulse on/off ratio \( (Q = T/\tau) \), i.e., the average temperature \( T_{1av} \) of the active layer tends to the value

\[
T_N = Ua_1 \tau / \lambda_1 .
\]

Thus, at \( Q > 100 \) the average temperature of the active layer can be found using formula (16).

4. CONCLUSIONS

Thermal model of the pulse IMPATT diode proposed in the present paper satisfactorily describes the dynamics of the active layer heating at the pulse duration, which is smaller than the time of thermal relaxation of the heat sink. We have obtained formulas, which describe the thermal process in the diode under periodic pulse action depending on the geometric and thermophysical parameters of the semiconductor and the heat sink, and which can be used for the engineering calculation of the pulse diodes.

REFERENCES