J. Nano- Electron. Phys.
1 (2009) No3, P. 52-57

PACS numbers: 05.45.Df, 05.45.Tp

MULTIFRACTAL ANALYSIS OF THE TIME SERIES OF ECONOMIC SYSTEMS

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Within the method of the multifractal detrended fluctuation analysis the time series of the currency exchange rate for a period including the world financial crisis is investigated. By the example of the growing demand for currency purchase during the crisis, the connection between time correlations in the distribution of the series terms and external factors is found. Crisis influence on the statistical characteristics of the time series is studied.

Keywords: *TIME SERIES, METHOD OF THE MULTIFRACTAL DETRENDED FLUCTUATION ANALYSIS, MULTIFRACTAL SPECTRUM.*

(Received 27 November 2009, in final form 15 December 2009)

1. INTRODUCTION

Time series is a sequence of values of the magnitude taken at equal time intervals. Time series are widespread in natural sciences and applied research as the most rational presentation method of the obtained data. Examples of such series give the meteorological data about the accidental variations of the air temperature, amount of precipitation and wind speed. Time series are also used in economics, where the currency exchange rates give the most striking example, in medicine (for example, electrocardiogram of the heart), in physics and chemistry, where they represent the sequences of measured values, and in other sciences [1].

Quantitative investigation of the random time series is possible in only that case if they are reduced to the self-affine sets, the fractal nature of which is the following: under proper change of scale any section looks like a whole series. As known from the fractal theory, the key features of such series are the Hurst exponent, spectrum of fractal dimensionalities and mass index [2]. Determination of these characteristics is achieved by the multifractal detrended fluctuation analysis (MF DFA) [3], which allowed to investigate the time series in physics [4], economics [5], biology [6] and meteorology [7].

The present paper investigates the influence of the external factors on the statistical properties of the time series of economic systems. For the purpose, at first we briefly describe the MF DFA method. Then the analysis of the fluctuations of exchange quotations during the world financial crisis is carried out. This analysis shows that the section of time series corresponding to the crisis period is characterized by the substantial strengthening of time correlations and broadening of the spectrum of statistical properties.

2. METHOD DESCRIPTION

The standard procedure of the MF DFA method is presented by the following stages [3].

1. For the analyzed time series $X(i)$ ($i = 1, 2, ..., N$) the fluctuation profile $Y(i) = \sum_{k=1}^{i}\left[x(k) - \bar{x}\right]$ with the average value $\bar{x}$ is selected.

2. Obtained series $Y(i)$ is divided into $N_s = (N/s)$ non-overlapping segments containing the equal number of points s. To take into account the last section, which contains a number of points less than s, we repeat the division procedure starting from the opposite end of the series. As a result we obtain $2N_s$ segments of the length s.

3. Within the least-squares method for each segment we determine the polynomial $y_\nu(i)$ of local trend, which interpolates profiles $Y(i)$. Then for the segments $\nu = 1, ..., N_s$ and $\nu = N_s + 1, ..., 2N_s$ we find the variances

$$F^2(\nu, s) = \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left[(\nu - 1)s + i\right] - y_\nu(i)\right\}^2, \quad (1)$$

$$F^2(\nu, s) = \frac{1}{s}\sum_{i=1}^{s}\left\{Y\left[N - (\nu - N_s)s + i\right] - y_\nu(i)\right\}^2. \quad (2)$$

4. Averaging the values of (2) deformed by the exponent q we obtain the fluctuation function

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\left[F^2(\nu, s)\right]^{q/2}\right\}^{1/q}, \quad (3)$$

for which at $q = 0$ one should use the expression

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{\nu=1}^{2N_s}\ln\left[F^2(\nu, s)\right]\right\}. \quad (4)$$

5. At fixed q we plot the dependence $F_q(s)$ in the log-log coordinates. For the self-affine series it is of the power form

$$F_q(s) \propto s^{h(q)}, \quad (5)$$

which the straight line with the slope equal to the generalized Hurst exponent $h(q)$ [2] corresponds to.

6. Repeat steps 1-5 for different values of the exponent q. Here we should keep in mind, that the fluctuation function $F_q(s)$ loses statistical informativeness for the scale $s > N/4$, since the number of segments N_s becomes very small; on the other hand, it is necessary to exclude the small segments $s < 10$. Testing of the MF DFA algorithm on the series admitting the analytical investigation of the multifractal spectrum has showed its high accuracy, which provides use for the real time series [8].

It is convenient to turn from the Hurst exponent $h(q)$ to the mass index $\tau(q)$ and the multifractal spectrum $f(\alpha)$ [2, 3] in description of the scaling properties:

$$\tau(q) = qh(q) - 1, \tag{6}$$

$$f(\alpha) = q\alpha - \tau(q), \ \alpha = \tau'(q). \tag{7}$$

Constant value of $h(q)$, linear increase of $\tau(q)$ and the δ-shaped dependence of $f(\alpha)$ correspond to the monofractal time series, while the decline of the function $h(q)$, non-linear increase of $\tau(q)$ and the blurred spectrum of $f(\alpha)$ indicate the transition to the multifractal series.

As known, multifractal properties of time series are conditioned by the spread in the series values, on the one hand, and by the correlations of their distribution, on the other hand [3]. Obviously, during the random rearrangement of these values the distribution function is not changed, while the time correlations are destroyed. Therefore, if mixing leads to the partial narrowing of the multifractal spectrum, one can state, that the time series has the correlations, and during the transition to the δ-shaped spectrum of $f(\alpha)$ such correlations are absent.

3. DATA UNDER INVESTIGATION

Obvious interest in the study of the currency exchange rates is conditioned by the attempt to predict their behavior. MF DFA of such series allows to determine the presence of time correlations between the series terms, which play the key role in prediction of its evolution.

As the subject of the investigation we have chosen the data of the change in the euro-to-dollar rate for the period from January 2007 to November 2009. The data is taken from the resource "Dealing center Forex EuroClub" (www.fxeuroclub.ru). Chosen time range is of interest since it includes both the period of relative financial stability and the world financial crisis.

In such investigations, except of the initial currency rates, the currency quotations are usually represented by the difference of logarithms $r(t) = \ln[P(t)] - \ln[P(t - \Delta t)]$, $P(t)$ is the currency price at the time t, Δt is the measurement interval (in our case $\Delta t = 15$ min). Time history of the initial currency rate is presented in Fig. 1a, and the corresponding changes in the difference of logarithms are in Fig. 1b. Moreover, the spread in probabilities of different currency quotations determined using the multifractal spectrum (see below) is displayed in Fig. 1c.

Feature of Fig. 1 is the presence of a section of anomalously big change in the currency rate, which is appeared in the beginning of 2009. For the quantitative representation of the crisis influence, the spectrums of fractal dimensions corresponding to the time intervals assigned in Fig. 1 are shown in Fig. 2. Seen, the spectral function $f(\alpha)$ of the crisis section acquires the anomalously wide interval of the change in fractal dimensions. Since the minimum $\alpha_{\min} \propto \ln(1/p_{\max})$ and the maximum $\alpha_{\max} \propto \ln(1/p_{\min})$ values of these dimensions are defined by the limiting probability values $p_{\max}$ and $p_{\min}$ of the currency quotation change, the shown in Fig. 2 broadening of the spectral function leads to the increase in the dispersion in probabilities presented in Fig. 1c.

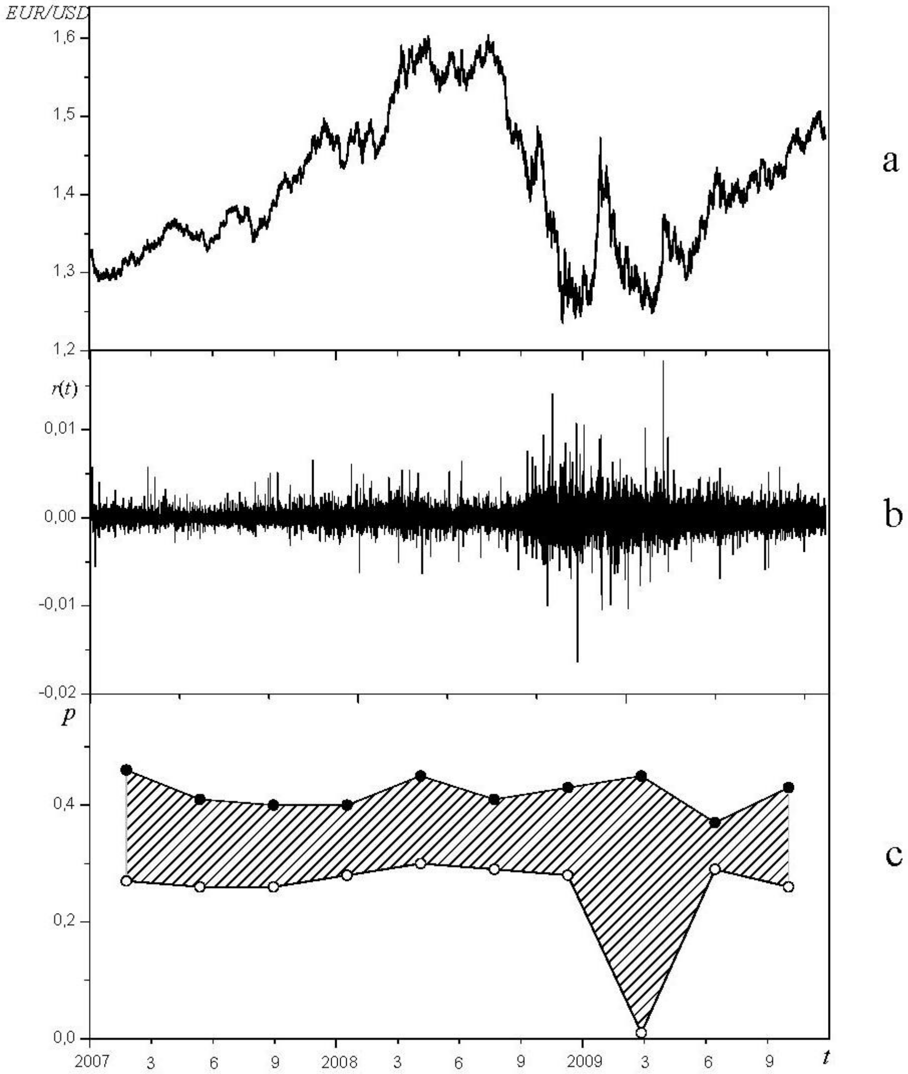

***Fig. 1** – Time history of the euro against the dollar (a); corresponding changes of the difference of logarithms r(t) (b); dispersion in probabilities of different currency quotations (c)*

According to Fig. 3 the series mixing preceding the crisis decreases the width of the multifractal spectrum by the value of $\Delta\alpha \approx \mathbf{0{,}1}$, while the spectrum of crisis series becomes narrower by the considerably more value $\Delta\alpha \approx \mathbf{0{,}31}$ (26% and 43%, respectively). Here we can conclude that the crisis leads to the substantial strengthening of time correlations.

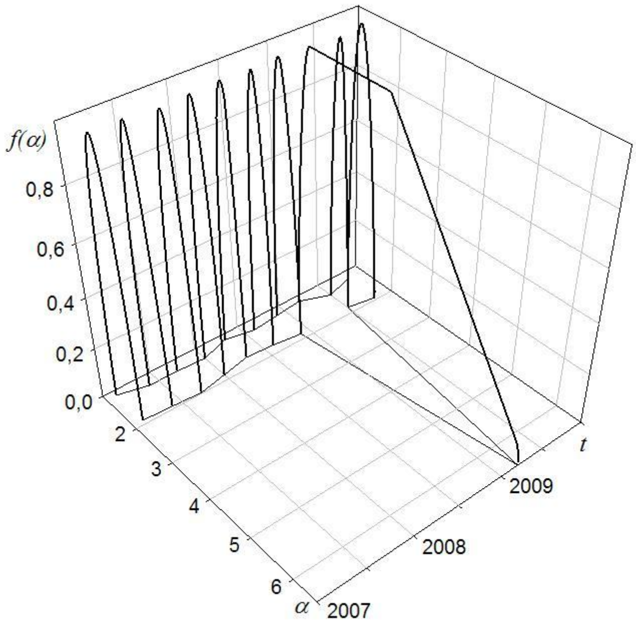

Fig. 2 *– Multifractal spectrums corresponding to the three-month intervals assigned in Fig. 1*

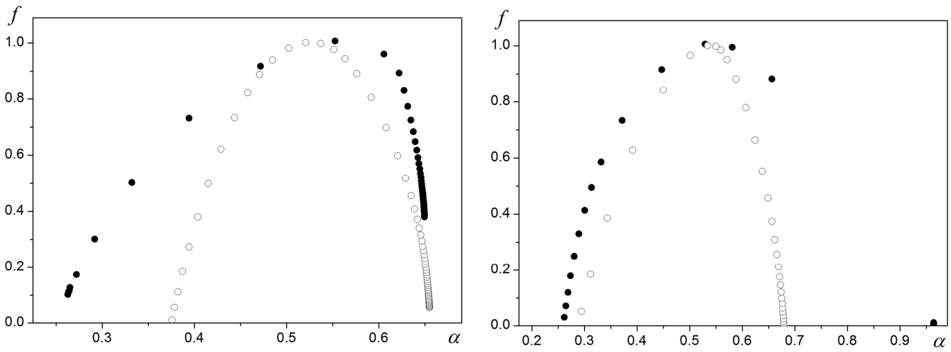

Fig. 3 *– Multifractal spectrums for the series sections before (a) and during (b) the crisis (heavy dots correspond to the initial series, hollow dots – to the mixing ones)*

4. CONCLUSIONS

Carried out investigations show, that the time history of the currency exchange rate during the high demand for the currency purchase is characterized by a wide spectrum of fractal dimensions and strong correlations in comparison with the period of the fluctuating exchange rate.

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