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ANALYSIS OF THE EFFECTIVENESS OF POWER EQUATIONS IN THE DESCRIPTION OF ELECTRON-WAVE PROCESSES IN THE AMPLIFIER MODEL BASED ON THE SMITH-PURCELL EFFECT

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We present the comparative analysis of different approximations used to obtain the dispersion equations in the form of power polynomials, which allow to describe the electron-wave processes in the amplifier system an open waveguide with periodic structure and an electron beam. Shown, that the amplifier model described by the analytical third- and seventh-order dispersion equations is the most evident one from the physical point of view. These equations allow to take into account the basic characteristics of electrodynamic system and electron beam, which interacts with a bulk wave of the investigated quasi-optical system.

Keywords*: DIFFRACTION RADIATION, ELECTRON FLOW, OPEN WAVEGUIDE, MAGNETIC FIELD, ELECTROMAGNETIC WAVE.*

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1. INTRODUCTION

Recently the questions of amplification and generation of electromagnetic oscillations on spatial harmonics of the coherent diffraction radiation (the Smith-Purcell radiation) [1] are actively discussed. This radiation is excited when the non-relativistic electron flow (EF) moves along the diffraction gratings placed in the quasi-optical systems of different modifications [2]. At the same time the questions of a wideband amplification of electromagnetic oscillations in millimeter- and submillimeter-wave bands, which can be realized in accordance with an amplifier scheme based on the Smith-Purcell effect [3], are the most topical ones. By now development of this problem has been performed in two directions: theoretical analysis of the electron-wave processes [4] and experimental simulation of the wave processes in different modifications of the amplifier electrodynamic systems [3]. In particular, results of Ref. [5] for linear approximation of the interaction between EF and electromagnetic field diffracted on the grating allowed to obtain (the self-consistent statement of the problem) the dispersion equation of amplifier in the form of the third-order polynomial and explain in the first approximation the electron-wave process physics at the final stage of excitation of electromagnetic wave. Further theoretical development [4] allowed to obtain the general transcendental dispersion equation, which takes into account the influence of the focusing magnetic field and other system parameters on the initial stage of wave excitation in amplifier. However, inconvenience and complexity of numerical analysis of the transcendental equation required the transition (using the correct simplifications) to the power analytical dispersion equations, which are more compact and effective in description of the physical processes in amplifier based on the Smith-Purcell effect.

In the present work by the example of the electron-wave processes the comparative analysis of the use effectiveness for some types of approximate dispersion equations of amplifier based on the Smith-Purcell effect, which allow to solve a number of specific problems, is carried out.

2. POWER DISPERSION EQUATIONS

In general case, when EF interacts with electromagnetic field in microwave devices, the space-time spectrum of the longitudinal and transverse electron waves [1, 6] is generated. Slow space-charge waves (SSCW) and fast space-charge waves (FSCW) propagating with different phase velocities are the longitudinal waves. Slow cyclotron waves (SCW) and fast cyclotron waves (FCW), which are excited in EF when it is focused by magnetic field, are the transverse waves. In contrast to the longitudinal waves the transverse ones have the polarization properties: FCW are the clockwise polarized and SCW are the counterclockwise polarized. In classical microwave devices with the long-term interaction the effectiveness of power-interchange is defined by the condition of quasi-synchronism of one or some electron waves with the surface wave field of slowing structure [6]. In diffraction electronics devices in addition to the condition of electron quasi-synchronism with the field of diffraction radiation the condition of coherent radiation should hold [1].

In general case the theoretical model of amplifier [4] represents the open waveguide (OW) formed by the metal mirror and diffraction grating of the "comb" type (the period is $2l$, the slot width is $2d$, and the slot depth is h), which are placed parallel at the distance H. Flat EF focused by the magnetic field $\vec{B}_0$ moves above the grating. Operating principle of amplifier as in the case of other devices of diffraction electronics is based on the radiation of coherent bunches of the electron charge densities. These bunches form the longitudinal (transverse, or superposition of the longitudinal and transverse) electron wave of the circulating current propagating along the system grating-EF with the constant phase velocity, which satisfies the excitation conditions of diffraction radiation. Such waves are excited in EF, when electrons interact with the field of slow spatial diffraction harmonic formed during the electromagnetic wave diffraction on the periodic structure. EF excites the maximal energy density of diffraction radiation under synchronism of the FSCW with the field of slow diffraction harmonic. In this case at the interaction with an incident field diffracted on the grating the regime of electromagnetic wave amplification can be realized.

For the model described above (the self-consistent statement of the problem) the general transcendental dispersion equation [4] is obtained, which can be solved by the numerical methods only, for example, by the iteration method, which allows to operate effectively by the complex numbers and achieve the good solution convergence. Under transformation of the transcendental dispersion equation to the power equations it is necessary to make some simplifying assumptions, which do not essentially touch the electron-wave physics:

– connection between the grating field and the EF is maximal;

– EF interacts with the first spatial harmonic, and the bulk wave radiation occurs on the zero harmonic.

Taking into account the small order of magnitude of the space charge q and the cyclotron parameter $\Omega_c = \omega_c/\omega$ ($\omega_c = \eta B_0$ is the cyclotron frequency; ω is the oscillation frequency; B_0 is the induction of the focusing magnetic field; $\eta = e/m_e$; e and m_e are the electron charge and mass, respectively), and using the dimensionless wave numbers and system geometric parameters, we obtain the simplified transcendental dispersion equation, which is transformed into the power seventh-order equation by the Taylor expansion and application of the graphical approximation method.

The given equation in a compact form can be written as follows:

$$\frac{\Gamma_{c1}^2(\kappa_0, \mu_0 + \delta\mu)}{\xi_{c1}^2(\kappa_0, \mu_0 + \delta\mu)} - 1 = \frac{\mu_0 \pi\chi \cdot \operatorname{cosec}^2 \pi\chi\sqrt{\kappa^2 - \mu_0^2} + \dfrac{\mu_0}{\sqrt{\kappa^2 - \mu_0^2}} \operatorname{ctg} \pi\chi\sqrt{\kappa^2 - \mu_0^2}}{(\kappa^2 - \mu_0^2)} \delta\mu, \quad (1)$$

where

$$\xi_{c1} = \left[\left(1 + (q^2(\kappa - \beta_0(\mu+1))\kappa^3\right) \Big/ \left(1 - \Omega_c^2\right)\left(\kappa - \frac{\beta_0(\mu+1)}{(1-\Omega_c)}\right)\left(\kappa - \frac{\beta_0(\mu+1)}{(1+\Omega_c)}\right)\right]^{1/2},$$

$$\Gamma_{c1} = \left(1 - \frac{q^2\kappa^2}{\left(\kappa - \beta_0\left(\mu+1\right)\right)^2}\right)^{1/2} \Big/ \left(1 - \frac{q^2\kappa^2}{(1-\Omega_c^2)\left(\kappa - \dfrac{\beta_0(\mu+1)}{(1-\Omega_c)}\right)\left(\kappa - \dfrac{\beta_0(\mu+1)}{(1+\Omega_c)}\right)}\right)^{1/2},$$

$\chi = H/l$, $\delta = h/l$ are the dimensionless geometric parameters of the system; $\mu = \mu_0 + \delta\mu$ is the wave number, the absolute value of which does not exceed 0,5, and $\mu_0 >> \delta\mu$; $\beta_0 = \upsilon_0/c$, υ_0 is the constant component of the unperturbed electron velocity, c is the light speed; $\kappa = kl/\pi$, $k = 2\pi/\lambda$ are the wave numbers, λ is the radiation wave length.

Without taking into account the magnetic field influence on the wave processes ($B_0 \to \infty$) equation (1) is transformed into the third-order equation in $\delta\mu$ [3, 7]:

$$-\beta_0^2\delta\mu^3 + 2\beta_0\left(\kappa - \beta_0\ (\mu_0+1)\right)\delta\mu^2 - \\ -\left(\kappa - \frac{\beta_0\ (\mu_0+1)}{1+q}\right)\left(\kappa - \frac{\beta_0\ (\mu_0+1)}{1-q}\right)\delta\mu - \frac{q^2\kappa^2\Lambda}{\Delta'_{0\mu}} = 0, \quad (2)$$

where

$$\Lambda = \begin{cases} 1 & , \ \operatorname{Re}(\Gamma_1) >> \operatorname{Im}(\Gamma_1) \\ 1 & , \ \Gamma_1 = 0 \\ 1 & , \ \tilde{\varepsilon}\widehat{\Gamma}_1 \le 1/2 \\ \sin\left(\pi\tilde{\varepsilon}\Gamma_1\right) & , \ \tilde{\varepsilon}\widehat{\Gamma}_1 > 1/2 \end{cases}, \qquad \Gamma_1 = \sqrt{1 - \frac{q^2\kappa^2}{\left(\kappa - \beta_0(\mu+1)\right)^2 - q^2\kappa^2}}, \qquad \widehat{\Gamma}_1 = \sqrt{\frac{q^2\kappa^2}{\left(\kappa - \dfrac{\beta_0(\mu+1)}{1+q}\right)\left(\kappa - \dfrac{\beta_0(\mu+1)}{1-q}\right)} - 1},$$

$\tilde{\varepsilon}$ is the EF permeability;

$$\Delta'_{0\mu}(\mu_0) = \frac{\varepsilon\mu_0 \left(\pi\chi \cdot \cos ec^2 \left(\pi\chi\sqrt{\varepsilon\kappa^2 - \mu_0^2} \right) + \frac{1}{\sqrt{\varepsilon\kappa^2 - \mu_0^2}} \operatorname{ctg} \left(\pi\chi\sqrt{\varepsilon\kappa^2 - \mu_0^2} \right) \right)}{\varepsilon\kappa^2 - \mu_0^2},$$

ε is the dielectric permittivity.

Equation (2) differs from the one obtained before [5] by the factor Λ and the function $\Delta'_{0\mu}(\mu_0)$, which take into account the dispersion properties of EF and the influence of dielectric layer on the amplifier characteristics.

Power equations (1) and (2) allow to analyze in detail the wave process physics for different amplifier models based on the Smith-Purcell effect.

3. EXAMPLES OF THE ANALYSIS OF THE ELECTRON-WAVE PROCESSES BY THE POWER EQUATIONS

The main aim of the analysis of power dispersion equations of type (1) and (2) consists in determination of the complex propagation coefficient μ and the range of EF velocity and other parameters of the amplifier electrodynamic system, for which the increment of the oscillation amplitude $|\operatorname{Im}\mu|$ growth will have the optimal values. The case of imaginary μ corresponds to the condition of interaction between the OW waves and the EF space-charge waves. At $\operatorname{Im}\mu < 0$ we will have the exponentially growing waves, which take energy from EF (the wave amplification regime), and at $\operatorname{Im}\mu > 0$ the electron velocity will increase due to the electromagnetic wave energy (the wave absorption regime). In general case the wave propagation of some types is possible. The first type is the surface waves of periodic structure, the presence of which is determined by imaginary values of the transverse wave numbers. The second type is the bulk waves, which correspond to regimes of diffraction radiation. The third type is the EF space-charge waves.

Analysis of dispersion equations (1) and (2) allowed to elucidate that waves with the eigen wave numbers μ, which define the direction, the value of the phase velocity, and the angle of diffraction radiation, propagate in a waveguide volume. Phase velocities of some waves coincide in direction with the EF velocity, but some of them are oppositely directed. These waves relate to the harmonics of periodic structure without EF. There are SSCW and FSCW in the system as well.

Thus, in particular, the numerical analysis of the dispersion equation (1) allows to find 5 waves (there are 7 roots and 2 of them are the complex-conjugate ones). Except of the waves with the wave numbers corresponding to the phase velocities of SSCW and FSCW, the SCW and FCW appear, which can essentially influence on the power-interchange processes with an OW wave. So, with decrease in the magnetic field value (parameter Ω_c) the values of the wave numbers of the cyclotron waves tend to μ_0, that leads to substantial decrease in the amplitude of growth increment and the interaction region of EF SSCW with the diffraction harmonic. In the vicinity of the value $\Omega_c \approx 0.01$ the OW excitation by EF is almost ceased that is illustrated in Fig. 1.

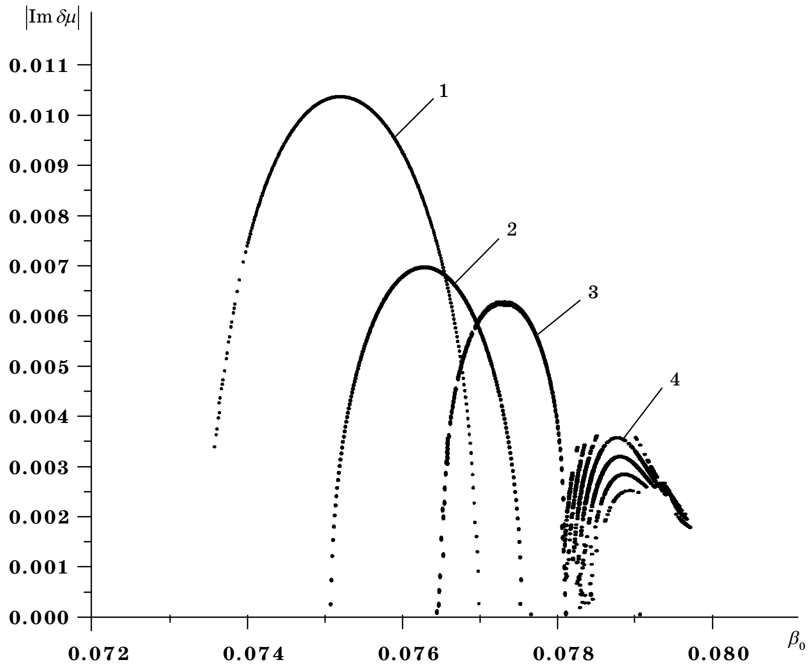

***Fig. 1** – Solution of the dispersion equation (1) at different values of the parameter* Ω_c*: 1* – $\Omega_c = 0{,}6$*; 2* – $\Omega_c = 0{,}5$*; 3* – $\Omega_c = 0{,}4$*; 4* – $\Omega_c = 0{,}025$

In some cases analysis of the dispersion equation (2) is more obvious from the point of view of the process physics. This equation does not take into account the influence of the magnetic field value ($B \to \infty$), but allows to demonstrate the limiting regimes of oscillation excitation and the "thin" radiation structure subject to the electrodynamic parameters of the system and electron beam, and the influence of additional dielectric layer on the wave processes as well [7].

Thus, divergence of the Taylor series near the wave number, which corresponds to the diffraction harmonic, allows to analyze the electron-wave processes with this wave only. In particular, in Fig. 2 we present the solution results of the cubic dispersion equation (2) without considering the magnetic field and the dielectric layer influence.

It is seen, that the given approximation allows to describe three waves with the wave numbers, which are close to the wave numbers of SSCW μ_{SSCW}, FSCW μ_{FSCW}, and the periodic structure wave μ_0. And in the regions I and II the periodic structure wave interacts with the EF space-charge waves (region I – with FSCW, region II – with SSCW). In this case the "thin" radiation structure is observed, which is conditioned by the influence of EF dispersion properties at its finite width. This is qualitatively agreed with the experimental investigation results of the EF interaction with the field diffracted on the periodic structure [8].

Analysis of equation (2) also shows that parameter χ (which is the distance between OW mirrors normalized to the grating period) has substantial influence on the wave propagation conditions. Distance change between the grating and the metal mirror leads to the change in the radiation angles and the phase velocity of the wave. As a result, the synchronism condition of EF SSCW with the diffraction harmonic is violated. This appears in the oscillation excitation region shift toward lesser β_0 and in decrease in the growth increment maximal value while χ increases. Physically this can be explained by the amplitude decrease in the reflected from mirror radiation, that influences on efficiency of EF bunching in the field of a waveguide traveling wave.

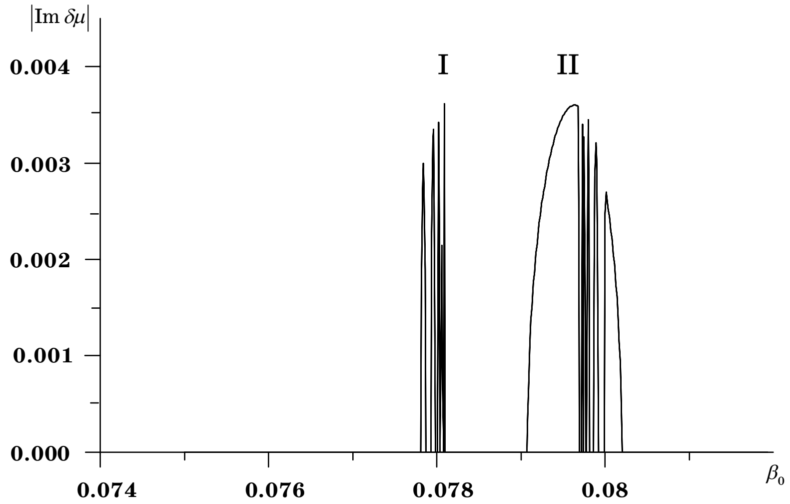

Fig. 2 *– Solution of the cubic dispersion equation (2) at* κ = 0,083, χ = 10, ζ = 3, *and* μ = 0,0524

By selection of the periodic structure parameters, in particular its period, the regimes of both bulk and surface waves can be realized. The first regime at the radiation angles, which are close to $\pi/2$, is characterized by the substantial increase in the increment of wave amplitude growth connected with the maximal energy conversion of EF to diffraction radiation. In this case the analyzed system becomes the resonant one and it does not allow to provide the wideband signal amplification having a tendency to the self-excitation. The second regime is typical for devices of the backward-wave tube (BWT) and the traveling-wave tube (TWT) types.

Established, that the EF thickness in the given approximation influences only on the amplitude value of the growth increment and on the width of interaction region of OW waves with EF over the parameter β_0. Therefore at calculation it is necessary to take into account that the mathematical model of amplifier implies the uniform distribution of the periodic structure field over the EF thickness and allows to increase it up to the distance between mirrors. At the same time the inverse proportion of the electromagnetic field penetration into EF versus the frequency is known. In fact, the bottom layer of EF only will interact with electromagnetic field. Optimal value of the EF thickness, according to [1], is determined by the formula $r = 0{,}19\beta_0\lambda$ that corresponds to the values r = 0,1-0,2 mm in the millimeter-wave range.

Introducing the additional dielectric layer, which can execute the input-output energy function [7], into the amplifier electrodynamic system, the wave process physics becomes more complicated. In particular, established that appearance of dielectric layer between EF and metal screen with small ε leads to additional wave generation with the values of propagation coefficient μ close to $\varepsilon = 1$. Further growth of ε values leads to increase in the number of waves propagating in OW, to change in their phase velocities and radiation angles. All waves satisfy the condition of the bulk wave propagation in dielectric layer. Increase in the number of waves in OW leads to the energy redistribution between them that is appeared in substantial decrease in the increment amplitude values of individual waves.

4. CONCLUSIONS

1. We present the transformation technique of the general transcendental dispersion equation of amplifier based on the Smith-Purcell effect to the power third- and seventh-order equations, which within the given approximations allow to visually analyze the wave process physics at the interaction between EF and OW wave.
2. Influence degree of cyclotron waves on the conditions of oscillation excitation in OW is established. Shown, that at the value of the cyclotron parameter $\Omega_c \approx 0{,}01$ the OW excitation by EF is almost ceased.
3. By the examples of the electron-wave processes shown, that the amplifier model, characteristics of which are described by the dispersion third-order equation (without considering the magnetic field influence), which takes into account the dispersion properties of EF and dielectric layer placed in a waveguide volume, is the most evident one from the physical point of view.

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