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Theory of Spin Waves in a Circular Nanotube Composed of an Easy-Plane Ferromagnet. Consideration of the Dissipation for a Non-metallic External Medium

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The paper extends study of dipole-exchange spin waves in a circular nanotube composed of an easy-plane ferromagnet started by the author in the previous paper. The proposed model considers the magnetic dipole-dipole interaction, the exchange interaction, the anisotropy effects, the damping effects and the general boundary conditions. An equation for the magnetic potential has been obtained for such waves and solved for the case of longitudinal-radial waves. As a result, the dispersion law for the investigated waves has been obtained. After implying boundary conditions, this dispersion law has been complemented with the relation between the wave vector components. This relation has been shown to degenerate into a quasi-one-dimensional values' spectrum of the orthogonal wave vector component for a thin nanotube. For the obtained spectral characteristics, graphical representations have been given and numerical evaluations have been performed. The spin wave frequency (calculated according to the obtained dispersion law) for typical values of the nanotube parameters corresponds to typical values observed in experiments, thus substantiating the obtained results. Comparative analysis of the dispersion law obtained in the paper and the analogous law for a nanotube composed of an easy-axis ferromagnet has been performed; differences and similarities have been outlined. It has been shown that branches (that correspond to different orthogonal modes) of the dependencies on the longitudinal wave vector components for both real and imaginary parts of the spin wave frequency are close to parabolic and are essentially apart from each other. The area of application of the obtained results is essentially extended compared to the previous paper. The method that is proposed in the paper can be applied to nanotubes of more complex configurations.

Keywords: Spin wave, Nanomagnetism, Dipole-exchange theory, Ferromagnetic nanotube, Easy-plane ferromagnet.

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1. INTRODUCTION

Magnetic dynamics of nanosystems become an actual and promising topic of research recently. In particular, spin waves in nanoscale systems are promicing for a variety of technical applications – both current and prospective – in different fields of technology. Most applications concern new devices for data storage, transfer and processing [1-3]. Magnonic devices allow for faster, more efficient, and more reliable signal processing and computation on higher frequencies than current computer technology [1]. Spin waves can propagate through magnetic materials with minimal energy loss and can be easily manipulated using magnetic fields, electric fields, spin currents, or thermal gradients thus making them prospective for novel data transfer technologies [1]. These applications require precise theoretical models of excitation and propagation of spin waves in various nanosystems, so these models are extensively developed recently.

Magnetic – in particular, spin-wave – properties of nanostructures depend essentially on their size and shape. Therefore, spin waves have been studied in different configurations of nanosystems individually. Synthesized recently magnetic nanotubes [4, 5] have found a wide range of technical applications – in particular, in magnetobiology. However, spin waves in nanotubes currently they attract little attention. Known theoretical papers on the subject investigate mostly spin solitons [6] and waves on magnetic domains interfaces [7, 8].

During research of spin waves in nanosystems, usually either isotropic or uniaxial easy-axis ferromagnets are considered as media for waves propagation. Uniaxial easy-plane ferromagnets possess a number of unique magnetic properties – in particular, due to a different degree of symmetry compared to similar systems composed of easy-axis ferromagnets. However, spin waves in nanosystems (in particular, nanotubes) composed of easy-plane ferromagnets currently remain poorly studied.

It is known that the effects associated with an energy dissipation can either significantly influence the pattern of spin waves in the system or be negligibly small (depending on the wave frequency, dimensions, shape and material of the system and other factors), see e.g., [9]. Therefore, study of spin waves in nanosystems in general case require taking into account dissipative effects.

The paper extends theoretical study of dipole-exchange spin waves in a circular nanotube composed of a uniaxial easy-plane ferromagnet started by the author in the previous paper [10]. The magnetic dipole-dipole interaction, the exchange interaction and the anisotropy effects are considered. Unlike in the previous paper, dissipation of the spin wave is taken into account and the external medium is not limited to a specific particular case of a non-magnetic high-conductivity metal. As a result, the dispersion law and the relation between the wave vector components for such waves are obtained and analyzed.

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2. SYSTEM AND MODEL DESCRIPTION. STARTING RELATIONS

2.1 Problem Statement. Model Description

Analogously to the previous paper [10] let us consider a ferromagnetic nanotube with inner and outer radii a and b, correspondingly, and with the medium outside the nanotube being non-magnetic. The ferromagnet is assumed to be uniaxial easy-plane type with its anisotropy axis directed along the axis of symmetry of the nanotube (the vector $\vec{n}$ on the Fig. 1), and the Oz axis of the coordinate system is also co-directed with $\vec{n}$. The ferromagnet parameters are denoted as follows: the exchange constant α, the uniaxial anisotropy parameter $\beta < 0$ (is considered constant), the gyromagnetic ratio γ (is considered constant). Unlike in the previous paper, let us consider the spin wave dissipation non-negligible and introduce the Gilbert damping constant of the ferromagnet α_G. The saturation magnetization $\vec{M}_0$ of the ferromagnet is assumed to be directed radially and have constant length in the entire volume of the tube. All components (in the cylindrical coordinate system) of the external magnetic field $\vec{H}^{(e)}$ are assumed to be stationary and homogeneous.

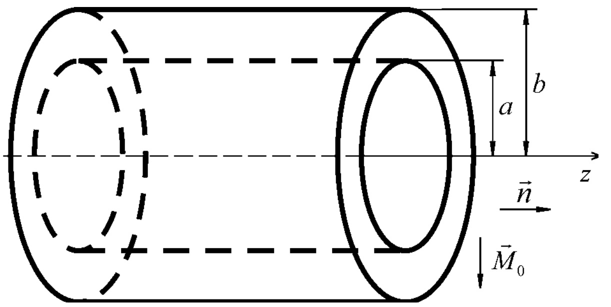

Fig. 1 – Ferromagnetic nanotube considered in the paper

Let us consider a spin wave propagating in the above-described nanotube and take into account both the magnetic dipole-dipole and exchange interactions (as they both are essential for a nanoscale system) as well as the anisotropy and damping in the Landau-Lifshitz equation. The wave is considered linear so the magnetization $\vec{m}$ and the magnetic field $\vec{h}$ of the wave are small perturbations of the overall magnetization $\vec{M}$ and the internal magnetic field (inside the ferromagnet) $\vec{H}^{(i)}$, correspondingly ($\vec{M} = \vec{M}_0 + \vec{m}$, $\vec{H}^{(i)} = \vec{H}_0^{(i)} + \vec{h}$, where $\vec{H}_0^{(i)}$ is the ground state internal magnetic field). Thus, the inequalities $|\vec{m}| << |\vec{M}_0|$, $|\vec{h}| << |\vec{H}_0^{(i)}|$ fulfill. The task of the paper is to obtain the dispersion relation for such wave and values' spectrum for the wave vector components.

2.2 Starting Relations

Let us introduce the cylindrical coordinate system (ρ,θ,z) and assume that the saturation magnetization $\vec{M}_0$ is directed radially: $\vec{M}_0 = M_0\vec{e}_\rho$, M_0=const, here $\vec{e}_\rho$ is a unit vector for the coordinate ρ. Then, a non-dissipative linearized Landau-Lifshitz equation (see, e.g., [11]) for the considered nanotube can be written as follows:

$$\frac{\partial \vec{m}}{\partial t} = \gamma\left(M_0\vec{e}_\rho \times \left(\vec{h} + \alpha\Delta\vec{m} + \beta m_z\vec{e}_z - \frac{H_0^{(i)}}{M_0}\vec{m}\right)\right), \quad (1)$$

here $\vec{e}_z$ is a unit vector for the coordinate z. In order to consider the dissipation effects, let use a damping term in the Gilbert form in the Landau-Lifshitz equation ($\vec{t}_G = \alpha_G\left[\vec{M}_0 \times \frac{\partial \vec{m}}{\partial t}\right]$ in the linearized form). Then, the Landau-Lifshitz equation (1) can be rewritten as follows:

$$\frac{\partial \vec{m}}{\partial t} = \gamma\left(M_0\vec{e}_\rho \times \left(\vec{h} + \alpha\Delta\vec{m} + \beta m_z\vec{e}_z - \frac{H_0^{(i)}}{M_0}\vec{m} + \frac{\alpha_G}{\gamma M_0}\frac{\partial \vec{m}}{\partial t}\right)\right). \quad (2)$$

For the perturbations of the magnetization and magnetic field in a form of the travelling waves

$$\vec{m} = \vec{m}_0(\vec{r})\exp(i\omega t) = \vec{m}_{\perp 0}(\rho,\theta)\exp(i\omega t - ik_{||}z),$$

$$\vec{h} = \vec{h}_0(\vec{r})\exp(i\omega t) = \vec{h}_{\perp 0}(\rho,\theta)\exp(i\omega t - ik_{||}z),$$

where ω is the spin wave frequency and $k_{||}$ is the longitudinal wave number, the equation (2) can be rewritten as follows:

$$i\omega\vec{m}_0 = \gamma\left(M_0\vec{e}_\rho \times \left(\vec{h}_0 + \alpha\Delta\vec{m}_0 + \beta m_{0z}\vec{e}_z - \frac{H_0^{(i)}}{M_0}\vec{m}_0 + i\omega\frac{\alpha_G}{\gamma M_0}\vec{m}_0\right)\right)$$

This equation combined with the Maxwell equation $div\vec{h} = -4\pi div\vec{m}$ forms a system of equations in which the spin wave magnetization vector can be eliminated. Let us use the magnetostatic approximation (that can be applied for typical spin waves, see, e.g., [11]) and introduce the magnetic potential $\Phi(\vec{r},t) = \Phi_0(\vec{r})\exp(i\omega t) = \Phi_{\perp 0}(\rho,\theta)\exp(i\omega t - ik_{||}z)$, so for the magnetic field the relations $\vec{h} = -\vec{\nabla}\Phi$, $\vec{h}_0 = -\vec{\nabla}\Phi_0$ fulfill. Then, after the above-mentioned elimination of the magnetization from the system of equations one can obtain the following equation for the magnetic potential:

$$\left(\frac{\omega^2}{\gamma^2 M_0^2} - \left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + i\omega\frac{\alpha_G}{\gamma M_0}\right)\left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + i\omega\frac{\alpha_G}{\gamma M_0} - 4\pi\right)\right) \times$$
$$\times\Delta\Phi_0 - 4\pi\left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + i\omega\frac{\alpha_G}{\gamma M_0}\right)\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi_0}{\partial\rho}\right) + \frac{\omega\beta k_{||}}{\gamma M_0\rho}\frac{\partial\Phi_0}{\partial\theta} + \quad (3)$$
$$+\beta k_{||}^2\left(\alpha\Delta - \frac{H_0^{(i)}}{M_0} + i\omega\frac{\alpha_G}{\gamma M_0}\right)\Phi_0 = 0$$

If the nanotube is thin ($(b–a)/a<<1$), for the internal magnetic field the relation for a flat film fulfills approximately: $\vec{H}_0^{(i)} \approx \vec{H}^{(e)} - 4\pi\vec{M}_0$. In the opposite case of a thick nanotube ($b–a$~b) the relation for a continuous cylinder fulfills approximately: $\vec{H}_0^{(i)} \approx \vec{H}^{(e)} - 2\pi\vec{M}_0$. In particular, if the external magnetic field is directed along the tube axis analogously to the previous papers by the author [12,13] that consider nanotubes made of an easy-axis ferromagnet, one can obtain $H_0^{(i)} \approx \sqrt{\left(H^{(e)}\right)^2 + \left(4\pi M_0\right)^2}$ in the first case (thin nanotube) and $H_0^{(i)} \approx \sqrt{\left(H^{(e)}\right)^2 + \left(2\pi M_0\right)^2}$ in the second case (thick nanotube).

3. SPECTRAL CHARACTERISTICS OF THE SPIN WAVES

3.1 Dispersion Relation

Unlike the case of nanotubes composed of an easy-axis ferromagnet [12, 13], for the case of an easy-plane ferromagnet in general case it is not possible to seek a solution of the Eq. (3) in the form of a linear combination of cylindrical functions because of the presence of the derivatives $\frac{\partial \Phi_0}{\partial \varphi}, \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Phi_0}{\partial \rho}\right)$. However, it becomes possible for the particular case of the absence of angular oscillations. Therefore, let us consider the case of longitudinal-radial waves for which, in particular, the relation $\partial\Phi_0/\partial\theta=0$ fulfills.

In such case, a solution of the Eq. (3) can be sought in the following form:

$$\Phi_0 = \left(A_1 J_0(k_\perp \rho) + A_2 N_0(k_\perp \rho)\right)\exp\left(-ik_{||} z\right), \tag{4}$$

here A_1 and A_2 are constants, J_n and N_n are the Bessel and Neumann functions of order n, correspondingly, $k_\perp$ is the transverse wave number. In the considered case n – the transverse-angular oscillatory mode number – can only be equal to 0 (zero transverse-angular mode). After substituting the solution (4) into the Eq. (3) one can obtain the following dispersion relation:

$$\omega = \frac{\gamma M_0}{2\left(1+\alpha_G^2\right)}\left(-i\alpha_G\left(2\alpha k^2 + 2\frac{H_0^{(i)}}{M_0} + \left(4\pi + |\beta|\right)\frac{k_{||}^2}{k^2}\right) \pm \right.$$
$$\left. \pm\sqrt{-\alpha_G^2\left(2\alpha k^2 + 2\frac{H_0^{(i)}}{M_0} + \left(4\pi+|\beta|\right)\frac{k_{||}^2}{k^2}\right)^2 + 4\left(1+\alpha_G^2\right)\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0}\right)\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0} + \left(4\pi+|\beta|\right)\frac{k_{||}^2}{k^2}\right)}\right),$$

here $k = \sqrt{k_{||}^2 + k_\perp^2}$ is the total wavenumber.

Let us note that spin waves can be excited only when the damping parameter is small: $\alpha_G \leq 0.1$. Therefore, the dispersion relation can be rewritten approximately in the following simplified form (the root with the positive real part):

$$\omega \approx \gamma M_0\left(\sqrt{\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0}\right)\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0} + \left(4\pi+|\beta|\right)\frac{k_{||}^2}{k^2}\right)} - \right.$$
$$\left. -i\alpha_G\left(\alpha k^2 + \frac{H_0^{(i)}}{M_0} + \frac{1}{2}\left(4\pi+|\beta|\right)\frac{k_{||}^2}{k^2}\right)\right) \tag{5}$$

The last equation represents the sought dispersion relation for longitudinal-radial spin waves in the investigated nanotube.

3.2 Relation Between the Wave Vector Components

The obtained dispersion relation (5) includes two components of the wave vector, longitudinal and transverse. Then, for more complete specification of the spin wave pattern this relation must be supplemented by either a spectrum of values of at least one of these components or a relation between them.

Unlike in the previous paper [10], let us not limit the investigation to a specific particular case of the external media being a non-magnetic high-conductivity metal. Instead, let us use more general considerations.

If the investigated nanotube is thin compared to its radius ($(b - a) / a \ll 1$), radial spin excitations – that can be considered standing waves – become quasi-one-dimensional. Therefore. the spectrum of transverse wave vector component

$$k_\perp = \frac{\pi s}{b-a}, \tag{6}$$

where s is an integer (number of the transverse mode). In order to obtain more precise expression for the spectrum of values of the transverse wave vector component or the relation between the wave vector components one should solve the equation for the magnetic potential (3) both inside the ferromagnet and in the external media and match the obtained solutions using boundary conditions.

Standard boundary conditions for the magnetic field $b_{1n} = b_{2n}$, $h_{1\tau} = h_{2\tau}$ (here $\vec{b}$ is the wave magnetic induction vector, medium 1 is the investigated ferromagnet, 2 is the external medium, n means the normal and τ – the tangential to the interface components of the vector) yield the following condition for the vector $\vec{h}$: $h_{1n} - h_{2n} = 4\pi m_n$, $h_{1\tau} = h_{2\tau}$, or

$$\begin{cases} \Phi_0\big|_1 = \Phi_0\big|_2 \\ \left(\nabla\Phi_0\right)_{1\tau} = \left(\nabla\Phi_0\right)_{2\tau} \\ \left.\frac{\partial\Phi_0}{\partial n}\right|_1 - \left.\frac{\partial\Phi_0}{\partial n}\right|_2 = 4\pi m_{0n} \end{cases} \tag{7}$$

for the magnetic potential. As the saturation magnetization is directed radially, the magnetization of the wave has the radial component $m_{0\rho} = 0$ and, therefore, the third condition in (7) can be rewritten as

$$\left(\partial\Phi_0/\partial n\right)\big|_1-\left(\partial\Phi_0/\partial n\right)\big|_2=\left(\partial\Phi_0/\partial\rho\right)\big|_1-\left(\partial\Phi_0/\partial\rho\right)\big|_2=0 .$$

As the external medium is non-magnetic, outside the ferromagnet the Laplace equation for the magnetic potential fulfills: $\Delta\Phi^e=0$, $\Delta\Phi_0^e=0$. Its solution (for the longitudinal-radial waves) that is bounded both on the tube axis and at infinity can be written as follows:

$$\Phi_0^e=\begin{cases}A_1^e I_0(k_{||}\rho)\exp(ik_{||}z), & \rho\le a\\ A_2^e K_0(k_{||}\rho)\exp(ik_{||}z), & \rho>b\end{cases}$$

here A_1^e, A_2^e are constants, I_0, K_0 are the modified Bessel and Neumann functions of order 0, correspondingly.

After substituting the above-written external magnetic potential and (4) into (7) one can obtain the sought relation between $k_{||}$ and $k_\perp$ (in an implicit form) as follows:

$$\left(k_\perp J_0'(k_\perp a)-\frac{k_{||}I_0'(k_{||}a)}{I_0(k_{||}a)}J_0(k_\perp a)\right)\left(\frac{k_{||}K_0'(k_{||}b)}{K_0(k_{||}b)}N_0(k_\perp b)-k_\perp N_0'(k_\perp b)\right)=$$
$$=\left(k_\perp J_0'(k_\perp b)-\frac{k_{||}K_0'(k_{||}b)}{K_0(k_{||}b)}J_0(k_\perp b)\right)\left(N_0(k_\perp a)\frac{k_{||}I_0'(k_{||}a)}{I_0(k_{||}a)}-k_\perp N_0'(k_\perp a)\right) \quad (8)$$

For a thin nanotube ($(b-a)/a\ll 1$) using the Bessel and Neumann functions' asymptotics allows to simplify the above-obtained implicit relation between $k_{||}$ and $k_\perp$ as follows:

$$tg\left(k_\perp(b-a)\right)=\frac{\dfrac{k_{||}}{k_\perp}\left(\dfrac{I_0'(k_{||}a)}{I_0(k_{||}a)}-\dfrac{K_0'(k_{||}b)}{K_0(k_{||}b)}\right)}{1+\left(\dfrac{1}{2k_\perp a}+\dfrac{k_{||}}{k_\perp}\dfrac{I_0'(k_{||}a)}{I_0(k_{||}a)}\right)\left(\dfrac{1}{2k_\perp b}+\dfrac{k_{||}}{k_\perp}\dfrac{K_0'(k_{||}b)}{K_0(k_{||}b)}\right)} \quad (9)$$

The component $k_{||}$ has the order of magnitude of the reciprocal nanotube length or more and the component $k_\perp$ has the order of magnitude of the reciprocal nanotube thickness. Therefore, on the most part of the ranges of values of the components $k_{||}$ and $k_\perp$ the relations $k_\perp a\gg 1$, $k_\perp b\gg 1$, $k_{||}\ll k_\perp$ fulfill. Analysis shows that when these relations fulfill simultaneously, the right hand part of the relation (9) becomes negligibly small. Therefore, nearly everywhere the relation (9) degenerates into the relation $\mathrm{tg}(k_\perp(b-a))=0$ that corresponds to the quasi-one-dimensional spectrum (6) for the component $k_\perp$.

Thus, the sought dispersion relation for the investigated spin waves can be written in the form (5) with $k^2=k_{||}^2+k_\perp^2$. The longitudinal wave vector component $k_{||}$ can be considered to change continuously while the orthogonal wave vector component $k_\perp$ is defined by the implicit relation (8). If the nanotube is thin ($(b-a)/a\ll 1$), this relation can be reduced to (9) that, in turn, can be reduced to quasi-one-dimensional spectrum (6) for $k_\perp$ nearly everywhere.

4. DISCUSSION

4.1 Comparison of the Results for Easy-Plane and Easy-Axis Ferromagnets

Let us compare the above-obtained dispersion relation for the easy-plane ferromagnet with the dispersion relation for the easy-axis ferromagnet (obtained in the previous paper of the author [13]).

Comparison of the real parts of the spin wave frequency has been already performed in the previous paper of the author [10]: these real parts for both easy-plane and easy-axis ferromagnet can be written in the form

$$\mathrm{Re}\,\omega=\gamma M_0\sqrt{\left(h_c+\alpha k^2\right)\left(h_c+\alpha k^2+F_1\left(k_{||}^2/k^2\right)\right)}$$

with $h_c=H_0^{(i)}/M_0$ for the easy-plane ferromagnet and $h_c=H_0^{(i)}/M_0+\beta=H^{(e)}/M_0+\beta$ for the easy-axis ferromagnet. The function $F_1\left(k_{||}^2/k^2\right)$ has different form for the two ferromagnet types; however, it is linear for both of them.

Let us now compare the imaginary parts of the frequency (that define the spin wave damping). Imaginary part of the dispersion relation (5) has the following form:

$$\mathrm{Im}\,\omega=-|\gamma|M_0\alpha_G\left(\alpha k^2+\frac{H_0^{(i)}}{M_0}+\frac{1}{2}\left(4\pi+|\beta|\right)\frac{k_{||}^2}{k^2}\right),$$

or

$$\mathrm{Im}\,\omega=-|\gamma|M_0\alpha_G\left(\alpha k^2+\frac{H_0^{(e)}}{M_0}-2\pi+\frac{1}{2}\left(4\pi+|\beta|\right)\frac{k_{||}^2}{k^2}\right)=$$
$$=-|\gamma|M_0\alpha_G\left(\alpha k^2+\frac{H_0^{(e)}}{M_0}+\frac{|\beta|}{2}-\left(2\pi+\frac{|\beta|}{2}\right)\frac{k_\perp^2}{k^2}\right)$$

for the case $(b-a)/a\ll 1$. For the easy-axis ferromagnet (see [12, 13])

$$\mathrm{Im}\,\omega=-|\gamma|M_0\alpha_G\left(\alpha k^2+\beta+\frac{H_0^{(i)}}{M_0}+2\pi\frac{k_\perp^2}{k^2}\right)=$$
$$=-|\gamma|M_0\alpha_G\left(\alpha k^2+\beta+\frac{H^{(e)}}{M_0}+2\pi\left(1-\frac{k_{||}^2}{k^2}\right)\right).$$

Therefore, the expressions for both easy-plane and easy-axis ferromagnets can be written in the form

$$\mathrm{Im}\,\omega=-|\gamma|M_0\alpha_G\left(\alpha k^2+h_c+F_2\left(\frac{k_{||}^2}{k^2}\right)\right).$$

Analogously to the real parts of the dispersion relation, the function $F_2\left(k_{||}^2/k^2\right)$ has different form for the two ferromagnet types but is linear for both of them.

Alternatively, for the case $(b-a)/\alpha << 1$ both imaginary parts can be expressed through the external magnetic field in the following way:

$$\mathrm{Im}\,\omega = -|\gamma| M_0 \alpha_G \left(\alpha k^2 + \frac{H_0^{(e)}}{M_0} + F_3\left(\frac{k_\perp^2}{k^2}\right)\right),$$

where the function F_3 – similarly to F_1, F_2 – has different form for the two ferromagnet types but is linear for both of them.

4.2 Analysis of the Spin Wave Spectral Characteristics

Now, let us analyze the obtained spectral characteristics themselves.

First, let us note that the relation between the wave vector components – in either of the forms (8), (9) or (6) – is similar to the analogous relations for an easy-axis ferromagnet [12] after limiting the mode number n to 0.

Graphical representation of the relation (9) can be seen on the Fig. 2.

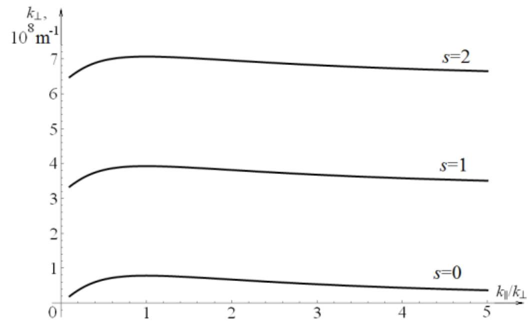

Fig. 2 – Dependence of $k_\perp$ on $k_{||}/k_\perp$ for the investigated nanotube with the radii $a = 50\ nm$, $b = 60\ nm$

As it can be seen from the Fig. 2, the relation between the wave vector components, really, is close to the quasi-one-dimensional spectrum of values of the component $k_\perp$ (6). This regularity becomes more pronounced not only for $k_{||} << k_\perp$ (the fact that has been mentioned earlier), but also for $k_\perp << k_{||}$.

Dependence of the real part of the spin wave frequency (given by the dispersion relation (5)) on $k_{||}$ for typical values of the nanotube parameters and with the spectrum of the transverse wave vector component in the form (6) is represented on the Fig. 3.

Dependence of the imaginary part of the spin wave frequency (given by the dispersion relation (5)) on $k_{||}$ for typical values of the nanotube parameters and with the spectrum of the transverse wave vector component in the form (6) is represented on the Fig. 4.

As it can be seen from the Figs. 3, 4, the spin wave damping, really, becomes small ($\mathrm{Im}\omega << \mathrm{Re}\omega$) for $\alpha_G \sim 0.1$ – and, therefore, for $\alpha_G \leq 0.1$. Branches of both dependencies ($\mathrm{Re}\omega(k_{||})$ and $\mathrm{Im}\omega(k_{||})$) that correspond to different orthoginal mode number s are essentially apart from each other. According to the relation (5), each branch for both Imω and Reω should be close to parabolic; this regularity can be observed on both Figs. 3, 4.

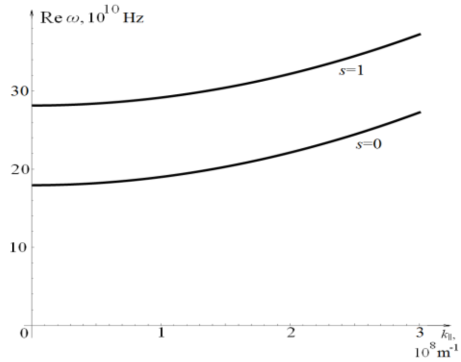

Fig. 3 – Dependence of Reω on $k_{||}$ for the investigated nanotube with the thickness $b - a = 10\ nm$ and the ferromagnet parameters $\alpha = 10^{-12}$ cm^{-2}, $\beta = -1$, $\gamma = 10^7\ Gs/Hz$, $M_0 = 10^3\ Gs$

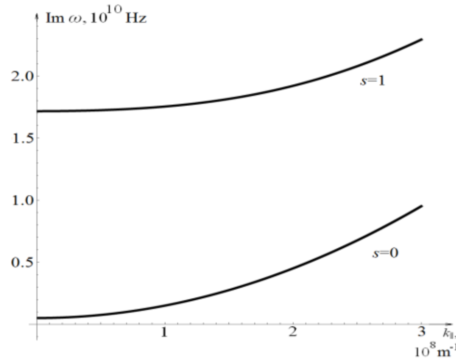

Fig. 4 – Dependence of Imω on $k_{||}$ for the investigated nanotube with the thickness $b - a = 10\ nm$ and the ferromagnet parameters $\alpha = 10^{-12}$ cm^{-2}, $\beta = -1$, $\gamma = 10^7\ Gs/Hz$, $M_0 = 10^3$ Gs, $\alpha_G = 0.1$

Finally, let us carry out numerical evaluations for spin wave frequency (both real and imaginary parts) given by (5) in the absence of the external field. The longitudinal wavenumber is restricted, on the one hand, by the nanotube length – unities of micrometers for typical nanotubes – and, on the other hand, by the interatomic distance – several angstroms for typical materials. After substitution of the typical values of nanotube parameters used for the graphs ($b - a = 10\ nm$, $\alpha = 10^{-12}$ cm^{-2}, $\beta = -1$, $\gamma = 10^7\ Gs/Hz$, $M_0 = 10^3$ Gs, $\alpha_G = 0.1$, k and $k_{||}$ vary from 10^2 cm^{-1} to 10^8 cm^{-1}) one can find that the real part of the frequency has the order of magnitude $10^{11} - 10^{12}$ Hz over the entire range of wave numbers (that corresponds to the typical spin waves' frequencies). The imaginary part, correspondingly, has the order of magnitude $10^9 - 10^{10}$ Hz (so the characteristic damping time is $10^{-9} - 10^{-8}$ s).

5. CONCLUSIONS

Thus, dipole-exchange spin waves in a circular nanotube composed of an easy-plane ferromagnet have been investigated in the paper. The magnetic dipole-dipole interaction, the exchange interaction, the anisotropy effects and (unlike in the previous paper) dissipation of the spin wave are taken into consideration. An equation for the magnetic potential has been obtained for such waves and solved for the case of longitudinal-radial waves. As a result, the dispersion law for the investigated waves has been obtained. After implying boundary conditions, this dispersion law has been complemented with the relation between the wave vector components. (Unlike in the previous paper of the author [10], general boundary conditions for the magnetic field have been used, so area of application of the obtained results is not limited to a specific particular case of a high-conductivity metal outside the nanotube.) This relation has been shown to degenerate into a quasi-one-dimensional values' spectrum of the orthogonal wave vector component for a thin nanotube.

For the obtained spectral characteristics, graphical representations have been given and numerical evaluations have been performed. The resulting spin wave frequency for typical values of the nanotube parameters corresponds to typical values observed in experiments, thus substantiating the obtained results. Comparative analysis of the dispersion relation obtained in the paper and the analogous dispersion relation for a nanotube composed of an easy-axis ferromagnet has been performed; differences and similarities have been outlined.

It has been shown that branches of the dependencies on the longitudinal wave vector components for both real and imaginary parts of the spin wave frequency are close to parabolic and are essentially apart from each other. (Different branches correspond to different orthogonal modes.)

The method proposed in the paper can be applied to nanotubes of more complex configurations – for instance, with an elliptic cross-section – as well as for more complex configurations of tube-type nanosystems in general. However, one have to bear in mind that for some configurations additional conditions (for instance, fixed boundary conditions for the magnetization) are required.

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Теорія спінових хвиль у коловій нанотрубці з легкоплощинного феромагнетику. Врахування дисипації для неметалічного зовнішнього середовища

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В роботі продовжується дослідження дипольно-обмінних спінових хвиль у коловій нанотрубці з легкоплощинного феромагнетику, розпочате автором у попередній статті. Запропонована модель враховує магнітну диполь-дипольну взаємодію, обмінну взаємодію, ефекти анізотропії, ефекти згасання та загальні граничні умови. Отримано рівняння для магнітного потенціалу зазначених хвиль; рівняння розв'язано для випадку поздовжньо-радіальних хвиль. Як результат, отримано закон дисперсії для досліджуваних хвиль. Після накладання граничних умов зазначений закон дисперсії доповнено співвідношенням між компонентами хвильового вектору. Показано, що для тонкої нанотрубки це співвідношення вироджується в квазіодновимірний спектр значень поперечних хвильових чисел. Для отриманих спектральних характеристик представлено графічне відображення та зроблено числові оцінки. Частота спінової хвилі (розрахована відповідно до отриманого закону дисперсії) для типових значень параметрів нанотрубок відповідає типовим значенням, що спостерігаються в експериментах – що підтверджує отримані результати. Проведено порівняльний аналіз закону дисперсії, отриманого в роботі, з аналогічним законом для нанотрубки з легкоосьового феромагнетику; окреслено відмінності та подібності. Показано, що гілки залежностей (які відповідають різним поперечним модам) як дійсної, так і уявної частин частоти спінової хвилі від поздовжньої компоненти хвильового вектора близькі до параболічних і суттєво віддалені одна від одної. Область застосування отриманих результатів є суттєво ширшою порівняно з попередньою роботою. Запропонований у роботі метод може бути застосований до нанотрубок більш складної конфігурації.

Ключові слова: Спінова хвиля, Наномагнетизм, Дипольно-обмінна теорія, Феромагнітна нанотрубка, Легкоплощинний феромагнетик.