The Exact Nonrelativistic Energy Eigenvalues for Modified Inversely Quadratic Yukawa Potential Plus Mie-type Potential

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The modified theories of noncommutative quantum mechanics have engrossed much attention in the last decade, especially its application to the fundamental three equations: Schrödinger, Klein-Gordon and Dirac equations. In this contextual exploration, we further investigate for modified quadratic Yukawa potential plus Mie-type potential (*MIQYM*) in the framework of modified nonrelativistic Schrödinger equation (MSE) using generalization of Bopp's shift method and standard perturbation theory instead of using directly the generalized Moyal-Weyl product method, we obtained modified energy eigenvalues and corresponding modified anisotropic Hamiltonian operator in both three dimensional noncommutative space and phase (NC-3D: RSP) symmetries.

Keywords: Inversely quadratic Yukawa potential plus Mie-type potential, Noncommutative space, Noncommutative phase and Bopp's shift method.

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1. INTRODUCTION

The exact analytic solutions of an arbitrary, or multidimensional, nonrelativistic Schrödinger equation and relativistic Klein-Gordon and Dirac equations has been the principal area of research interest in the field of quantum mechanics, many authors have used different methods like asymptotic iteration method, improved AIM, Laplace integral transform, factorization method, proper quantization rule and exact quantization rule, Nikiforov–Uvarov method , supersymmetry quantum mechanics in two, three and *D*-dimensional spaces to study the central and non central potentials [1-9]. The algebraic physical structure of ordinary quantum mechanics based on the following fundamental three canonical commutations relations (CCRs), which plays as fundamental postulates of quantum mechanics, $\left[x_i, p_j\right]$, $\left[x_i,x_j\right]$ and $\left[p_i,p_j\right]$, in both Schrödinger and Heisenberg pictures, respectively, as $\left(c=\hbar=1\right)$:

$$\left[x_i, p_j\right]=i\delta_{ij} \quad \text{and} \quad \left[x_i, x_j\right]=\left[p_i, p_j\right]=0 \tag{1}$$

and

$$\begin{gathered}\left[x_i\left(t\right),p_j\left(t\right)\right]=i\delta_{ij} \quad \text{and} \\ \left[x_i\left(t\right),x_j\left(t\right)\right]=\left[p_i\left(t\right),p_j\left(t\right)\right]=0\end{gathered} \tag{2}$$

Furthermore, the two timely operators $x_i\left(t\right)$ and $p_i\left(t\right)$ are determined from the projection relations:

$$\begin{gathered}x_i\left(t\right)=\exp(iH\left(t-t_0\right))x_i\exp(-iH\left(t-t_0\right)) \\ p_i\left(t\right)=\exp(iH\left(t-t_0\right))p_i\exp(-iH\left(t-t_0\right))\end{gathered} \tag{3}$$

Here H denote to the Hermitian Hamiltonian operators on a Hilbert space of physical states, by differentiating eq. (3), we find the Heisenberg equation of motions:

$$\frac{dx_i\left(t\right)}{dt}=i\left[H,x_i\left(t\right)\right] \quad \text{and} \quad \frac{dp_i\left(t\right)}{dt}=i\left[H,p_i\left(t\right)\right] \tag{4}$$

Recently, theoretical physicists have shown a great deal of interest in solving two and three-dimensional nonrelativistic Schrödinger equation and relativistic Klein-Gordon and Dirac equations for various spherically symmetric potentials in the case of new structure of quantum mechanics namely noncommutative quantum mechanics, which know firstly by H. Snyder [6], to obtain an profound physical interpretations in the microscopic scales. In this recently work we attempt to investigate the problem of the generalized (*IQYM*) potential within the framework of the (*MSE*) in (*NC-3D: RSP*) symmetries with the interaction of modified Yukawa potential and modified Mie-type potential using both Bopp's shift method and standard perturbation theory methods:

$$\begin{gathered}V_{iqym}\left(\hat{r}\right)=\frac{B-V_0}{r^2}+\frac{2V_0\delta-A}{r}+\left(C-2V_0\delta^2\right)+ \\ +\left[\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right]\vec{\mathbf{L}}\vec{\Theta}+\frac{\vec{\mathbf{L}}\bar{\vec{\theta}}}{2\mu}\end{gathered} \tag{5}$$

On based to the work of the author B.I. Ita [9] to the case of the noncommutative space and space phase in addition to the our previously works [11-23]. It is worth to mentioning that the (CCRs) will be changes in noncommutative three dimensional spaces and phases to the new canonical commutations relations (NCCRs), in both Schrödinger (*SP*) and Heisenberg (*HP*) pictures, as follows [7-11]:

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$$\left[\hat{x}_i \overset{*}{,} \hat{p}_j\right] = i\delta_{ij}, \left[\hat{x}_i \overset{*}{,} \hat{x}_j\right] = i\theta_{ij} \text{ and } \left[\hat{p}_i \overset{*}{,} \hat{p}_j\right] = i\bar{\theta}_{ij}$$
$$\left[\hat{x}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\delta_{ij}, \left[\hat{x}_i(t) \overset{*}{,} \hat{x}_j(t)\right] = i\theta_{ij} \text{ and} \quad (6)$$
$$\left[\hat{p}_i(t) \overset{*}{,} \hat{p}_j(t)\right] = i\bar{\theta}_{ij}$$

The very small two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$ (compared to the energy) are elements of two antisymmetric real matrixes and $(*)$ denote to the new star product (the generalized Moyal-Weyl product), which is generalized between two arbitrary functions $f(x,p)$ and $g(x,p)$ to $(f*g)(x,p)$, in the first order of two parameters $\theta^{\mu\nu}$ and $\bar{\theta}^{\mu\nu}$,instead of the old product $(fg)(x,p)$ [12]:

$$(f*g)(x,p) = \left(fg - \frac{i}{2}\theta^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial g}{\partial x^\nu} - \frac{i}{2}\bar{\theta}^{\mu\nu}\frac{\partial f}{\partial p^\mu}\frac{\partial g}{\partial p^\nu}\right)(x,p) + O\left(\theta^2,\bar{\theta}^2\right) \quad (7)$$

Here $O\left(\theta^2,\bar{\theta}^2\right)$ stands for the second and higher order terms of θ and $\bar{\theta}$, the new canonical coordinates $\hat{x}_i(t)$ and new momentum $\hat{p}_i(t)$ in (*HP*) are determined from two corresponding operators $\hat{x}_i$ and $\hat{p}_i$ in (*SP*) from the projection relations, respectively [12-14]:

$$\hat{x}_i(t) = \exp(iH_{nc}(t-t_0)) * \hat{x}_i * \exp(-iH_{nc}(t-t_0))$$
$$\hat{p}_i(t) = \exp(iH_{nc}(t-t_0)) * \hat{p}_i * \exp(-iH_{nc}(t-t_0)) \quad (8)$$

Which are satisfying the new Heisenberg motion equations [11-12]:

$$\frac{d\hat{x}_i(t)}{dt} = i\left[H_{nc} \overset{*}{,} \hat{x}_i(t)\right] \text{ and } \frac{d\hat{p}_i(t)}{dt} = i\left[H_{nc} \overset{*}{,} \hat{p}_i(t)\right] \quad (9)$$

The formalism of star product, Bopp's shift method and the Seiberg-Witten map were played crucial roles in this new theory. The Bopp's shift method will be apply in this paper instead of solving the Schrödinger equation in (*NC-3D:RSP*) with star product, the Schrödinger equation will be treated by using directly the two new commutators, in addition to usual commutator on quantum mechanics, in the both Schrödinger and Heisenberg representations [12-14]:

$$\left[\hat{x}_i,\hat{x}_j\right] = \left[\hat{x}_i(t),\hat{x}_j(t)\right] = i\theta_{ij}$$
$$\left[\hat{p}_i,\hat{p}_j\right] = \left[\hat{p}_i(t),\hat{p}_j(t)\right] = i\bar{\theta}_{ij}. \quad (10)$$

It is important to noticing that the new operators $\hat{x}_i$ and $\hat{p}_i$ in (*NC-3D: RSP*) are depended with ordinary operator x_i and p_i from the projections relations:

$$\hat{x} = x - \frac{\theta_{12}}{2}p_y - \frac{\theta_{13}}{2}p_z, \quad \hat{y} = y - \frac{\theta_{21}}{2}p_x - \frac{\theta_{23}}{2}p_z$$
$$\hat{z} = z - \frac{\theta_{31}}{2}p_x - \frac{\theta_{32}}{2}p_y, \quad (11)$$

and

$$\hat{p}_x = p_x - \frac{\bar{\theta}_{12}}{2}y - \frac{\bar{\theta}_{13}}{2}z,$$
$$\hat{p}_y = p_y - \frac{\bar{\theta}_{21}}{2}x - \frac{\bar{\theta}_{23}}{2}z \text{ and } \hat{p}_z = p_z - \frac{\bar{\theta}_{31}}{2}x - \frac{\bar{\theta}_{32}}{2} \quad (12)$$

So the purpose of this present work is to study the (*IQYM*) potential in noncommutative three dimensional spaces and phase to generate accurate energy spectrum in this new symmetries, which plays an important role in many fields of physics such as molecular physics, solid state and chemical physics [8-9]. The rest of this paper is organized as follows: in the next section we briefly review the basic of eigenvalues and eigenfunctions for: (*IQYM*) potential in ordinary three dimensional spaces. In section 3, we give a brief review of Bopp's shift method and then, we derive the spin-orbital noncommutative Hamiltonians for (*MIQYM*) potential in (*NC-3D: RSP*) symmetries, we find the exact spectrum produced by noncommutative spin-orbital Hamiltonians $\hat{H}_{so-iqym}$ for (*MIQYM*) potential by applying ordinary standard perturbation theory and then we deduce the exact spectrum produced by noncommutative magnetic Hamiltonian $\hat{H}_{m-iqym}$ for (*MIQYM*) potential in (NC: 3D- RSP) symmetries. In section four we summarize the global spectrums for (*MIQYM*) potential. The conclusion of the present work comes at the section 5.

2. THE (IQYM) POTENTIALS IN ORDINARY THREE DIMENSIONAL SPACES

A First we give a briefly review of eigenvalues and eigenfunctions for ordinary (*IQYM*) potential $V_{iqym}(r)$, on based to the principal reference [5]:

$$V_{iqym}(r) = \frac{B-V_0}{r^2} + \frac{2V_0\delta - A}{r} + \left(C - 2V_0\delta^2\right) \quad (13)$$

Where r represent inter nuclear distance, the potential parameters (A,B,C) are constants, δ is the screening parameter and V_0 is the dissociation energy. The ordinary Schrödinger equations (SE) with above potential can be written in spherical coordinate (r,θ,ϕ) as [5]:

$$\frac{1}{2\mu}\left(-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin(\theta)}\frac{\partial}{\partial\theta}\left(\sin(\theta)\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial\phi^2}\right)\times \Psi(r,\theta,\phi) + V_{iqym}(r)\Psi(r,\theta,\phi) = E_{iqym}\Psi(r,\theta,\phi) \quad (14)$$

Where E_{iqym} represent the ordinary energy in ordinary three dimensional spaces and μ is the rest mass of the confined particle. The method of separation of variable has been applied in reference [5]:

$$\Psi(\vec{r}) = \frac{R_{nl}(r)}{r}\varphi_{ml}(\theta)\Phi_m(\phi) \quad (15)$$

The radial function $R_{nl}(r)$ and the spherical functions $\varphi_{ml}(\theta)$ and $\Phi_m(\phi)$ for ordinary (IQYM) potential are satisfying the following three differential equations, respectively [5]:

$$\frac{d^2R_{nl}(r)}{dr^2} + 2\mu\left(E_{iqym} - V_{iqym}(r) - \frac{\lambda}{2\mu r^2}\right)R_{nl}(r) = 0 \quad (16)$$

and

$$\frac{d^2\varphi_{ml}(\theta)}{d\theta^2} + \cot(\theta)\frac{d\varphi_{ml}(\theta)}{d\theta}\left(\lambda - \frac{m^2}{\sin^2(\theta)}\right)\varphi_{ml}(\theta) = 0$$
$$\frac{d^2\Phi_m(\phi)}{d\phi^2} + m^2\frac{d\Phi_m(\phi)}{d\phi} = 0 \quad (17)$$

here $\lambda = l(l+1)$, according Nikiforov-Uvarov method, the normalized energy eignenfunctions $\Psi(\vec{r})$ and corresponding eigenvalues E_{iqym} for ordinary (*IQYM*) potential [5]:

$$\Psi(\vec{r}) = N_n z^{-\frac{1}{2}+\left(-1+\sqrt{1+4\gamma}\right)/2} e^{-\sqrt{\alpha}z} L_n^{\sqrt{1+4\gamma}} \times$$
$$\times\left(-2\sqrt{\alpha}z\right)\varphi_{ml}(\theta)\Phi_m(\phi) \quad (18)$$

and

$$E_{yh} = C + 2V_0\delta^2 - \frac{\mu\left(2V_0\delta - A\right)^2/2}{\left(n + \frac{1}{2} + \sqrt{2\mu B + \left(l+1/2\right)^2}\right)^2} \quad (19)$$

where $z = r^2$, N_n is the normalization constant and the factor γ is given by [5]:

$$\gamma = 2\mu\left(B - V_0\right) + l(l+1) \quad (20)$$

3. NONCOMMUTATIVE THREE DIMENSIONAL PHASE-SPACES (NC-3D: RSP) HAMILTONIAN FOR (MIQYM) POTENTIAL

3.1 Formalism of Bopp's Shift Method

In this formulation we may now proceed to present the fundamental bases of (*MSE*) in (*NC-3D: RSP*) on based to essentially our previously works [15-17], to achieve this goal, we apply the important 4-steps on the ordinary Schrödinger equation:

1. Ordinary three dimensional Hamiltonian operators $\hat{H}_{iqym}(p_i, x_i)$ will be replace by new Hamiltonian operators $\hat{H}_{iqym}(\hat{p}_i, \hat{x}_i)$.

2. Ordinary complex wave function $\Psi(\vec{r})$ will be replacing by new two complex wave functions $\hat{\Psi}(\vec{\hat{r}})$.

3. Ordinary two energies E_{iqym} will be replace by new values $E_{nc-iqym}$.

And the forth steps correspond to replace the ordinary old product by the new generalized Moyal-Weyl product$(*)$, which allow us to constructing the (*MSE*) in both (*NC-3D: RSP*) as:

$$\hat{H}_{iqym}(\hat{p}_i, \hat{x}_i) * \hat{\Psi}(\vec{\hat{r}}) = E_{nc-iqym}\hat{\Psi}(\vec{\hat{r}}) \quad (21)$$

The Bopp's shift method allows finding the reduced above (*MSE*) without generalized Moyal-Weyl product as:

$$H_{iqym}(\hat{p}_i, \hat{x}_i)\Psi(\vec{r}) = E_{nciqym}\Psi(\vec{r}) \quad (22)$$

Where the modified Hamiltonian $H_{iqym}(\hat{p}_i, \hat{x}_i)$ for (*MIQYM*) potential defined as a function of the two operators $\hat{x}_i$ and $\hat{p}_i$ which can be expressed as a function of generalized coordinates $x_i(x, y, z)$ and generalized momentums $p_i(p_x, p_y, p_z)$ in usual quantum mechanics:

$$H_{iqym}(\hat{p}_i, \hat{x}_i) = \frac{\hat{p}_i^{\,2}}{2\mu} + V_{iqym}(\hat{r}) \quad (23)$$

here the modified potential $V_{iqym}(\hat{r})$ is obtained by replace the old position r by new operator $\hat{r}$ in the expression of the ordinary potential $V_{iqym}(r)$ to obtain the following new potential:

$$V_{iqym}(\hat{r}) = \frac{(B - V_0)}{\hat{r}^2} + \frac{(2V_0\delta - A)}{\hat{r}} + \left(C - 2V_0\delta^2\right) \quad (24)$$

On based to our references [15-17], we can write the two operators $\hat{r}^2$ and $\hat{p}^2$ in (NC-3D: RSP) as follows:

$$\hat{r}^2 = r^2 - \mathbf{L}\vec{\Theta}$$
$$\frac{\hat{p}^2}{2\mu} = \frac{p^2}{2\mu} + \frac{\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}}{2\mu} \quad (25)$$

Where the two couplings $\mathbf{L}\Theta$ and $\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}}$ are given by, respectively [31-44]:

$$\mathbf{L}\Theta \equiv L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13}$$
$$\vec{\mathbf{L}}\vec{\bar{\boldsymbol{\theta}}} \equiv L_x\bar{\theta}_{12} + L_y\bar{\theta}_{23} + L_z\bar{\theta}_{13} \quad (26)$$

With $\Theta_{ij} \equiv \theta_{ij}/2$, after straightforward calculations one can obtains the different terms for (*MIQYM*) potential in (*NC-3D: RSP*) as follows:

$$\frac{B - V_0}{\hat{r}^2} = \frac{B - V_0}{r^2} + \frac{(B - V_0)}{r^4}\vec{\mathbf{L}}\vec{\Theta}$$
$$\frac{2V_0\delta - A}{\hat{r}} = \frac{2V_0\delta - A}{r} + \frac{(2V_0\delta - A)}{2r^3}\vec{\mathbf{L}}\vec{\Theta}. \quad (27)$$

Which allow us to writing the (*MIQHM*) global potential $V_{iqym}(\hat{r})$ in (*NC-3D: RSP*) as follows:

$$V_{iqym}(\hat{r}) = \frac{B-V_0}{r^2} + \frac{2V_0\delta - A}{r} + \left(C - 2V_0\delta^2\right) + +V_{pert-iqym}\left(r,\Theta,\bar{\theta}\right) \tag{28}$$

Where the o additive operator $V_{pert-iqym}\left(r,\Theta,\bar{\theta}\right)$ is given by:

$$V_{pert-yh}\left(r,\Theta,\bar{\theta}\right) = \left[\frac{B-V_0}{r^4} + \frac{2V_0\delta - A}{2r^3}\right]\vec{\mathbf{L}}\vec{\Theta} + \frac{\vec{\mathbf{L}}\bar{\bar{\boldsymbol{\theta}}}}{2\mu} \tag{29}$$

It is obvious that the above operator is proportional with two infinitesimals parameters Θ and $\bar{\theta}$, which allows us to considering as a perturbative terms.

3.1 The Spin-orbital Noncommutative Hamiltonian for (*MIQYM*) Potential in (*NC- 3D: RSP*)

In order to discover the new contribution of the perturbative terms $V_{pert-iqym}\left(r,\Theta,\bar{\theta}\right)$ for (*MIQYM*) potential, we turn to the case of spin ½ particles described by the (*MSE*), we make the two simultaneously transformations:

$$\vec{\mathbf{L}}\vec{\Theta} \to 2\Theta\vec{S}\vec{L} \quad \text{and} \quad \vec{\mathbf{L}}\bar{\bar{\boldsymbol{\theta}}} \to 2\bar{\theta}\vec{S}\vec{L} \tag{30}$$

Then the above two perturbed operators $V_{pert-iqym}\left(r,\Theta,\bar{\theta}\right)$ becomes as:

$$V_{pert-iqym}\left(r,\Theta,\bar{\theta}\right) = = 2\left[\frac{(b-V_0)}{r^4} + \frac{(2V_0 - a - b\delta)}{2r^3} + \frac{1}{2\mu}\right]\vec{\mathbf{L}}\vec{\mathbf{S}} \tag{31}$$

Here $\vec{S}$ denote to the spin of a fermionic particle (like electron). It is possible to replace the spin-orbital interaction $\vec{\mathbf{L}}\vec{S}$ by $G^2 = \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$ to obtain directly the corresponding eigenvalues, and then new physical form of the eq. (26) can be expressed as:

$$V_{pert-yh}\left(r,\Theta,\bar{\theta}\right) = = \left[\frac{B-V_0}{r^4} + \frac{2V_0\delta - A}{2r^3} + \frac{1}{2\mu}\right]\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right) \tag{32}$$

It is well known that the, the 4-operators ($\vec{J}^2$, $\vec{L}^2$, $\vec{S}^2$ and J_z) formed a complete basis on ordinary quantum mechanics, then the operator $\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$ will be gives 2-eigenvalues $k_{\pm} \equiv \frac{1}{2}\left\{\left(l \pm \frac{1}{2}\right)(l + \frac{1}{2} + 1) + l(l+1) - \frac{3}{4}\right\} = \frac{l}{2}$, and $k_{-} \equiv \frac{1}{2}\left\{\left(l - \frac{1}{2}\right)(l - \frac{1}{2} + 1) + l(l+1) - \frac{3}{4}\right\} = -\frac{l+1}{2}$ corresponding $j = l + \frac{1}{2}$ and $j = l - \frac{1}{2}$ respectively and then, we can form a diagonal matrixes $\hat{H}_{so-iqym}$ of order (3×3), with non null elements $\left(\hat{H}_{so-iqym}\right)_{11}$, $\left(\hat{H}_{so-iqym}\right)_{22}$ and $\left(\hat{H}_{so-iqym}\right)_{33}$ for (*MIQYM*) potential in (*NC-3D: RSP*) symmetries:

$$\left(\hat{H}_{so-iqym}\right)_{11} = k_{+}\left\{\Theta\left(\frac{B-V_0}{r^4} + \frac{2V_0\delta - A}{2r^3}\right) + \frac{\bar{\theta}}{2\mu}\right\} \text{if } j = l + \frac{1}{2} \Rightarrow \text{spin up}$$
$$\left(\hat{H}_{so-iqym}\right)_{22} = k_{-}\left\{\Theta\left(\frac{B-V_0}{r^4} + \frac{2V_0\delta - A}{2r^3}\right) + \frac{\bar{\theta}}{2\mu}\right\} \text{ if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \tag{33}$$
$$\left(\hat{H}_{so-iqym}\right)_{33} = 0$$

After profound calculation, one can show that, the radial function $R_{nl}(r)$ for (*MIQYM*) potential and two spherical functions $\varphi_{ml}(\theta)$ and $\Phi_m(\phi)$ are satisfying the following differential equations, in new structure (*NC-3D: RSP*):

$$\frac{d^2R_{nl}(r)}{dr^2} + +2\mu\begin{pmatrix} E - \frac{B-V_0}{r^2} - \frac{2V_0\delta - A}{r} - \left(C - 2V_0\delta^2\right) - \frac{\lambda}{2\mu r^2} \\ -2\left[\Theta\frac{B-V_0}{r^4} + \Theta\frac{2V_0\delta - A}{2r^3} + \frac{\bar{\theta}}{2\mu}\right]\vec{\mathbf{L}}\vec{\mathbf{S}} \end{pmatrix} R_{nl}(r) \tag{34}$$

and

$$\frac{d^2\varphi_{ml}(\theta)}{d\theta^2} + \cot(\theta)\frac{d\varphi_{ml}(\theta)}{d\theta}\left(\lambda - \frac{m^2}{\sin^2(\theta)}\right)\varphi_{ml}(\theta) = 0$$
$$\frac{d^2\Phi_m(\phi)}{d\phi^2} + m^2\frac{d\Phi_m(\phi)}{d\phi} = 0 \tag{35}$$

3.2 The Exact Spectrum Produced by Noncommutative Spin-orbital Hamiltonian $\hat{H}_{so-iqym}$ for (*MIQYM*) Potential by Using the Standard Perturbation Method in (*NC- 3D: RSP*)

The aim of this subsection is to obtain the modifications to the energy levels for n^{th} excited states $E_{\text{u-}iqym}$ and $E_{\text{d-}iqym}$ corresponding a fermionic particle with two polarizations spin up and spin down, respectively, at first order of two infinitesimal parameters Θ and $\bar{\theta}$. In order to achieve this goal, we apply the standard perturbation theory using eq. (18) for (MIQYM) potential:

$$E_{\text{u-}iqym} = \frac{1}{2}|N_n|^2 k_{+}\int_0^{+\infty} z^{-1/2+\left(-1+\sqrt{1+4\gamma}\right)} e^{-2\sqrt{\alpha}z} \times \times\left[L_n^{\sqrt{1+4\gamma}}\left(-2\sqrt{\alpha}z\right)\right]^2\left(\left\{\Theta\left(\frac{B-V_0}{z^2} + \frac{2V_0\delta - A}{2z^{3/2}}\right) + \frac{\bar{\theta}}{2\mu}\right\}\right)dz \tag{36}$$

$$E_{\text{d-}iqym} = \frac{1}{2}|N_n|^2 k_- \int_0^{+\infty} z^{-1/2+\left(-1+\sqrt{1+4\gamma}\right)} e^{-2\sqrt{\alpha}z} \times \\ \times\left[L_n^{\sqrt{1+4\gamma}}\left(-2\sqrt{\alpha}z\right)\right]^2\left(\left\{\Theta\left(\frac{B-V_0}{z^2}+\frac{2V_0\delta-A}{2z^{3/2}}\right)+\frac{\bar{\theta}}{2\mu}\right\}\right)dz \tag{37}$$

It is possible to write both $E_{\text{u-}iqym}\left(n,l,s,A,B,V_0,\delta\right)$ and $E_{\text{d-}iqym}\left(n,l,s,A,B,V_0,\delta\right)$ as functions of three terms T_{iqym}^1, T_{iqym}^2 and $\overline{T}_{iqym}$ as follow:

$$E_{\text{u-}iqym}\left(n,l,s,A,B,V_0,\delta\right) = \\ = \frac{1}{2}|N_n|^2 k_+ \left\{\Theta\left(T_{iqym}^1 + T_{iqym}^2\right) + \frac{\bar{\theta}}{2\mu}\overline{T}_{iqym}\right\} \tag{38}$$

and

$$E_{\text{d-}iqym}\left(n,l,s,A,B,V_0,\delta\right) = \\ = \frac{1}{2}|N_n|^2 k_- \left\{\Theta\left(T_{iqym}^1 + T_{iqym}^2\right) + \frac{\bar{\theta}}{2\mu}\overline{T}_{iqym}\right\} \tag{39}$$

The explicit mathematical forms of three terms T_{iqym}^1, T_{iqym}^2 and $\overline{T}_{iqym}$ are given by:

$$T_{iqym}^1 = \left(B-V_0\right)\int_0^{+\infty} z^{\left[-\frac{3}{2}+\left(-1+\sqrt{1+4\gamma}\right)\right]-1} e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma}}\left(-2\sqrt{\alpha}z\right)\right]^2 dz, \\ T_{iqym}^2 = \frac{\left(2V_0\delta-A\right)}{2}\int_0^{+\infty} z^{\left[-1+\left(-1+\sqrt{1+4\gamma}\right)\right]-1} e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma}}\left(-2\sqrt{\alpha}z\right)\right]^2 dz, \\ \overline{T}_{iqym} = \int_0^{+\infty} z^{\left[-\frac{1}{2}+\sqrt{1+4\gamma}\right]-1} e^{-2\sqrt{\alpha}z}\left[L_n^{\sqrt{1+4\gamma}}\left(-2\sqrt{\alpha}z\right)\right]^2 dz \tag{40}$$

To obtain the modifications to the energy levels for n^{th} excited states we apply the following special integration [26]:

$$\int_0^{+\infty} t^{\alpha-1}\exp\left(-\delta t\right)L_m^{\lambda}\left(\delta t\right)L_n^{\beta}\left(\delta t\right)dt = \\ = \frac{\delta^{-\alpha}\Gamma\left(n-\alpha+\beta+1\right)\Gamma\left(m+\lambda+1\right)}{m!n!\Gamma\left(1-\alpha+\beta\right)\Gamma\left(1+\lambda\right)}\,{}_3F_2 \times \\ \times\left(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1\right) \tag{41}$$

where ${}_3F_2\left(-m,\alpha,\alpha-\beta;-n+\alpha,\lambda+1;1\right)$ denote to the hypergeometric function, obtained from ${}_pF_q\left(\alpha_1,\ldots,\alpha_p,\beta_1,\ldots,\beta_q,z\right)$ for $p=3$ and $q=2$. After straightforward calculations, we can obtain the explicitly results:

$$T_{iqym}^1\left(n,A,B,V_0,\delta\right) = \\ = \left(B-V_0\right)\frac{\left(2\sqrt{\alpha}\right)^{\left(-5/2+\sqrt{1+4\gamma}\right)}\Gamma\left(n+7/2\right)\Gamma\left(n+\sqrt{1+4\gamma}+1\right)}{\left(n!\right)^2\Gamma\left(1+5/2\right)\Gamma\left(1+\sqrt{1+4\gamma}\right)} \\ {}_3F_2\left(-n,-5/2+\sqrt{1+4\gamma},-5/2;-n+-5/2+\sqrt{1+4\gamma},\sqrt{1+4\gamma}+1;1\right) \tag{42}$$

and

$$T_{iqym}^2\left(n,A,B,V_0,\delta\right) = \\ = \frac{\left(2V_0\delta-A\right)}{2}\frac{\left(2\sqrt{\alpha}\right)^{-\sqrt{1+4\gamma}}\Gamma\left(n+1\right)\Gamma\left(n+\sqrt{1+4\gamma}+1\right)}{\left(n!\right)^2\Gamma\left(1\right)\Gamma\left(1+\sqrt{1+4\gamma}\right)} \\ {}_3F_2\left(-n,\sqrt{1+4\gamma},0;\sqrt{1+4\gamma}-n,\sqrt{1+4\gamma}+1;1\right) \tag{43}$$

and

$$\overline{T}_{iqym} = \\ = \frac{\left(2\sqrt{\alpha}\right)^{\frac{1}{2}-\sqrt{1+4\gamma}}\Gamma\left(n+2\sqrt{1+4\gamma}+1\right)\Gamma\left(n+\sqrt{1+4\gamma}+1/2\right)}{\left(n!\right)^2\Gamma\left(1/2\right)\Gamma\left(1/2\sqrt{1+4\gamma}\right)} \\ {}_3F_2\left(-n,\sqrt{1+4\gamma}-\frac{1}{2},-\frac{1}{2};-n-\frac{1}{2}+\sqrt{1+4\gamma},\sqrt{1+4\gamma}+1;1\right) \tag{44}$$

Inserting the above obtained expressions (42), (43) and (44) into equations (38) and (39), gives the following results for exact modifications of $E_{\text{u-}iqym}$ and $E_{\text{d-}iqym}$ produced by new spin-orbital effect $V_{pert-yh}\left(r,\Theta,\bar{\theta}\right)$ for (*MIQYM*) potential [15-18]:

$$E_{\text{u-}iqym}\left(n,l,s,A,B,V_0,\delta\right) = \\ = \frac{1}{2}|N_n|^2 k_+ \left\{\Theta T_{iqym}\left(n,A,B,V_0,\delta\right) + \frac{\bar{\theta}}{2\mu}\overline{T}_{iqym}\right\} \tag{45}$$

$$E_{\text{d-}iqym}\left(n,l,s,A,B,V_0,\delta\right) = \\ = \frac{1}{2}|N_n|^2 k_- \left\{\Theta T_{iqym}\left(n,A,B,V_0,\delta\right) + \frac{\bar{\theta}}{2\mu}\overline{T}_{iqym}\right\} \tag{46}$$

Where the new factor $T_{iqym}\left(n,A,B,V_0,\delta\right)$ is given by:

$$T_{iqym}\left(n,A,B,V_0,\delta\right) = \\ = T_{iqym}^1\left(n,A,B,V_0,\delta\right) + T_{iqym}^2\left(n,A,B,V_0,\delta\right). \tag{47}$$

3.3 The Exact Spectrum Produced by Noncommutative Magnetic Hamiltonian $\hat{H}_{m-iqym}$ for (*MIQYM*) Potential in (*NC-3D: RSP*) Symmetries

Having found out how to calculate the corrections of energies for the automatically produced spin-orbital, $V_{pert-iqym}\left(r,\Theta,\bar{\theta}\right)$ we can discover a second symmetry produced by the effect and influence of the noncommutativity of space-phase by modified Zeeman Effect for (*MIQYM*) potential, to found this physical symmetry we apply the same strategy in our previously works as follows:

$$\Theta \to \chi B \text{ and } \bar{\theta} \to \bar{\sigma} B \tag{48}$$

The two parameters χ and $\bar{\sigma}$ are just only infinitesimal real proportional's constants and B is a uniform external magnetic field, we orient it to $\left(Oz\right)$ axis and then we can make the following two translations for (*MIQYM*) potential:

$$\left\{\Theta\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}\overline{B}\overline{L}\rightarrow$$
$$\rightarrow B\left(\chi\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)L_z \quad (49)$$

Which allow us to introduce the two modified new magnetic Hamiltonians $\hat{H}_{m-iqym}$ in (NC-3D: RSP) for (MIQYM) potential, as:

$$\hat{H}_{m-iqym}=\left(\chi\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)\left(\overline{B}\overline{J}+\hat{H}_z\right) \quad (50)$$

Where $\hat{H}_z\equiv-\overline{S}\overline{B}$ denote to the ordinary operator of Hamiltonian for Zeeman Effect in ordinary quantum mechanics. To obtain the exact noncommutative magnetic modification of energy $E_{\text{mag-}iqym}$ for modified (IQYM) potential, it is sufficient to replace the 3-parameters: k_+, Θ and $\bar{\theta}$ in the eq.(45) by the following new parameters: m, χ and $\bar{\sigma}$, respectively:

$$E_{\text{mag-}iqym}\left(n,m,A,B,V_0,\delta\right)=\frac{1}{2}\left|N_n\right|^2B\times$$
$$\times\left\{\chi T_{iqym}\left(n,A,B,V_0,\delta\right)+\frac{\bar{\sigma}}{2\mu}\overline{T}_{iqym}\left(n,A,B,V_0,\delta\right)\right\}m \quad (51)$$

Where m denote to the eigenvalues of the operator L_z which can be taking the values $-l$, $-l+1,\ldots,0,\ldots,l$.

4. RESULTS

Let us now resume the global exact spectrum of n^{th} excited states: $E_{\text{ncu-}iqym}\left(n,m,l,s,A,B,V_0,\delta\right)$, $E_{\text{ncd-}iqym}\left(n,m,l,s,A,B,V_0,\delta\right)$ and $E_{\text{com-}iqym}\left(n,A,B,V_0,\delta\right)$ for (*MIQYM*) potential in (*NC-3D: RSP*) which produced by the diagonal elements $\left(\hat{H}_{nc-iqym}\right)_{11}$, $\left(\hat{H}_{nc-iqym}\right)_{22}$ and $\left(\hat{H}_{nc-iqym}\right)_{33}$ of non-commutative Hamiltonians operator $\hat{H}_{nc-iqym}$. The original eigenvalue E_{iqym} in ordinary three dimensional spaces for (*IQYM*) potential and the obtained results (45), (46), (51) allow us to getting the following global results:

$$E_{\text{ncu-}iqym}\left(n,m,l,s,A,B,V_0,\delta\right)=$$
$$C+2V_0\delta^2-\frac{\mu\left(2V_0\delta-A\right)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu B+\left(l+1/2\right)^2}\right)^2}$$
$$+\frac{\left|N_n\right|^2}{2}k_+\left\{\Theta T_{iqym}+\frac{\bar{\theta}}{2\mu}\overline{T}_{iqym}\right\}+$$
$$+\frac{1}{2}\left|N_n\right|^2B\left\{\chi T_{iqym}+\frac{\bar{\sigma}}{2\mu}\overline{T}_{iqym}\right\}m \quad (52)$$

$$E_{\text{ncd-}iqym}\left(n,m,l,s,A,B,V_0,\delta\right)=$$
$$=C+2V_0\delta^2-\frac{\mu\left(2V_0\delta-A\right)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu B+\left(l+1/2\right)^2}\right)^2}$$
$$+\frac{\left|N_n\right|^2}{2}k_-\left\{\Theta T_{iqym}+\frac{\bar{\theta}}{2\mu}\overline{T}_{iqym}\right\}+$$
$$+\frac{1}{2}\left|N_n\right|^2B\left\{\chi T_{iqym}+\frac{\bar{\sigma}}{2\mu}\overline{T}_{iqym}\right\}m \quad (53)$$

$$E_{\text{com-}iqym\text{h}}\left(n,A,B,V_0,\delta\right)=$$
$$=C+2V_0\delta^2-\frac{\mu\left(2V_0\delta-A\right)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu B+\left(l+1/2\right)^2}\right)^2} \quad (54)$$

The explicit diagonal elements $\left(\hat{H}_{nc-iqym}\right)_{11}$, $\left(\hat{H}_{nc-iqym}\right)_{22}$ and $\left(\hat{H}_{nc-iqym}\right)_{33}$ of operator $\hat{H}_{nc-iqym}$ for (*MIQYM*) potential in (*NC-3D: RSP*) can deduced as follows:

$$\left(\hat{H}_{nc-iqym}\right)_{11}=-\frac{\Delta}{2\mu}+\frac{B-V_0}{r^2}+\frac{2V_0\delta-A}{r}+$$
$$\left(C-2V_0\delta^2\right)+k_+\left\{\Theta\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}+$$
$$+\left(\chi\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)BL_z \quad (55)$$
$$\text{if } j=l+\tfrac{1}{2}\ \Rightarrow \text{spin -up}$$

$$\left(\hat{H}_{nc-iqym}\right)_{22}=-\frac{\Delta}{2\mu}+\frac{B-V_0}{r^2}+\frac{2V_0\delta-A}{r}+$$
$$+\left(C-2V_0\delta^2\right)+k_-\left\{\Theta\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right\}+$$
$$+\left(\Theta\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\theta}}{2\mu}\right)BL_z \quad (56)$$
$$\text{if } j=l-\tfrac{1}{2}\ \Rightarrow \text{spin -down}$$

$$\left(\hat{H}_{nc-iqym}\right)_{33}=-\frac{\Delta}{2\mu}+\frac{B-V_0}{r^2}+\frac{2V_0\delta-A}{r}+\left(C-2V_0\delta^2\right) \quad (57)$$

It is well known that the atomic quantum number m can be takes $(2l+1)$ values and we have also two possible values for eigenvalues $j=l\pm\frac{1}{2}$, thus every state in usually three dimensional space for (*MIQYM*) potential will be replace, in (*NC-3D: RSP*) by $2(2l+1)$ sub-states and then the degenerated state can be take $2\sum_{i=0}^{n-1}(2l+1)\equiv 2n^2$ values. It is important to notice that our recent study can be extended to apply to molecular with spin $s\neq\frac{1}{2}$, we replace the factors $k_\pm\equiv\frac{1}{2}\left\{\left(l\pm\frac{1}{2}\right)(l+\frac{1}{2}\pm1)+l(l+1)-\frac{3}{4}\right\}$ by new factor $k(j,l,s)$:

$$k(j,l,s) \equiv \tfrac{1}{2}\{j(j+1)+l(l+1)-s(s+1)\} \qquad (58)$$

With $|l-s| \le j \le |l+s|$, which allow us to obtaining the modifications to the energy levels $E_{\text{nc-}iqym}(n,m,j,l,s,A,B,V_0,\delta)$ for (*MIQYM*) potential:

$$\begin{aligned} &E_{\text{nc-}iqym}(n,m,j,l,s,A,B,V_0,\delta)= \\ &= C+2V_0\delta^2-\frac{\mu(2V_0\delta-A)^2/2}{\left(n+\frac{1}{2}+\sqrt{2\mu B+(l+1/2)^2}\right)^2}+ \\ &+\frac{|N_n|^2}{2}k(j,l,s)\left\{\Theta T_{\text{nc-}iqym}+\frac{\bar{\theta}}{2\mu}\bar{T}_{\text{nc-}iqym}\right\}+ \\ &+\frac{1}{2}|N_n|^2 B\left\{\chi T_{\text{nc-}iqym}+\frac{\bar{\sigma}}{2\mu}\bar{T}_{\text{nc-}iqym}\right\}m \end{aligned} \qquad (59)$$

And the corresponding noncommutative two Hamiltonian operators $\hat{H}_{nc-iqym}$ can be fixed by the following results:

$$\begin{aligned} &\hat{H}_{nc-iqym}= \\ &=-\frac{1}{2\mu}\left(-\tfrac{1}{r^2}\tfrac{\partial}{\partial r}\left(r^2\tfrac{\partial}{\partial r}\right)+\tfrac{1}{r^2\sin(\theta)}\tfrac{\partial}{\partial\theta}\left(\sin(\theta)\tfrac{\partial}{\partial\theta}\right)+\tfrac{1}{r^2\sin^2(\theta)}\tfrac{\partial}{\partial\phi^2}\right)+\frac{B-V_0}{r^2} \\ &+\frac{2V_0\delta-A}{r}+\left(C-2V_0\delta^2\right)+\left\{\Theta\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\theta}}{\mu}\right\}\vec{\mathbf{L}}\vec{\mathbf{S}} \\ &+\left(\chi\left(\frac{B-V_0}{r^4}+\frac{2V_0\delta-A}{2r^3}\right)+\frac{\bar{\sigma}}{2\mu}\right)BL_z \end{aligned} \qquad (60)$$

It is important to noticing that these results are excellent agreement with our reference [12]. Furthermore, the appearance of the polarization states of a fermionic particle for (*MIQYM*) potential indicates the validity of obtained results at very high energy where the two relativistic equations Klein-Gordon and Dirac will be applied, which allowing to apply these results of various Nano-particles at Nanoscales. Finally, if we make the two simultaneously limits $(\theta,\bar{\theta})\to(0,0)$ we obtain all results of ordinary quantum mechanics.

5. CONCLUSION

To find the analytic solutions for the modified inversely quadratic Yukawa potential plus modified Mie-type potential, the general method Bopp's shift has been used and we are investigated the spectrum perturbatively around the solution of the standard inversely quadratic Yukawa potential plus Mie-type potential in the case of (*NC-3D: RSP*) symmetries, we showed the obtained degenerated spectrum depended by new discrete atomic quantum numbers (m, $j=l\pm\frac{1}{2}$ and $s_z=\pm\frac{1}{2}$) and the validity of obtained corrections can be prolonged to Nano-particles at Nano and Plank's scales. In addition, we recover the ordinary commutative spectrums when, we make the two simultaneously limits: $(\theta,\bar{\theta})\to(0,0)$ for (*MIQYM*) potential. Finally, because of the accuracy and simplicity of the elegant method presented in this study (the general method Bopp's shift method and standard perturbation theory), we recommend its application in finding bound state solutions of some other model of central potentials in different fields.

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