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New Relativistic Atomic Mass Spectra of Quark (*u*, *d* and *s*) for Extended Modified Cornell Potential in Nano and Plank's Scales

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A novel study for the exact solvability of relativistic quantum spectrum systems for extended Cornell potential is discussed used both Boopp's shift method and standard perturbation theory in noncommutativity three dimensional real space (NC-3DS), furthermore the exact corrections for the spectrum of studied potential was depended on infinitesimal parameter θ and a new discreet quantum numbers and we have also found the corresponding noncommutative Hamiltonian.

Keywords: The Cornell potential, Quark (*u*, *d* and *s*), Noncommutative space, Boopp's shift method.

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1. INTRODUCTION

During the last few years, a new scientific revolution appears a result of the productions of three fundamentals equations: Schrödinger, Klein-Gordon and Dirac, it's well known, that the fundamental dynamical systems based essentially on the provisos' equations [1-28]. Many physical effort has been produced to extended the previously study to the noncommutative spaces and phases to obtains a profound and a new applications [29-46]. The notions of noncommutativity of space and phase based essentially on Seiberg-Witten map and Boopp's shift method and the star product, defined on the first order of infinitesimal antisymmetric parameter $\theta^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{ijk}\theta_k$ as [29-46]:

$$f(x)*g(x)=f(x)g(x)-\frac{i}{2}\theta^{\mu\nu}\partial_\mu^x f(x)\partial_\nu^x g(x) \quad (1)$$

Which allow us to obtaining the new non nulls commutator:

$$\left[\hat{x}_\mu,\hat{x}_\nu\right]_* = i\theta_{\mu\nu} \quad (2)$$

The Boopp's shift method will be apply in this work instead of solving the (NC-3DS) spaces with star product, the Dirac equation and reduced Schrödinger equation will be treated by using directly the commutator, in addition to usual commutator on quantum mechanics [29-46]:

$$\left[\hat{x}_i,\hat{x}_j\right] = i\theta_{ij} \quad (3)$$

The main goal of this work is to extend our study in reference [32] for the potential $V(r)=ar-\frac{b}{r}+cr^2$ into relativistic noncommutative three dimensional space on based to the two principal references [46,47] to discover the new spectrum and a possibility of obtain new applications to the modified extended Cornell potential $V_{is}(\hat{r})=ar-\frac{b}{r}+cr^2+V_{pert}(r,\Theta)$ in different fields.

The rest of present search is organized as follows: In next section, we briefly review the Dirac equation with extended Cornell potential in three dimensional spaces. In section 3, we review and applying Boopp's shift method to derive: the deformed potential and corresponding relativistic noncommutative spin-orbital Hamiltonian. In section 4, we apply perturbation theory to find the spectrum for ground stat and first excited states corresponding spin-orbital operator. In next section we deduce the spectrum produced automatically by the external magnetic field. In section 6, we resume the global spectrum for modified extended Cornell potential and we conclude the corresponding global noncommutative Hamiltonian in (NC-3DS) in first order of infinitesimal parameter' Θ. Finally, the important found results and the conclusions are discussed in the last section.

2. REVIEW THE DIRAC EQUATION FOR EXTENDED CORENELL POTENTIAL IN THREE DIMENSIONAL SPAPACES

In this section, we shall review the eigenvalues and eignenfunctions for spherically symmetric for the potential $V(r)$ [47]:

$$V(r)=ar-\frac{b}{r}+cr^2 \quad (4)$$

Where a, b and c are arbitrary positive constants. The Dirac equation in the presence of a above Cornell potential [47]:

$$\left(\alpha \mathrm{P}+\beta m_\alpha+\frac{1}{2}(1+\beta)V(r)\right)\Psi(r,\theta,\phi)=E\Psi(r,\theta,\phi) \quad (5)$$

Here m_α denote to the quark $(u,d$ and $s)$ mass and (α,β) are the usual Dirac matrices, the spinor $\Psi(r,\theta,\phi)=\frac{1}{r}\begin{pmatrix}u(r)\Omega_k^m(\theta,\phi)\\ iv(r)\Omega_k^m(\theta,\phi)\end{pmatrix}$ and the radial functions $(u(r),\ v(r))$ are obtained by solving [47]:

$$\begin{aligned}\frac{du(r)}{dr} &= -\frac{k}{r}u(r)+\left[E-m_\alpha\right]v(r)=0\\ \frac{dv(r)}{dr} &= -\frac{k}{r}u(r)+\left[E-m_\alpha-V(r)\right]u(r)\end{aligned} \quad (6)$$

Where k is related to the total angular momentum quantum number j [47]:

$$k=\begin{cases}-(l+1) & \text{if } j=l+\frac{1}{2}\\ l & \text{if } j=l-\frac{1}{2}\end{cases} \quad (7)$$

The two parts of eq. (6) permuted to obtain the reduced Schrödinger equation [47]:

$$\frac{d^2u(r)}{dr^2}+\left[2\varepsilon_0\left(\varepsilon_1-V(r)\right)-\frac{k(k+1)}{r^2}\right]u(r)=0 \quad (8)$$

Where $\varepsilon_0=\frac{1}{2}(E+m_\alpha)$ and $\varepsilon_1=\frac{1}{2}(E-m_\alpha)$. On based to two references [47, 48], the radial wave function $u(r)$ can be expressed in the following:

$$u_{nk}(r)=C_{nk}r^{-\frac{B}{\sqrt{2A}}-1}e^{\sqrt{2A}r}\left(-r^2\frac{d}{dr}\right)^n\left(r^{-2n+\frac{2B}{\sqrt{2A}}}e^{-2\sqrt{2A}r}\right) \quad (9)$$

Where C_{nk} is a normalization constant and

$$A=-\varepsilon_0\left(\varepsilon_1-\frac{3a}{\delta}-\frac{6c}{\delta^2}\right),\ B=\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right) \text{ and } \delta=\frac{1}{r_0} \quad (10)$$

Here r_0 is a characteristic radius of the meson. The technique of Nikiforov-Uvarov allows us to obtain the energy [46, 47]:

$$\varepsilon_1=\frac{3a}{\delta}+\frac{6c}{\delta^2}-\frac{2\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right)^2}{\left[(2n+1)\pm\sqrt{1+\frac{8\varepsilon_0 a}{\delta^3}+4k(k+1)}+\frac{24\mu c}{\delta^4}\right]^2} \quad (11)$$

For the ground state and the first existed states, the corresponding radial function $u_{0k}(r)$ and $u_{1k}(r)$, respectively, are given by:

$$\begin{aligned}u_{0k}(r)&=C_{0k}r^{\frac{B}{\sqrt{2A}}-1}e^{-\sqrt{2A}r}\\ u_{1k}(r)&=C_{1k}\left(-\left(-2+\frac{2B}{\sqrt{2A}}\right)r^{\frac{B}{\sqrt{2A}}-2}+2\sqrt{2A}r^{\frac{B}{\sqrt{2A}}}\right)e^{-\sqrt{2A}r}\end{aligned} \quad (12)$$

Then, the complete normalized wave functions and corresponding energies for the ground state and the first existed states, respectively:

$$\Psi^{(0)}(\vec{r})=C_{0k}r^{\frac{B}{\sqrt{2A}}-2}e^{-\sqrt{2A}r}\Omega_k^m(\theta,\phi)$$

and $$\varepsilon_{10}=\frac{3a}{\delta}+\frac{6c}{\delta^2}-\frac{2\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right)^2}{\left[1\pm\sqrt{1+\frac{8\varepsilon_0 a}{\delta^3}+4k(k+1)}+\frac{24\mu c}{\delta^4}\right]^2} \quad (13.a)$$

$$\Psi^{(1)}(\vec{r})=C_{1k}\left(-\eta r^{\frac{B}{\sqrt{2A}}-3}+\sigma r^{\frac{B}{\sqrt{2A}}-1}\right)e^{-\sqrt{2A}r}\Omega_k^m(\theta,\phi) \text{ and}$$

$$\varepsilon_{11}=\frac{3a}{\delta}+\frac{6c}{\delta^2}-\frac{2\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right)^2}{\left[3\pm\sqrt{1+\frac{8\varepsilon_0 a}{\delta^3}+4k(k+1)}+\frac{24\mu c}{\delta^4}\right]^2} \quad (13.b)$$

Where $\eta=\left(-2+\frac{2B}{\sqrt{2A}}\right)$ and $\sigma=2\sqrt{2A}$.

3. NON-COMMUTATIVE RELATIVISTIC HAMILTONIAN FOR EXTENDED MODIFIED CORNELL POTENTIAL

$$V(\hat{r})=ar-\frac{b}{r}+cr^2+V_{pert}(r,\Theta):$$

3.1 Formalism of Boopp's Shift

Know, we shall review some fundamental principles of the quantum non-commutative Dirac equation which resumed in the following steps for modified potential $V(\hat{r})$ [29-46]:

$$\begin{aligned}&\text{Ordinary Hamiltonian:}\hat{H}(p_i,x_i)\rightarrow\\ &\text{NC-Hamiltonian:}\hat{H}(\hat{p}_i,\hat{x}_i)\\ &\text{Ordinary complex wave function:}\Psi(\vec{r})\rightarrow\\ &\text{NC-complex-spinor:}\hat{\Psi}(\vec{\hat{r}})\\ &\text{Ordinary energy:E}\rightarrow NC-Energy:E_{nc}\\ &\text{Ordinary product}\rightarrow\text{New star producr:}*\end{aligned} \quad (14)$$

Which allow us to writing the noncommutative Dirac equation as follows:

$$\hat{H}(\hat{p}_i,\hat{x}_i)*\hat{\Psi}(\vec{\hat{r}})=E_{nc}\hat{\Psi}(\vec{\hat{r}}) \quad (15)$$

The Boopp's shift method permutes to reduce the above equation to simplest the form:

$$H_{nc-3is}(\hat{p}_i,\hat{x}_i)\psi(\vec{r})=E_{nc-is}\psi(\vec{r}) \quad (16)$$

The new modified Hamiltonian $H_{nc}(\hat{p}_i,\hat{x}_i)$ defined as a function of $\hat{x}_i$ and $\hat{p}_i$:

$$H_{nc-3is}(\hat{p}_i,\hat{x}_i)=\alpha\mathrm{P}+\beta m_\alpha+\frac{1}{2}(1+\beta)V(\hat{r}) \quad (17)$$

And the modified extended Cornell potential $V(\hat{r})$:

$$V_{is}(\hat{r}) = a\hat{r} - \frac{b}{\hat{r}} + c\hat{r}^2 \quad (18)$$

The operator $\hat{x}_i$ in (NC-3DS) space is given by [29-42]:

$$\hat{x}_i = x_i - \frac{\theta_{ij}}{2} p_j \quad (19)$$

On based to our references [31-42], we can write the operator $\hat{r}^2$ in noncommutative three dimensional spaces as follows:

$$\hat{r}^2 = r^2 - \mathbf{L}\vec{\Theta} \quad (20)$$

Where $\mathbf{L}\Theta$ denotes to $\left(L_x\Theta_{12} + L_y\Theta_{23} + L_z\Theta_{13}\right)$, with $\Theta \equiv \frac{\theta}{2}$. After straightforward calculations one can obtains the different terms in (NC-3DS) spaces as follows:

$$\begin{aligned} a\hat{r}^2 &= ar^2 - a\vec{\mathbf{L}}\vec{\Theta} \\ b\hat{r} &= br - \frac{b}{2r}\vec{\mathbf{L}}\vec{\Theta} \\ \frac{c}{\hat{r}} &= \frac{c}{r} + \frac{c}{2r^3}\vec{\mathbf{L}}\vec{\Theta} \end{aligned} \quad (21)$$

Which allow us to writing the studied potential $V(\hat{r})$ in (NC-3DS) spaces as follows:

$$V_m(\hat{r}) = ar^2 + br - \frac{c}{r} + V_{pert}(r,\Theta) \quad (22)$$

It's clearly that, the first 3-terms in eq. (22) represent the ordinary extended Cornell potential while the rest term is produced by the deformation of space. The global perturbative potential operators $V_{pert}(r,\Theta)$ for studied potential in both (NC-3DS) spaces will be written as:

$$V_{pert}(r,\Theta) = \left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)\vec{\mathbf{L}}\vec{\Theta} \quad (23)$$

4. THE RELATIVISTIC SPIN-ORBITAL NONCOMMUTATIVE HAMILTONIAN FOR EXTENDED CORNELL POTENTIAL

We replace $\vec{\mathbf{L}}\vec{\Theta}$ by $2\Theta\vec{S}\vec{L}$ to obtain the new forms of $H_{\text{pert}}(r,\Theta)$ for extended Cornell potential $V(\hat{r})$:

$$H_{\text{pert}}(r,\Theta,\bar{\theta}) = 2\Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)\vec{\mathbf{L}}\vec{S} \quad (24)$$

Here $\vec{S} = \vec{\frac{1}{2}}$ denote the spin of a fermionic particle; it's possible also to rewriting eq. (24) to the equivalent physical form:

$$H_{\text{pert}}(r,\Theta,\bar{\theta}) = \Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right) \quad (25)$$

It is well known, ($\vec{J}^2$, $\vec{L}^2$, $\vec{S}^2$ and s_z) formed complete basis on quantum mechanics, then the operator $\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$ will be gives 2-eigenvalues $k_\pm \equiv \frac{1}{2}\left\{\left(l \pm \frac{1}{2}\right)(l + \frac{1}{2} \pm 1) + l(l+1) - \frac{3}{4}\right\}$, corresponding $j = l \pm \frac{1}{2}$ respectively [31-42]. Then, one can form a diagonal matrix $H_{so}(r,\vec{p},\Theta)$ of order (3×3), with non-null elements $(H_{so})_{11}$, $(H_{so})_{22}$ and $(H_{so})_{33}$ for in both (NC-3DS):

$$\begin{aligned} (H_{so})_{11} &= k_+\Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right) \text{if } j = l + \tfrac{1}{2}: \text{ spin up} \\ (H_{so})_{22} &= k_-\Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right) \text{if } j = l - \tfrac{1}{2}: \text{spin down} \\ (H_{so})_{33} &= 0 \end{aligned} \quad (26)$$

After profound straightforward calculation, one can show that, the radial functions $u(r)$ and $v(r)$ are satisfying the following differential equation, in (NC-3DS) for modified extended Cornell potential $V(\hat{r})$:

$$\begin{aligned} \frac{du(r)}{dr} &= -\frac{k}{r}u(r) + \left[E_{nc} - m_\alpha\right]v(r) = 0 \\ \frac{dv(r)}{dr} &= -\frac{k}{r}u(r) + \left[E_{nc} - m_\alpha - V(r) - V_{pert}(r,\Theta)\right]u(r) \end{aligned} \quad (27)$$

The two parts of eq. (25) permuted to obtain the reduced Schrödinger equation:

$$\frac{d^2u(r)}{dr^2} + \left[2\varepsilon_0\left(\varepsilon_1 - V(r) - V_{pert}(r,\Theta)\right) - \frac{k(k+1)}{r^2}\right]u(r) = 0 \quad (28)$$

4.1 The Exact Spin-orbital Spectra for Fundamental States Produced by Modified Extended Cornell Potential in (NC:3DS):

The modifications to the energy levels for fundamental states E_{u0} and E_{d0} for spin up and spin down, respectively, at first order of the parameter θ obtained by applying the standard perturbation theory:

$$E_{\text{u0}} = \left|C_{0k}\right|^2 k_+\Theta \int_0^{+\infty} r^{\frac{2B}{\sqrt{2A}}-2} e^{-2\sqrt{2A}r}\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)dr \quad (29)$$

$$E_{\text{d0}} = \left|C_{0k}\right|^2 k_-\Theta \int_0^{+\infty} r^{\frac{2B}{\sqrt{2A}}-2} e^{-2\sqrt{2A}r}\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)dr \quad (30)$$

A direct simplification gives:

$$E_{\text{u0}} = \left|C_{0k}\right|^2 k_+\Theta\sum_{i=1}^{3} T_i \quad (31.a)$$

$$E_{\text{d0}} = \left|C_{0k}\right|^2 k_-\Theta\sum_{i=1}^{3} T_i \quad (33)$$

Where $\Gamma\left(\frac{m+1}{n}, \beta x^n\right)$ is incomplete Gamma function, it's evidently, that the first term, the second term and the third term in the eq. (32) are corresponding the values

$$\left(m=\frac{2B}{\sqrt{2A}}-5,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-3,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right)$$

and $\left(m=\frac{2B}{\sqrt{2A}}-2,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right)$, respectively, then, by applying the above special integral, we obtain:

$$T_1=\frac{c}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-4,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-4}}$$
$$T_2=-\frac{b}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-2,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-2}} \qquad (34)$$
$$T_3=-a\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-1,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-1}}$$

Thus, the modifications to the energy levels for fundamental states E_{u} and E_{d} for spin up and spin down, respectively, at first order of the parameter θ :

$$E_{\mathrm{u0}}=\left|C_{0k}\right|^2 k_+\Theta L_o \qquad (35.a)$$

$$E_{\mathrm{d0}}=\left|C_{0k}\right|^2 k_-\Theta L_o \qquad (35.b)$$

Where the factor L_o is determined by:

$$L_o=\frac{c}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-4,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-4}}-$$
$$-\frac{b}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-2,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-2}}-a\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-1,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-1}} \qquad (36)$$

The noncommutative energy E_{NC0} of the fundamental state is the sum of obtained above correction and the usual energy (13.a) in commutative 3D space:

$$E_{NC0}=m_\alpha+\frac{3a}{\delta}+\frac{6c}{\delta^2}-\frac{2\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right)^2}{\left[1\pm\sqrt{1+\frac{8\varepsilon_0 a}{\delta^3}+4k(k+1)+\frac{24\mu c}{\delta^4}}\right]^2} \qquad (37)$$
$$+\begin{cases}\left|C_{0k}\right|^2 k_+\Theta L_o \text{ if } j=l+\frac{1}{2} & \Rightarrow \text{spin up}\\ \left|C_{0k}\right|^2 k_-\Theta L_o \text{ if } j=l-\frac{1}{2} & \Rightarrow \text{spin down}\end{cases}$$

4.2 The Exact Spin-orbital Spectra for First Excited States Produced by Modified Extended Cornell Potential in (NC:3D- RS)

Now, we turn to the modifications to the energy levels for first excited states E_{u1} and E_{d1} corresponding spin up and spin down, respectively, at first order of two parameter θ, which obtained by applying the standard perturbation theory:

$$E_{\mathrm{u1}}=\left|C_{1k}\right|^2 k_+\Theta\int_0^{+\infty}\left(-\eta^2 r^{\frac{2B}{\sqrt{2A}}-6}+\sigma^2 r^{\frac{2B}{\sqrt{2A}}-2}-2\eta\sigma r^{\frac{2B}{\sqrt{2A}}-4}\right)\times$$
$$\times e^{-2\sqrt{2A}r}\left(\frac{c}{2r^3}-\frac{b}{2r}-a\right)dr \qquad (38.a)$$

$$E_{\mathrm{d1}}=\left|C_{0k}\right|^2 k_-\Theta\int_0^{+\infty}\left(-\eta^2 r^{\frac{2B}{\sqrt{2A}}-6}+\sigma^2 r^{\frac{2B}{\sqrt{2A}}-2}-2\eta\sigma r^{\frac{2B}{\sqrt{2A}}-4}\right)\times$$
$$\times e^{-2\sqrt{2A}r}\left(\frac{c}{2r^3}-\frac{b}{2r}-a\right)dr \qquad (38.b)$$

A direct simplification gives:

$$E_{\mathrm{u1}}=\left|C_{1k}\right|^2 k_+\Theta\sum_{i=1}^{9}T_i \qquad (39.a)$$

$$E_{\mathrm{d1}}=\left|C_{1k}\right|^2 k_-\Theta\sum_{i=1}^{9}T_i \qquad (39.b)$$

Where, the three terms $T_i\left(i=\overline{1,9}\right)$ are given by:

$$T_1=-\frac{c}{2}\eta^2\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-9}e^{-2\sqrt{2A}r}dr$$
$$T_2=-\frac{b}{2}\eta^2\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-7}e^{-2\sqrt{2A}r}dr \qquad (40.a)$$
$$T_3=a\eta^2\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-6}e^{-2\sqrt{2A}r}dr$$

$$T_4=\frac{c}{2}\sigma^2\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-5}e^{-2\sqrt{2A}r}dr$$
$$T_5=\frac{b}{2}\sigma^2\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-3}e^{-2\sqrt{2A}r}dr \qquad (40.b)$$
$$T_6=-a\sigma^2\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-2}e^{-2\sqrt{2A}r}dr$$

$$T_7=-c\eta\sigma\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-7}e^{-2\sqrt{2A}r}dr$$
$$T_8=-b\eta\sigma\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-5}e^{-2\sqrt{2A}r}dr \qquad (40.c)$$
$$T_9=2\eta a\sigma\int_0^{+\infty}r^{\frac{2B}{\sqrt{2A}}-4}e^{-2\sqrt{2A}r}dr$$

It's evidently, that the three parameters of every term, in the Eqs. (40.a, b, c) are corresponding the values:

$$\left(m=\frac{2B}{\sqrt{2A}}-9,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-7,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-6,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-5,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-3,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-2,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-7,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

$$\left(m=\frac{2B}{\sqrt{2A}}-5,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right)\quad \text{and}$$

$$\left(m=\frac{2B}{\sqrt{2A}}-4,\quad n=1\quad \text{and}\quad \beta=2\sqrt{2A}\right),$$

respectively, then, by applying the previously special integral, we obtain:

$$T_1=-\frac{c\eta^2}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-8,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-8}},$$

$$T_2=-\frac{b\eta^2}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-6,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-6}}\quad . \tag{41.a}$$

$$\text{and}\quad T_3=a\eta^2\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-5,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-5}}$$

$$T_4=\frac{c\sigma^2}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-4,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-4}},$$

$$T_5=\frac{b\sigma^2}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-2,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-2}} \tag{41.b}$$

$$\text{and}\quad T_6=-a\sigma^2\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-1,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-1}}$$

$$T_7=-c\eta\sigma\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-6,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-6}},$$

$$T_8=-b\eta\sigma\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-4,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-4}} \tag{41.c}$$

$$\text{and}\quad T_9=2\eta a\sigma\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-3,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-3}}$$

Thus, the modifications to the energy levels for fundamental states E_{u1} and E_{d1} for spin up and spin down, respectively, at first order of the parameter θ :

$$E_{u1}=\left|C_{0k}\right|^2 k_+\Theta L_1 \tag{42.a}$$

$$E_{d1}=\left|C_{0k}\right|^2 k_-\Theta L_1 \tag{42.b}$$

Where the factor L_1 is given by:

$$\begin{aligned}L_1=&\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-13,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-13}}+\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-11,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-11}}\\&+\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-5,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-5}}+\frac{c\sigma^2}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-4,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-4}}\\&+\frac{b\sigma^2}{2}\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-2,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-2}}-a\sigma^2\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-1,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-1}}\\&-c\eta\sigma\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-6,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-6}}-b\eta\sigma\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-4,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-4}}\\&+2\eta a\sigma\frac{\Gamma\left(\frac{2B}{\sqrt{2A}}-3,2\sqrt{2A}r\right)}{\left(2\sqrt{2A}\right)^{\frac{2B}{\sqrt{2A}}-3}}\end{aligned} \tag{43}$$

Thus, the noncommutative energy E_{NC1} of the first excited states is the sum of obtained above correction and the usual energy (13.b) in commutative 3D space:

$$E_{NC1}=m_\alpha+\frac{3a}{\delta}+\frac{6c}{\delta^2}-\frac{2\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right)^2}{\left[3\pm\sqrt{1+\frac{8\varepsilon_0 a}{\delta^3}+4k(k+1)+\frac{24\mu c}{\delta^4}}\right]^2} \tag{44}$$

$$+\begin{cases}\left|C_{1k}\right|^2 k_+\Theta L_1 \text{ if } j=l+\frac{1}{2} & \Rightarrow \text{spin up}\\ \left|C_{1k}\right|^2 k_-\Theta L_1 \text{ if } j=l-\frac{1}{2} & \Rightarrow \text{spin down}\end{cases}$$

5. THE EXACT RELATIVISTIC MAGNETIC SPECTRUM MODIFICATIOBS FOR EXTENDED MODIFIED CORNELL POTENTIAL $V(\hat{r})$

IN (NC:3DS)

On another hand, it's possible to found another automatically symmetry for modified potential $V(\hat{r})$ related to the influence of an external uniform magnetic field, it's deduced by the following replacements:

$$\Theta \to \chi B \tag{45}$$

Here χ is infinitesimal real proportional' constants and we choose the magnetic field $\vec{B} = B\vec{k}$, and then we can make the following translation:

$$\Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)\vec{B}\vec{L} \to \chi\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)BL_z \tag{46}$$

Which allow us to introduce the new relativistic modified magnetic Hamiltonian H_{rm} in noncommutative three dimensional spaces as:

$$H_{rm} = \chi\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)\left(\vec{B}\vec{J} - \vec{S}\vec{B}\right) \tag{47}$$

Here $\left(-\vec{S}\vec{B}\right)$ denote to the ordinary Hamiltonian of Zeeman Effect. To obtain the exact relativistic noncommutative magnetic modifications of energy ($E_{\text{mag-0}}$, $E_{\text{mag-1}}$) for modified extended Cornell potential $V(\hat{r})$, we replace: k_+ and Θ in the Eqs.(35.a) and (42.a) by the following parameters: m and χ, respectively:

$$E_{\text{mag-0}} = |C_{0k}|^2 \Theta B m L_0 \tag{48.a}$$

$$E_{\text{mag-1}} = |C_{1k}|^2 \Theta B m L_1 \tag{48.b}$$

Where $E_{\text{mag-0is}}$ and $E_{\text{mag-1is}}$ are the exact magnetic modifications of spectrum corresponding the fundamental states and first excited states, respectively and $-l \le m \le +l$.

6. THE EXACT RELATIVISTIC EXTENDED MODIFIED ENERGY OF THE LOWEST EXCITATIONS SPECTRUM FOR EXTENDED MODIFIED CORNELL POTENTIAL $V(\hat{r})$ IN (NC: 3DS)

The total modified energies E_{NC0} ($E_{\text{nc u0}} - E_{\text{nc d0}}$) and E_{NC1} ($E_{\text{nc u1}} - E_{\text{nc d1}}$) of a fermionic particle with spin up and spin down corresponding fundamental states and first excited states, respectively, for modified potential $V(\hat{r})$ in (NC: 3DS) are determined on based to the Eqs. (13.a), (13.b), (35.a), (35.b), (42.a), (42.b), (48.a) and (48.b):

$$E_{NC0} = m_\alpha + \frac{3a}{\delta} + \frac{6c}{\delta^2} - \frac{2\varepsilon_0\left(\frac{3a}{\delta^2} + b + \frac{8c}{\delta^3}\right)^2}{\left[1 \pm \sqrt{1 + \frac{8\varepsilon_0 a}{\delta^3} + 4k(k+1)} + \frac{24\mu c}{\delta^4}\right]^2} \tag{49.a}$$

$$+\begin{cases} |C_{0k}|^2 k_+\Theta L_o + |C_{0k}|^2 \Theta B m L_0 & \text{if } j = l + \frac{1}{2} \Rightarrow \text{spin up} \\ |C_{0k}|^2 k_-\Theta L_o + |C_{0k}|^2 \Theta B m L_0 & \text{if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \end{cases}$$

And

$$E_{NC1} = m_\alpha + \frac{3a}{\delta} + \frac{6c}{\delta^2} - \frac{2\varepsilon_0\left(\frac{3a}{\delta^2} + b + \frac{8c}{\delta^3}\right)^2}{\left[3 \pm \sqrt{1 + \frac{8\varepsilon_0 a}{\delta^3} + 4k(k+1)} + \frac{24\mu c}{\delta^4}\right]^2} \tag{49.b}$$

$$+\begin{cases} |C_{1k}|^2 k_+\Theta L_1 + |C_{1k}|^2 \Theta B m L_1 & \text{if } j = l + \frac{1}{2} \Rightarrow \text{spin up} \\ |C_{1k}|^2 k_-\Theta L_1 + |C_{1k}|^2 \Theta B m L_1 & \text{if } j = l - \frac{1}{2} \Rightarrow \text{spin down} \end{cases}$$

The quantum number m can be takes $(2l+1)$ values and we have also two values for $j = l \pm \frac{1}{2}$, thus every state in usually three dimensional space of modified extended Cornell potential $V(\hat{r})$ will be in (NC: 3DS): $4(2l+1)$ sub-states. It's clearly, that the obtained eigenvalues of energies are real's and then the noncommutative diagonal Hamiltonian H_{nc} is Hermitian. Regarding the previous obtained results, we can deduce the global diagonal noncommutative Hamiltonian matrix H_{nc} of order (3×3), with elements $(H_{nc})_{11}$, $(H_{nc})_{22}$ and $(H_{com})_{33}$ in (NC-3DS):

$$(H_{nc})_{11} = \alpha \mathrm{P} + \beta m_\alpha + \frac{1}{2}(1+\beta)V(r) + k_+\Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right) + \chi\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)BL_z \quad \text{if } j = l + \tfrac{1}{2} \Rightarrow \text{spin up} \tag{50.a}$$

$$\left(H_{nc-3is}\right)_{22} = \alpha \mathrm{P} + \beta m_\alpha + \frac{1}{2}(1+\beta)V(r) + k_-\Theta\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right) + +\chi\left(\frac{c}{2r^3} - \frac{b}{2r} - a\right)BL_z \quad \text{if } j = l - \tfrac{1}{2} \Rightarrow \text{spin -down} \tag{50.b}$$

And

$$\left(H_{com-3is}\right)_{33} = \alpha \mathrm{P} + \beta m_\alpha + \frac{1}{2}(1+\beta)V(r) \to \text{Non} - \text{polarised} - \text{electron} \tag{50.c}$$

Which allow us to deduce the spectra $\bar{E}_{NC0}$ ($\bar{E}_{\text{nc u0}} - \bar{E}_{\text{nc d0}}$) and $\bar{E}_{NC1}$ ($\bar{E}_{\text{nc u1}} - \bar{E}_{\text{nc d1}}$) of anti quarks $\left(\bar{u}, \bar{d}\ \text{and}\ \bar{s}\right)$.

Know for spin $s \ne 1/2$, we have $|l-s| \le j \le |l+s|$, then the values of $k(j,l,s)$ will be taking new form $\frac{1}{2}\{j(j+1) + l(l+1) - s(s+1)\}$, which allow us obtaining the two extremely values of spectrum E_{NC0} and E_{NC1} for $j = |l-s|$ and $j = l+s$, respectively, as follows:

$$E_{NC0} = m_\alpha + \frac{3a}{\delta} + \frac{6c}{\delta^2} - \frac{2\varepsilon_0\left(\frac{3a}{\delta^2} + b + \frac{8c}{\delta^3}\right)^2}{\left[1 \pm \sqrt{1 + \frac{8\varepsilon_0 a}{\delta^3} + 4k(k+1)} + \frac{24\mu c}{\delta^4}\right]^2} \tag{51.a}$$

$$+\begin{cases} |C_{0k}|^2 k(l+s,l,s)\Theta L_o + |C_{0k}|^2 \Theta B m L_0 & \text{if } j = l + s \\ |C_{0k}|^2 k(l-s,l,s)\Theta L_o + |C_{0k}|^2 \Theta B m L_0 & \text{if } j = |l - s| \end{cases}$$

And

$$E_{NC1} = m_\alpha + \frac{3a}{\delta} + \frac{6c}{\delta^2} - \frac{2\varepsilon_0\left(\frac{3a}{\delta^2}+b+\frac{8c}{\delta^3}\right)^2}{\left[3\pm\sqrt{1+\frac{8\varepsilon_0 a}{\delta^3}+4k(k+1)}+\frac{24\mu c}{\delta^4}\right]^2} \quad (51.b)$$

$$+\begin{cases}|C_{1k}|^2 k(l+s,l,s)\Theta L_1 + |C_{1k}|^2\Theta BmL_1 & \text{if } j=l+s\\ |C_{1k}|^2 k(l-s,l,s)\Theta L_1 + |C_{1k}|^2\Theta BmL_1 & \text{if } j=|l-s|\end{cases}$$

Thus, every old state, in ordinary three dimensional spaces, will be replaced by $2j(j+1)(2l+1)$ sub-states.

Finally, we can write the new noncommutative Hamiltonian operator H_{nc} for extended modified Cornell potential in (NC-3DS):

$$H_{nc} = \alpha P + \beta m_\alpha + \frac{1}{2}(1+\beta)V(r) + +k(j,l,s)\Theta\left(\frac{c}{2r^3}-\frac{b}{2r}-a\right)\vec{L}\vec{S} + \chi\left(\frac{c}{2r^3}-\frac{b}{2r}-a\right)BL_z \quad (52)$$

7. CONCLUSIONS

In this recent work, we have obtained exact degenerated bound state solution in noncommutative three dimensional spaces by applying perturbation theory and Boopp's Shift method instead to solving Dirac equation directly with star product for extended modified Cornell potential $V(\hat{r}) = ar - \frac{b}{r} + cr^2 + V_{pert}(r,\Theta)$. We showed the obtained degenerated spectrum for the extended modified Cornell potential can described the family quarks and anti-quarks $(u,d$ and $s)$ and $(\bar{u},\bar{d}$ and $\bar{s})$ respectively.

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